Operator's Economy of Device-to-Device Offloading in Underlaying Cellular Networks

Bodong Shang, *Student Member, IEEE*, Liqiang Zhao, *Member, IEEE*, and Kwang-Cheng Chen, *Fellow, IEEE*

***Abstract*— Offloading onto device-to-device (D2D) communications is considered a promising technology to alleviate traffic load of cellular networks. However, the success of D2D communications relies on user's willingness to share contents and operator's incentives to tolerate interference in networks, which has been overlooked in most literature. We propose an incentive framework of D2D offloading, where the operator encourages some users acting as D2D transmitters to broadcast their popular contents to nearby region to improve operators, overall economic efficiency. A two-stage Stackelberg game is employed to analytically model such interaction: The operator (leader) determines the incentive price to maximize its interests and D2D transmitters (followers) choose appropriate traffic volume to be offloaded. The transmit powers at the base station and D2D transmitters are simultaneously characterized by considering the intra-tier and inter-tier interferences in complex cellular networks. Simulations confirm that the cellular operator can fully utilize the spectrum and significantly increase its profit by incorporating this D2D offloading incentive mechanism.**

***Index Terms*— D2D communications, offloading, network economy, Stackelberg game, stochastic geometry.**

I. INTRODUCTION

RECENTLY, with the ever-growing traffic in cellular networks, device-to-device (D2D) communication has attracted great attention to cover poorly received signal strength and to aimed at providing high-speed data rate by facilitating the effective physical proximity of communicating devices [1]. The traffic offloading of D2D communication not only improves the network spectral and energy efficiency, but also enhances the end-users throughput and system capacity [2]. Since some users carry popular contents that might be desirable for nearby mobile users the operator can formulate a commendable pricing scheme to stimulate the users possessing popular contents to broadcast their information within a certain local area, in order to further increase operator's profit, while still guaranteeing users quality of service (QoS).

The hybrid access policy within macrocell and femtocell BSs is analyzed by a proposed rate-based pricing framework in [3]. Pricing strategies for operators in cognitive femtocell network with static and dynamic pricing models are discussed in [4]. However, operator's incentives at the system level like profits have not been well taken into consideration for the economic aspects of D2D communications. Furthermore, pricing incentives and thus transmit power at both BS and D2D transmitters to control interference shall be interactive at the system level, which still remains unknown. Therefore, to maximize the incentives of cellular operator with appropriate transmission power control to satisfy users' desired data rates becomes our unique view on D2D communications.

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B. Shang and L. Zhao are with the State Key Laboratory of ISN, Xidian University, Xi'an 710071, China (e-mail: bdshang@stu.xidian.edu.cn; lqzhao@mail.xidian.edu.cn).

K.-C. Chen is with the Department of Electrical Engineering, University of South Florida, Tampa, FL 33647, USA (e-mail: kwangcheng@usf.edu).

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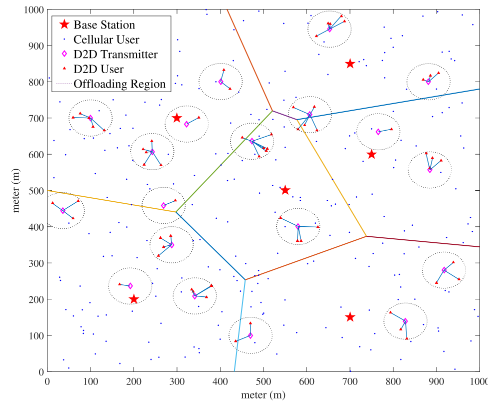

Fig. 1. Cellular downlinks with D2D transmitters which can broadcast information for users in the circular region of radius R_D.

II. SYSTEM MODEL

We consider cellular downlinks underlaid with D2D communications (Fig. 1). BSs are modeled as a homogeneous Poisson Point Process (PPP) on the entire plane $\mathbb{R}^2$ with density λ_B, and that can be denoted as the set of $\Psi_B = \{b_j, j = 0, 1, 2, \ldots\}$. Each BS has the maximum transmit power represented as P_M. The users are classified into cellular users (CUEs, i.e., attached to BSs or D2D transmitters) and donor users (DUEs, i.e., D2D transmitters), which are spatially scattered in $\mathbb{R}^2$ according to independent homogeneous PPPs Ψ_u and Ψ_D of various density λ_u and λ_D, respectively. At each DUE, the device has its maximum allowable transmit power indicated by P_D.

Towards cellular user's association, each cellular user connects to the closest BS $(b_j \in \Psi_B)$ if it cannot be offloaded onto D2D links, and the cell area can be defined as the set of $V_j = \{x \in \mathbb{R}^2 \,|\, \|x - b_j\| \leq \|x - b_k\|, b_k \in \Psi_B \backslash b_j\}$, where $\|a - b\|$ represents the distance between a and b. In D2D communications, the i^{th} offloaded user $u_{i,j}^d$ connects to the j^{th} DUE u_j^{DT}, when their distance is within the communication range of radius R_D and the user's required contents are the same with DUE's broadcast information. In addition, the offloading region of u_j^{DT} is $\Omega_j^{DT} = \left\{x \in \mathbb{R}^2 \,\middle|\, \left\|x - u_j^{DT}\right\| \leq R_D, u_j^{DT} \in \Psi_D\right\}$, and the probability, which DUE's broadcasting information likes with the required contents of an arbitrary cellular user, can be denoted by $\mathbb{P}_{con}$. If a user moves outside the designated offloading area, it will connect to traditional cellular link, and such vertical handover process is transparently performed for users.

A. Cellular Downlink

Cellular users in cell V_j can be denoted as a set of $\Psi_{u,j}^c$, where $\left|\Psi_{u,j}^c\right| = N_j^c$ is the number of cellular users. The bandwidth for downlink in each cell is B_C MHz, and user $u_{i,j}^c$ (i^{th} cellular user in j^{th} cell) obtains $B_{i,j}^C = \mu_{i,j}^c B_C / N_j^c$ MHz, where $\mu_{i,j}^c$ denotes the allocation factor for user $u_{i,j}^c$. Suppose that the bandwidth B_C is fully used, and thus we have $\sum_{i=1}^{N_j^c} \mu_{i,j}^c = N_j^c$. Within a cell, the transmission links are orthogonal. We assume each BS can be capable of performing adaptive power control according to zero-delay *channel state information* (CSI). According to Shannon theorem, the BS transmit power $P_{i,j}^B$ for $u_{i,j}^c$ on its sub-band is allocated to ensure the required data rate $R_{i,j}^c$ as follows:

$$R_{i,j}^c = B_{i,j}^C \log_2\left(1 + \frac{P_{i,j}^B g_{i,j}^c \left\|u_{i,j}^c - b_j\right\|^{-\alpha}}{I_{c,i,j}^C + I_{c,i,j}^D + \sigma^2}\right), \tag{1}$$

where $g_{i,j}^c$ represents the fast fading coefficient, α expresses the path-loss exponent, $I_{c,i,j}^C$ indicates the interference from cellular tier at $u_{i,j}^c$, $I_{c,i,j}^D$ denotes the cross-tier interference at $u_{i,j}^c$, and σ^2 denotes additive noise. The objective function of cellular operator can be expressed as the difference between income and total cost which includes the power consumption of BSs and incentive expenditure to DUEs, as follows:

$$\mathcal{U}_O = \lambda_u \tau R_u - \lambda_B \xi_B P_B^{total} - \lambda_D \varepsilon R_u N_{user}^D, \tag{2}$$

where τ denotes the income per unit data rate, and $R_u = \left(\sum_{j=1}^K \sum_{i=1}^{N_j} R_{i,j}^c\right) / \left(\sum_{j=1}^K N_j\right)$ indicates the average required throughput of users with K cells. Although each cellular user $u_{i,j}^c$ has a specific data rate requirement denoted by $R_{i,j}^c$, for tractable analysis, we can assume that all the users $\forall u_{i,j}^c \in \Psi_u$ have an identical rate requirement of $R_{i,j}^c = R_u$. In addition, ξ_B represents a cost factor regarding to power consumption, and $P_B^{total} = P_B^{non} + P_B^{agg}$ is the total power at BS which includes the non-transmission power P_B^{non} and aggregated transmit power P_B^{agg}. The incentive price per unit data rate is denoted by ε, and $N_{user}^D = \mathbb{P}_{con} \lambda_u \pi R_D{}^2$ is the average number of offloaded users in an offloading region Ω_j^{DT}. The first term in (2) represents the operator's revenue per unitary area, the second term expresses the power cost and the last term denotes incentive cost of operator.

B. D2D Link

We suppose that the available bandwidth D2D communications B_D is uniformly divided into several sub-bands by the parameter β, and each u_j^{DT} can randomly access to one of the sub-bands which are not actually used by its closest n D2D transmitters. For an offloaded user $u_{i,j}^d$, the data rate is

$$R_{i,j}^d = \frac{B_D}{\beta} \log_2\left(1 + \frac{P_j^D h_{i,j}^d \left\|u_{i,j}^d - u_j^{DT}\right\|^{-\alpha}}{I_{d,i,j}^C + I_{d,i,j}^D + \sigma^2}\right), \tag{3}$$

where B_D denotes the total D2D bandwidth underlaid with B_C, P_j^D ($P_j^D \leqslant P_D$) represents the transmit power at u_j^{DT}, and accordingly $h_{i,j}^d$ is the fading coefficient, $I_{d,i,j}^C$ denotes the interference from cellular tier at $u_{i,j}^d$, $I_{d,i,j}^D$ indicates the co-tier interference at $u_{i,j}^d$. The data rate requirement of the offloaded user $u_{i,j}^d$ should not be less than its previous cellular data rate, otherwise, throughput degradation suggests falling back to pure cellular operation. Therefore, to facilitate analysis of the networks performance, we assume that offloaded users have an identical rate of $R_{i,j}^d = R_u$.

Correspondingly, DUE's payoff can be defined as the difference between its income and payment, i.e.,

$$\mathcal{U}_{DUE} = \varepsilon R_u N_{user}^D - \xi_D \mathbb{E}\left[P_j^D\right], \tag{4}$$

where ε, R_u and N_{user}^D are the same as defined in (2), ξ_D denotes the payment factor of the power consumption at devices, $\mathbb{E}\left[P_j^D\right]$ is the average DUE's transmit power.

III. Game-Theoretical Approach for Offloading

Since the participants (i.e., operator and DUEs) are rational and actually act in order, the leader (i.e., operator) can take into full account the response of follower (DUE) to maximize its profit by determining an appropriate incentive price, and this suggests the formation of a two-stage Stackelberg game. In stage I, the operator announces an incentive price of a single user per unit data rate. In stage II, each DUE decides how much traffic volume to offload (i.e., the offloading radius R_D). We will analyze this game by using backward induction.

A. Stage II: Followers Game - DUE's Offloading Radius

The average DUE's transmit power can be obtained by a transformation of equation (3) as

$$\mathbb{E}\left[P_j^D\right] = \frac{2^{\beta R_u / B_D} - 1}{x^{-\alpha}} \left(I_{d,i,j}^C + I_{d,i,j}^D + \sigma^2\right), \tag{5}$$

where $x = \left\|u_{i,j}^d - u_j^{DT}\right\|$ is the distance for service link, and we have utilized $\mathbb{E}\left[h_{i,j}^d\right] = 1$. The worst-case scenario will be analyzed where the interferers transmit on their maximum power, and the average aggregated interference $I_{d,i,j}^C$ can be obtained by applying Campbell's Theorem, as follows:

$$\begin{aligned} I_{d,i,j}^C &= \mathbb{E}_{\Psi_B, g}\left[\sum\nolimits_{b_j \in \Psi_B} P_{c-d} g_{i,j}^d \left\|u_{i,j}^d - b_j\right\|^{-\alpha}\right] \\ &= P_{c-d} \lambda_B \int_{x \in \mathbb{R}^2} \left\|u_{i,j}^d - x\right\|^{-\alpha} dx \\ &= \frac{2\pi \lambda_B P_{c-d}}{2 - \alpha} \left(d_{\min}^{I_{c-d}}\right)^{2-\alpha}, \end{aligned} \tag{6}$$

where $P_{c-d} = \frac{B_D P_M}{B_C \beta}$. More specifically, a BS interferes a typical D2D link with the probability of $\frac{B_D}{B_C \beta}$. Note that $d_{\min}^{I_{c-d}}$ denotes the radius of a circular region centered at the receiver where the interferers are located outside the region, and here

$d_{\min}^{I_{c-d}}$ is the distance between the D2D receiver and its nearest BS. Similarly, the average co-tier interference $I_{d,i,j}^{D}$ is

$$I_{d,i,j}^{D} = \frac{2\pi \lambda_D P_{d-d}}{\alpha - 2} \left(d_{\min}^{I_{d-d}}\right)^{2-\alpha}, \tag{7}$$

where $P_{d-d} = \frac{P_D}{\beta}$, which indicates that two D2D links interfere each other with the probability of $\frac{1}{\beta}$, and $d_{\min}^{I_{d-d}}$ is the minimum distance of interferers, i.e., the distance between the D2D user $u_{i,j}^{d}$ and its n^{th} closest D2D transmitter.

Since the offloaded users can be viewed to follow uniform distribution on Ω_j^{DT}, the PDF of the distance x between user $u_{i,j}^{d}$ and DUE u_j^{DT} is $f(x) = \frac{2x}{{R_D}^2}$. Although the cell's boundary forms a Voronoi tessellation, it can be accurately approximated by a circle area with the radius of R_B, where $R_B = \frac{1}{\sqrt{\pi \lambda_B}}$. Therefore, the PDF of the distance between user $u_{i,j}^{d}$ and its nearest BS is $f(y) = \frac{2y}{{R_B}^2}$. In addition, the PDF of $d_{\min}^{I_{d-d}}$ for a typical D2D user $u_{i,j}^{d}$ is $f_{d_{\min}^{I_{d-d}}}(z) = \frac{2(\pi\lambda_D)^n}{\Gamma(n)} z^{2n-1} e^{-\pi \lambda_D z^2}$ [5], and thus, according to (5), the average transmit power of DUE u_j^{DT} is

$$\begin{aligned}
\mathbb{E}\left[P_j^D\right] &= \int_0^{\infty}\int_0^{R_D}\int_0^{R_B} \frac{\left(2^{\beta R_u/B_D} - 1\right)2\pi}{x^{-\alpha}(\alpha-2)} \\
&\quad \cdot \left[\frac{\lambda_B B_D P_M}{B_C \beta y^{\alpha-2}} + \frac{\lambda_D P_D}{\beta} z^{2-\alpha} + \frac{\sigma^2(\alpha-2)}{2\pi}\right] \\
&\quad \cdot \frac{2y}{{R_B}^2}\frac{2x}{{R_D}^2} f_{d_{\min}^{I_{d-d}}}(z)\, dy\, dx\, dz \\
&= \frac{2\left(2^{\beta R_u/B_D} - 1\right)\sigma^2}{(\alpha+2)\, {R_D}^{-\alpha}} + \frac{\left(2^{\beta R_u/B_D} - 1\right)}{\left(\alpha^2 - 4\right){R_D}^{-\alpha}} \\
&\quad \cdot \left[\frac{8(\pi\lambda_B)^{\frac{\alpha}{2}} B_D P_M}{B_C \beta (4-\alpha)} + \frac{4 P_D \Theta(n,\alpha)}{\beta (\pi\lambda_D)^{-\frac{\alpha}{2}}}\right],
\end{aligned} \tag{8}$$

where $\Theta(n,\alpha) = \Gamma\left(n + 1 - \frac{\alpha}{2}\right)/\Gamma(n)$, and $\Gamma(x)$ is the standard gamma function.

In equation (8), we note that the DUE's transmit power increases with the offloading radius R_D. By plugging (8) into (4), we can get a closed-form DUE's payoff function. We consider the first derivative of $\mathcal{U}_{DUE}$ on the price ε as

$$\frac{\partial \mathcal{U}_{DUE}}{\partial R_D} = 2\varepsilon R_u \lambda_u \pi R_D \mathbb{P}_{con} - \xi_D \partial \mathbb{E}\left[P_j^D\right] \Big/ \partial R_D, \tag{9}$$

In order to get the optimal value of the offloading radius R_D^* that can maximize DUE's payoff, we let the first derivative equal to zero (i.e., $\frac{\partial \mathcal{U}_{DUE}}{\partial R_D} = 0$), and calculate R_D^* as:

$$R_D^* = \left[\frac{\varepsilon R_u \lambda_u \mathbb{P}_{con} \pi^{1-\frac{\alpha}{2}} \frac{(\alpha+2)}{4\xi_D \alpha \Upsilon}(4-\alpha)}{\frac{B_D P_M {\lambda_B}^{\frac{\alpha}{2}}}{B_C \beta(\alpha-2)} + \frac{P_D {\lambda_D}^{\frac{\alpha}{2}} \Theta(n,\alpha)(4-\alpha)}{2\beta(\alpha-2)} + \mathcal{T}}\right]^{\frac{1}{\alpha-2}}, \tag{10}$$

where $\Upsilon = 2^{\frac{\beta R_u}{B_D}} - 1$ and $\mathcal{T} = \frac{\sigma^2(4-\alpha)}{4\pi^{\frac{\alpha}{2}}}$. According to (10), the optimal offloading radius R_D^* increases with the incentive price ε and the density of users λ_u, while decreases with the payment factor of power consumption ξ_D and the density of DUEs λ_D. Combining (10), (8) and (4) by substituting R_D^* for R_D, we can obtain the DUE's maximum payoff as:

$$\mathcal{U}_{DUE}^* = \varepsilon R_u \mathbb{P}_{con} \lambda_u \pi \left(R_D^*\right)^2 \left(1 - \frac{2}{\alpha}\right), \tag{11}$$

where R_D^* is the same as above (10).

B. Stage I: Leader Game - Operator's Pricing

Now, we study the design of stage I, where operator chooses the optimal incentive price ε^* to maximize its profit. Note that a BS interferes user $u_{i,j}^{c}$ with a fraction of power $\frac{\mu_{i,j}^c P_M}{N_j^c}$. In addition, the user's service bandwidth $B_{i,j}^{C}$ could contain $N_{sub} = \left(\mu_{i,j}^c B_C / N_j^c\right)/(B_D/\beta)$ D2D sub-bands, and an arbitrary DUE can access to these sub-bands with the probability of $\frac{N_{sub}}{\beta}$. Therefore, similar to (5), the average BS transmit power $P_{i,j}^{B}$ conditioned on the service link distance for user $u_{i,j}^{c}$ on its sub-band $B_{i,j}^{C}$ is

$$\begin{aligned}
&\mathbb{E}\left[P_{i,j}^B \,\middle|\, l_{i,j}^c = \left\|u_{i,j}^c - b_j\right\|\right] \\
&= \frac{\left(2^{N_j^c R_u/\left(\mu_{i,j}^c B_C\right)} - 1\right)2\pi}{\left(l_{i,j}^c\right)^{-\alpha}(\alpha-2)} \\
&\quad \cdot \left[\frac{\lambda_B \mu_{i,j}^c P_M}{N_j^c \left(l_{i,j}^c\right)^{\alpha-2}} + \frac{\lambda_D N_{sub} \mathbb{E}\left[P_j^D\right]}{\beta {R_D}^{\alpha-2}} + \frac{\sigma^2(\alpha-2)}{2\pi}\right],
\end{aligned} \tag{12}$$

where $l_{i,j}^{c}$ is the service link distance between $u_{i,j}^{c}$ and its associated BS b_j. We assume that the cellular user would not be assigned to the identical radio resource of its closest D2D transmitter, and thus the nearest distance of interfering DUE should not be less than R_D. Since the transmit power $P_{i,j}^{B}$ is conditioned on $l_{i,j}^{c}$ and $\mu_{i,j}^{c}$, the aggregated transmit power $\mathbb{E}\left[P_j^B\right]$ at a BS can be expressed by the summation form, i.e., $\mathbb{E}\left[P_j^B\right] = \sum_{u_{i,j}^c \in \Psi_{u,j}} \mathbb{E}\left[P_{i,j}^B \,\middle|\, l_{i,j}^c, \mu_{i,j}^c\right]$. For tractable analysis of interference from other cells and for the sake of maintaining the maximum fairness, we invoke the classic round-robin scheduling at BSs (i.e., $\forall \mu_{i,j}^c = 1$).

Therefore, by adopting the PDF of $l_{i,j}^{c}$ (i.e., $f_{l_{i,j}^c}(l) = \frac{2l}{{R_B}^2}$), we can obtain the average transmit power at a BS for an arbitrary cellular user given that the required data rate R_u, by calculating $\int_0^{R_B} \mathbb{E}\left[P_{i,j}^B \,\middle|\, l\right] f_{l_{i,j}^c}(l) dl$ as follows:

$$\begin{aligned}
\mathbb{E}\left[P_{i,j}^B\right] &= \frac{2\sigma^2\left(2^{N_j^c R_u / B_C} - 1\right)}{(\alpha+2)(\pi\lambda_B)^{\frac{\alpha}{2}}} \\
&\quad + \frac{2^{\frac{N_j^c R_u}{B_C}} - 1}{\alpha - 2}\left[\frac{P_M}{N_j^c} + \frac{4\lambda_D B_C \mathbb{E}\left[P_j^D\right]{R_D}^{2-\alpha}}{B_D N_j^c (\alpha+2)\pi^{\frac{\alpha-2}{2}}{\lambda_B}^{\frac{\alpha}{2}}}\right].
\end{aligned} \tag{13}$$

Since the users are uniformly distributed on the network plane $\mathbb{R}^2$, the probability that a user can be offloaded onto D2D link is characterized by a ratio of D2D region to the total area (i.e., $\mathbb{P}_{OL} = \frac{\mathcal{A}_{D2D}}{\mathcal{A}_{total}}\mathbb{P}_{con} = \lambda_D \pi {R_D}^2 \mathbb{P}_{con}, (\mathbb{P}_{OL} \leqslant 1)$). Therefore, the density of cellular users can be approximately

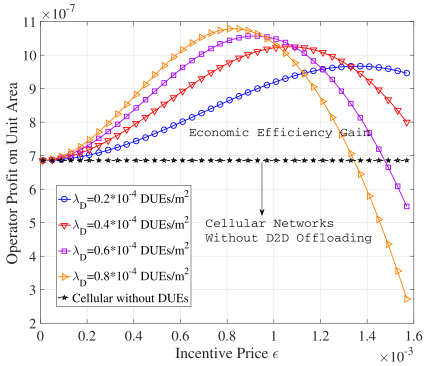

Fig. 2. Operator's profit per unit area with respect to the incentive price ε under varying densities of DUEs λ_D.

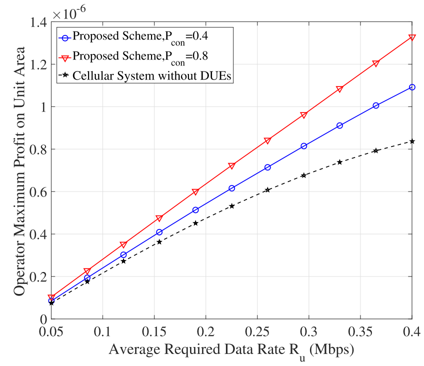

Fig. 3. Operator's profit per unit area with respect to the average required data rate of users.

obtained, i.e., $\lambda_u^c = (1 - \mathbb{P}_{OL})\lambda_u$. From the relations, the average number of cellular users per BS is given by $N_j^c = \frac{\lambda_u^c}{\lambda_B} = \frac{(1-\lambda_D \pi R_D{}^2 \mathbb{P}_{con})\lambda_u}{\lambda_B}$. Moreover, the aggregated transmission power per BS can be obtained by calculating the sum of per user's power, i.e., $P_B^{agg} = N_j^c \times \mathbb{E}\left[P_{i,j}^B\right]$.

We plug the DUE's reaction function $R_D^*(\varepsilon)$ (10) into the operator's profit function (2) as equation (14), shown below, where $\mathcal{A}_D^* = \lambda_D \pi \left(R_D^*\right)^2 \mathbb{P}_{con}$ and $\mathbb{E}\left[P_j^D\right]$ can be found in (8).

Thus, the operator's profit-maximization problem is

$$\begin{aligned} &\max_{\varepsilon \geqslant 0} \ : \mathcal{U}_O \\ &s.t. \ C_1 : P_B^{agg}\big|_{R_D = R_D^*} \leqslant P_M \\ &\quad\ \ C_2 : \mathbb{E}\left[P_D\right]\big|_{R_D = R_D^*} \leqslant P_D \\ &\quad\ \ C_3 : \lambda_D \pi \left(R_D^*\right)^2 \leqslant 1. \end{aligned} \tag{15}$$

The constraint C_1 indicates a lower bound of ε when the network parameters and user's quality of service are given. More specifically, when ε is lower than the limitation, the operator could not guarantee the user's required traffic data rate with maximum transmit power. Correspondingly, constraint C_2 gives an upper bound of ε, where each device has its corresponding maximum transmit power. Constraint C_3 satisfies the assumption of non-overlapping offloading regions. The unique existence of Nash equilibrium can be proved with some algebraic efforts. Moreover, when D2D offloading is disabled, the operator's profit can be given by

$$\begin{aligned} \mathcal{U}_O^{no-DUE} &= \lambda_u \tau R_u - \lambda_B \xi_B \left(P_B^{non} + P_j^B\right) \\ P_j^B &= \left(2^{\frac{\lambda_u R_u}{\lambda_B B_C}} - 1\right)\left[\frac{P_M}{\alpha - 2} + \frac{2\sigma^2 \lambda_u \lambda_B{}^{-\frac{\alpha+2}{2}}}{(\alpha+2)\,\pi^{\frac{\alpha}{2}}}\right], \end{aligned} \tag{16}$$

where P_j^B is the average transmit power of a BS. The optimal incentive price ε^* can be obtained by solving (15), and the corresponding profit is $\mathcal{U}_O^*$. If $\mathcal{U}_O^* > \mathcal{U}_O^{no-DUE}$, it is economically viable for operator to introduce D2D offloading.

IV. Numerical Simulations and Conclusions

In this section, we evaluate the results of D2D offloading by numerical simulations. We generally select $B_C = 20MHz$, $B_D = 10MHz$, $\alpha = 3$, $\beta = 10$, $R_u = 0.3Mbps$, $\lambda_u = 10\lambda_D = 0.4 \times 10^{-3} users/m^2$, $P_M = 40W$, $P_{om} = 10W$, $P_D = 20mW$, $\tau = 1 \times 10^{-2}$, $\xi_B = 1.5 \times 10^{-3}$, $\xi_D = 4 \times 10^{-2}$, $n = 5$, $\mathbb{P}_{con} = 0.8$, unless specified otherwise. Fig.2. shows the unit area profit of operator versus incentive price ε in different densities of DUEs. Intuitively, there exits an optimal price ε^* that can maximize operator's profit, and the maximum value increases with the density of DUEs as the amount of traffic can be offloaded from cellular networks which results in a reduction of aggregated power consumption at BSs.

In Fig. 3., we observe that an increasing average required data rate leads to the improvement of operator's profit, and significant economic efficiency gain can be obtained by the proposed incentive D2D offloading scheme.

In conclusion, we study the economics of D2D offloading in the large-scale networks. The theoretical model can assist the operator to maximize its profit by encouraging users to share contents in proximity, and D2D transmitter can improve its payoff according to the derived optimal offloading radius.

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$$\mathcal{U}_O = \frac{\lambda_u R_u}{\left(\tau - \varepsilon \mathcal{A}_D^*\right)^{-1}} - \lambda_B \xi_B \left\{ P_B^{non} + \frac{2^{\frac{\left(1-\mathcal{A}_D^*\right)\lambda_u R_u}{\lambda_B B_C}} - 1}{(\alpha - 2)\left(\pi \lambda_B\right)^{\frac{\alpha}{2}}} \cdot \left[\frac{P_M}{\left(\pi \lambda_B\right)^{-\frac{\alpha}{2}}} + \frac{\mathbb{E}\left[P_j^D\right]\left(R_D^*\right)^{2-\alpha}}{\frac{(\alpha+2)B_D}{4\pi \lambda_D B_C}} + \frac{\sigma^2\left(1 - \mathcal{A}_D^*\right)}{\frac{\alpha+2}{2(\alpha-2)}\frac{\lambda_B}{\lambda_u}} \right] \right\} \tag{14}$$