CGN 6933 Groundwater Engineering

MIDTERM EXAM | University of South Florida
Thurs., Oct. 17, 2019 | Fall 2019
100 points total | J. A. Cunningham

Instructions:

(1) You may read these instructions, but do not turn the page or begin working until instructed to do so.

(2) Work on your own paper. Write your name on each piece of paper. At the end of the time period, staple your papers together before submitting. Be sure to include every page! – if you discover tomorrow that you forgot to turn in one of the pages of your work, it will be too late.

(3) You are allowed one sheet of 8.5-by-11-inch paper with hand-written notes. You may write on both sides of that paper. However, mechanical reproductions (photocopying, laser printing, scanning, faxes, etc.) are not allowed; all notes must be hand-written. A calculator is recommended, but it may not be pre-programmed with formulae from the class.

(4) Time limit: 60 minutes. Stop working when asked. If you continue working after time has been called, you will be penalized at a rate of 2 points per minute.

(5) Show all work and state all assumptions in order to receive maximum credit for your work. I cannot award partial credit if I cannot follow what you did.

(6) Make sure your answers include units if appropriate. Watch your units!!

(7) This exam contains 6 questions. The point value of each question is indicated. The total number of points is 100.

(8) You may read all the information on this side of the page, but do not start working on the exam until instructed to do so.

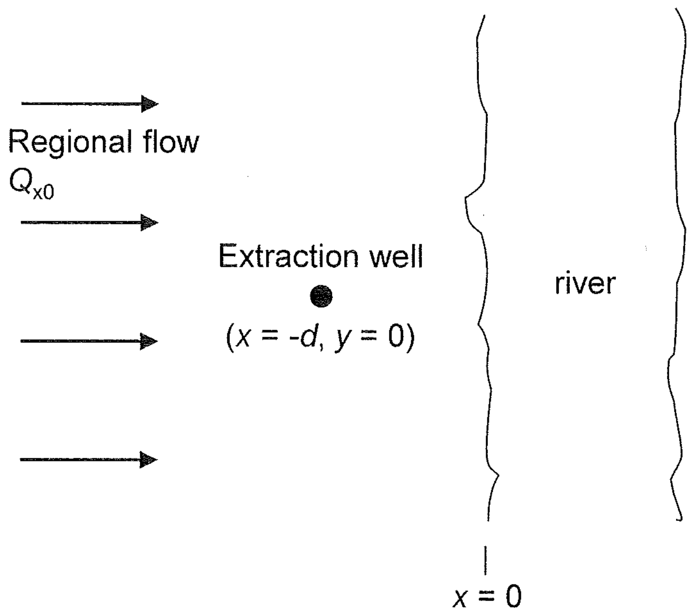
(9) Use a reasonable number of significant digits when reporting numerical answers. *You are likely to be graded down* if you report an excessive number of significant digits.

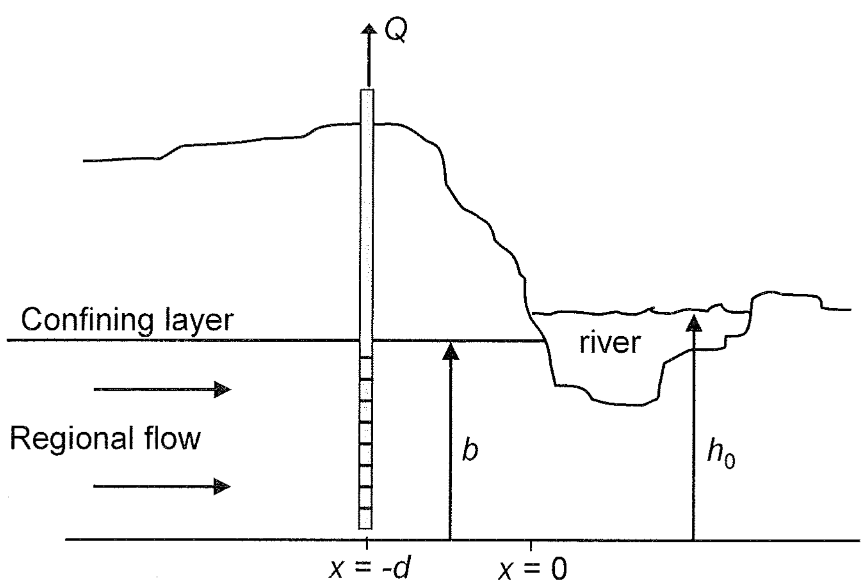
(10) Don't cheat. Cheating will result in disciplinary action consistent with USF System policies. Just as important, cheating indicates a lack of personal integrity.

(11) Hints:

- Read each question carefully and answer the question that is asked.
- Watch your units. If you take good care of your units, they will take good care of you.
- Work carefully and don't rush.
- Do not panic if you cannot finish the exam.
- If you do not know how to solve a problem, skip it and move to a problem that you can solve. That will be the best use of your time.

A river flows along the line $x = 0$. An extraction well is located at $(x = -d, y = 0)$. The radius of the well bore is r_w. The well extracts water from a confined aquifer at a volumetric flow rate Q. There is regional groundwater flow towards the river (i.e., in the $+x$-direction), and the discharge rate of the regional flow is Q_{x0}. The aquifer is underlain by a horizontal impervious base, and the thickness of the confining layer is b. The height of the river is h_0 above the base. The aquifer is homogeneous and isotropic, and the hydraulic conductivity in the aquifer is K.





(1) (10 pts) Write the appropriate expression for the discharge potential as a function of position, $\Phi(x, y)$. Hint: make sure that you satisfy the proper boundary condition along the river.

(2) (25 pts) Find the locations(s) of the stagnation point(s). Your answer should be expressed in terms of the physical parameters of the problem, which might include d, r_w, Q, Q_{x0}, h_0, b, and/or K.
Hint for problem 2: for the function

$$f(x,y) = \ln\left(\sqrt{(x-a)^2 + (y-b)^2}\ \right)$$

the partial derivatives of f are as follows.

$$\frac{\partial f}{\partial x} = \frac{x-a}{(x-a)^2 + (y-b)^2} \qquad \text{and} \qquad \frac{\partial f}{\partial y} = \frac{y-b}{(x-a)^2 + (y-b)^2}$$

(3) (5 pts) Using your answer from problem 2, find the pumping rate Q that would result in the stagnation point being located on the river, i.e., at $x = 0$. This value of Q is the maximum pumping rate allowable if you want to avoid drawing water out of the river. Your answer should be expressed in terms of the physical parameters of the problem, e.g., d, Q_{x0}, etc.

(4) (25 pts) Find the pumping rate Q that would result in a hydraulic head $h = b$ at the extraction well. This is the maximum pumping rate Q that enables the aquifer to still remain confined. If $h < b$, the aquifer is unconfined in the region of the well. Hint for problem 4: remember that the well has a finite radius r_w.

(5) (25 pts) Suppose we know $d = 200$ m, $r_w = 10$ cm $= 0.10$ m, $Q_{x0} = 2 \times 10^{-6}$ m^2/sec, $h_0 = 30$ m, and $b = 20$ m. There is a piezometer at $(x = -200, y = 300)$ m. When the pumping rate Q at the extraction well is $Q = 1.0$ gallon per minute $= 6.3 \times 10^{-5}$ m^3/sec, the hydraulic head at the piezometer is $h = 36$ m. Use this information to estimate/calculate K.

(6) (10 pts) Suppose that we want to maximize the rate of extraction from the well, but we have some constraints. We don't want to pull water out of the river, and we want to make sure that the aquifer remains confined at all locations. Use your answers from problems (3), (4), and (5) to determine the maximum allowable pumping rate Q at the well. The parameters given in problem 5 apply here.

(1) (10 pts) Write the appropriate expression for the discharge potential as a function of position, $\Phi(x, y)$.

Hint: make sure that you satisfy the proper boundary condition along the river.

$$\Phi(x,y) = \underbrace{-Q_{x0}\,x}_{\text{regional flow}} + \underbrace{\frac{Q}{2\pi}\ln\sqrt{(x+d)^2+y^2}}_{\text{extraction well at }(-d,0)} - \underbrace{\frac{Q}{2\pi}\ln\sqrt{(x-d)^2+y^2}}_{\text{image well at }(+d,0)} + C$$

... but what is the constant C?

We know that at $x=0$, $h=h_0$ $\Rightarrow$ $\Phi = Kbh_0 + C_c$

Define $\Phi_0 = Kbh_0 + C_c$ $\Rightarrow$ $\Phi = \Phi_0$ when $x=0$ (at the river)

Now plug this in to the equation above to find C

$$\Phi_0 = -Q_{x0}(0) + \cancel{\frac{Q}{2\pi}\ln\sqrt{d^2+y^2}} - \cancel{\frac{Q}{2\pi}\ln\sqrt{d^2+y^2}} + C$$

$\Rightarrow$ $C = \Phi_0$

$$\boxed{\Phi(x,y) = -Q_{x0}\,x + \frac{Q}{2\pi}\ln\sqrt{(x+d)^2+y^2} - \frac{Q}{2\pi}\ln\sqrt{(x-d)^2+y^2} + \Phi_0}$$

where $\Phi_0 = Kbh_0 + C_c$

(2) (25 pts) Find the locations(s) of the stagnation point(s). Your answer should be expressed in terms of the physical parameters of the problem, which might include d, r_w, Q, Q_{x0}, h_0, b, and/or K.

Hint for problem 2: for the function

this should say $f(x,y)$ ... minor typo

$$f(x) = \ln\left(\sqrt{(x-a)^2 + (y-b)^2}\,\right)$$

the partial derivatives of f are as follows.

$$\frac{\partial f}{\partial x} = \frac{x-a}{(x-a)^2 + (y-b)^2} \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{y-b}{(x-a)^2 + (y-b)^2}$$

$$Q_x = -\frac{\partial \Phi}{\partial x} = -\left[-Q_{x_0} + \frac{Q}{2\pi}\frac{x+d}{(x+d)^2+y^2} - \frac{Q}{2\pi}\frac{x-d}{(x-d)^2+y^2}\right] = 0$$

$$Q_y = -\frac{\partial \Phi}{\partial y} = -\left[0 + \frac{Q}{2\pi}\frac{y}{(x+d)^2+y^2} - \frac{Q}{2\pi}\frac{y}{(x-d)^2+y^2}\right] = 0$$

These come from differentiating $\Phi(x,y)$

Clearly the second of these equations is satisfied by $y=0$.

Now put $y=0$ into the first equation to find x.

$$Q_{x_0} - \frac{Q}{2\pi}\frac{1}{x+d} + \frac{Q}{2\pi}\frac{1}{x-d} = 0$$

$$Q_{x_0} = \frac{Q}{2\pi}\left[\frac{1}{x+d} - \frac{1}{x-d}\right] \Rightarrow Q_{x_0} = \frac{Q}{2\pi}\left[\frac{(x-d)-(x+d)}{(x+d)(x-d)}\right]$$

$$Q_{x_0} = \frac{Q}{2\pi}\left[\frac{-2d}{x^2-d^2}\right] \Rightarrow x^2 - d^2 = \frac{-\cancel{2}dQ}{\cancel{2}\pi Q_{x_0}}$$

$$x^2 = d^2 - \frac{Qd}{\pi Q_{x_0}} \Rightarrow x = \pm\sqrt{d^2 - \frac{Qd}{\pi Q_{x_0}}}$$

... but we only want the negative root because we are only looking in the region $x<0$ in this problem.

$$\boxed{\text{Stagnation point is at } \left(x = -\sqrt{d^2 - \frac{Qd}{\pi Q_{x_0}}}\,,\ y=0\right)}$$

(3) (5 pts) Using your answer from problem 2, find the pumping rate Q that would result in the stagnation point being located on the river, i.e., at $x = 0$. This value of Q is the maximum pumping rate allowable if you want to avoid drawing water out of the river. Your answer should be expressed in terms of the physical parameters of the problem, e.g., d, Q_{x0}, etc.

We know the stagnation point is at $x = -\sqrt{d^2 - \frac{Qd}{\pi Q_{x0}}}$ from problem 2

So set $x=0$ $\Rightarrow$ $0 = -\sqrt{d^2 - \frac{Qd}{\pi Q_{x0}}}$

$$0 = d^2 - \frac{Qd}{\pi Q_{x0}}$$

$$\frac{Qd}{\pi Q_{x0}} = d^2 \quad \Rightarrow \quad Q = \frac{\pi Q_{x0} d^2}{d}$$

$$\boxed{Q = \pi Q_{x0} d}$$

max. extraction rate to avoid drawing water from the river

(4) (25 pts) Find the pumping rate Q that would result in a hydraulic head $h = b$ at the extraction well. This is the maximum pumping rate Q that enables the aquifer to still remain confined. If $h < b$, the aquifer is unconfined in the region of the well. Hint for problem 4: remember that the well has a finite radius r_w.

From problem 1,

$$\Phi(x,y) = -Q_{x0}x + \frac{Q}{2\pi}\ln(r_1) - \frac{Q}{2\pi}\ln(r_2) + \Phi_0$$

where r_1 = distance from extraction well, r_2 = distance from image well

Re-write that in terms of head ... $\Phi = Kbh + C_c$

$$Kbh + \cancel{C_c} = -Q_{x0}x + \frac{Q}{2\pi}\ln(r_1) - \frac{Q}{2\pi}\ln(r_2) + Kbh_0 + \cancel{C_c}$$

Divide entire equation by Kb

$$h = h_0 - \frac{Q_{x0}}{Kb}x + \frac{Q}{2\pi Kb}\ln(r_1) - \frac{Q}{2\pi Kb}\ln(r_2)$$

Now we want to find Q that gives $h=b$ at the extraction well.

But "at the extraction well" means: $x = -d$, $r_1 = r_w$, $r_2 = 2d$

$$\therefore\ b = h_0 - \frac{Q_{x0}}{Kb}(-d) + \frac{Q}{2\pi Kb}\ln(r_w) - \frac{Q}{2\pi Kb}\ln(2d)$$

$$b - h_0 = \frac{Q_{x0}d}{Kb} + \frac{Q}{2\pi Kb}\ln\left(\frac{r_w}{2d}\right)$$

$$\frac{Q}{2\pi Kb}\ln\left(\frac{r_w}{2d}\right) = (b-h_0) - \frac{Q_{x0}d}{Kb}$$

$$Q = \frac{2\pi Kb\left[(b-h_0) - \frac{Q_{x0}d}{Kb}\right]}{\ln(r_w/2d)} = \frac{2\pi Kb(h_0-b) + 2\pi Q_{x0}d}{\ln(2d/r_w)}.$$

$$\boxed{Q = \frac{2\pi\left[Kb(h_0-b) + Q_{x0}d\right]}{\ln(2d/r_w)}}$$

max extraction rate that maintains confined conditions at the extraction well

(5) (25 pts) Suppose we know $d = 200$ m, $r_w = 10$ cm $= 0.10$ m, $Q_{x0} = 2 \times 10^{-6}$ m^2/sec, $h_0 = 30$ m, and $b = 20$ m. There is a piezometer at ($x = -200, y = 300$) m. When the pumping rate Q at the extraction well is $Q = 1.0$ gallon per minute $= 6.3 \times 10^{-5}$ m^3/sec, the hydraulic head at the piezometer is $h = 36$ m. Use this information to estimate/calculate K.

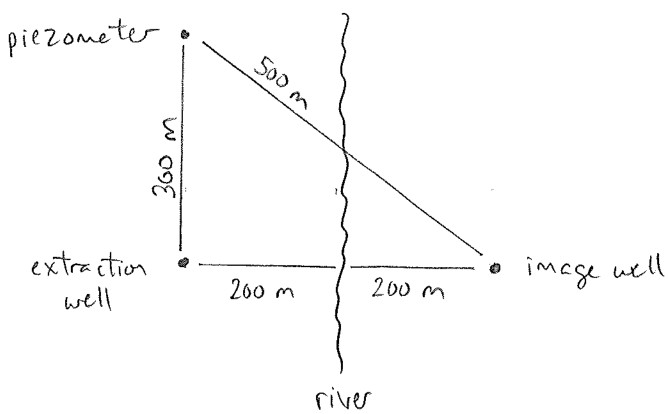
From problem 4 we have

$$h = h_0 - \frac{Q_{x_0}}{Kb} x + \frac{Q}{2\pi K b} \ln(r_1) - \frac{Q}{2\pi K b} \ln(r_2)$$

r_1 = distance from extraction well,
r_2 = distance from image well

$$\therefore \; K(h - h_0) = -\frac{Q_{x_0}}{b} x + \frac{Q}{2\pi b} \ln(r_1) - \frac{Q}{2\pi b} \ln(r_2)$$

What are x, r_1, r_2 for the piezometer?



$x = -200$ m
$r_1 = 300$ m
$r_2 = 500$ m

$$K(36\text{ m} - 30\text{ m}) = \left(\frac{-2.0 \times 10^{-6}\ \text{m}^2/\text{s}}{20\text{ m}}\right)(-200\text{ m}) + \frac{(6.3 \times 10^{-5}\ \text{m}^3/\text{s})}{2\pi\,(20\text{ m})} \ln\left(\frac{300\text{ m}}{500\text{ m}}\right)$$

$$K(6\text{ m}) = 2.0 \times 10^{-5}\ \frac{\text{m}^2}{\text{s}} - 2.56 \times 10^{-7}\ \frac{\text{m}^2}{\text{s}}$$

$$\boxed{K = 3.3 \times 10^{-6}\ \frac{\text{m}}{\text{s}}} = 3.3 \times 10^{-4}\ \frac{\text{cm}}{\text{s}} = 0.28\ \frac{\text{m}}{\text{d}}$$

This would probably be a silty sand ... reasonable

(6) (10 pts) Suppose that we want to maximize the rate of extraction from the well, but we have some constraints. We don't want to pull water out of the river, and we want to make sure that the aquifer remains confined at all locations. Use your answers from problems (3), (4), and (5) to determine the maximum allowable pumping rate Q at the well. The parameters given in problem 5 apply here.

We now have values of all the physical parameters.

From (3), max $Q = \pi Q_{x0} d$

$$= \pi (2.0 \times 10^{-6} \ m^2/s)(200 \ m)$$

$$= 0.00126 \ m^3/s = 110 \ m^3/d$$

From (4), max $Q = \dfrac{2\pi [K b (h_0 - b) + Q_{x0} d]}{\ln(2d/r_w)}$

$$= \frac{2\pi [(3.29 \times 10^{-6} \ \frac{m}{s})(20 \ m)(10 \ m) + (2.0 \times 10^{-6} \ \frac{m^2}{s})(200 \ m)]}{\ln(400 \ m / 0.1 \ m)}$$

$$= \frac{2\pi [6.58 \times 10^{-4} \ \frac{m^3}{s} + 4.00 \times 10^{-4} \ \frac{m^3}{s}]}{\ln(4000)}$$

$$= 8.0 \times 10^{-4} \ m^3/s = 69 \ m^3/d$$

The lower value of Q must be selected in order to meet both conditions.

$$\therefore \ \boxed{\text{Max allowable } Q = 8.0 \times 10^{-4} \ \frac{m^3}{s}} = 69 \ m^3/d$$

$$= 12.7 \ \text{gpm}$$