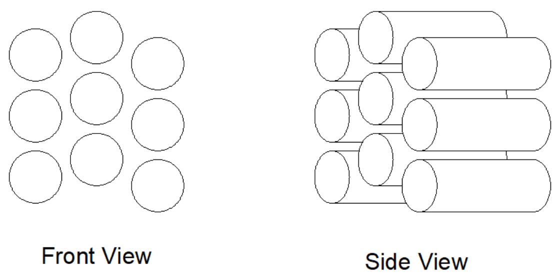
CGN 6933 Groundwater Engineering

Homework #2 — Fall 2019

Due Tues., Sept. 17, 2019 — Prof. J. A. Cunningham

This assignment will be collected and graded.
Complete this assignment in a team of 2 or 3 students.

(1) (20 pts) Perhaps you wonder why hydraulic conductivity is lower for fine-grained porous media than for coarse-grained media. Keep in mind that, for a porous medium, the size of the pores between grains is about the same as the size of the grains. Now, let's pretend that the pores of the medium look like a bundle of tubes, like this:



This is a pretty good (not perfect) model of the pores in a porous medium. Suppose each tube has a radius R and a length L. There is a pressure gradient ΔP across the bundle of tubes. Water flows through the tubes in response to this pressure gradient. You may assume that the flow is laminar, and you may treat the water as incompressible. We can use this model to investigate the permeability of porous media.

(a) Find the average velocity of water flow through a single tube. You can derive it, or you can look it up in a fluid-mechanics text-book under Hagen-Poiseuille flow, whichever you prefer.

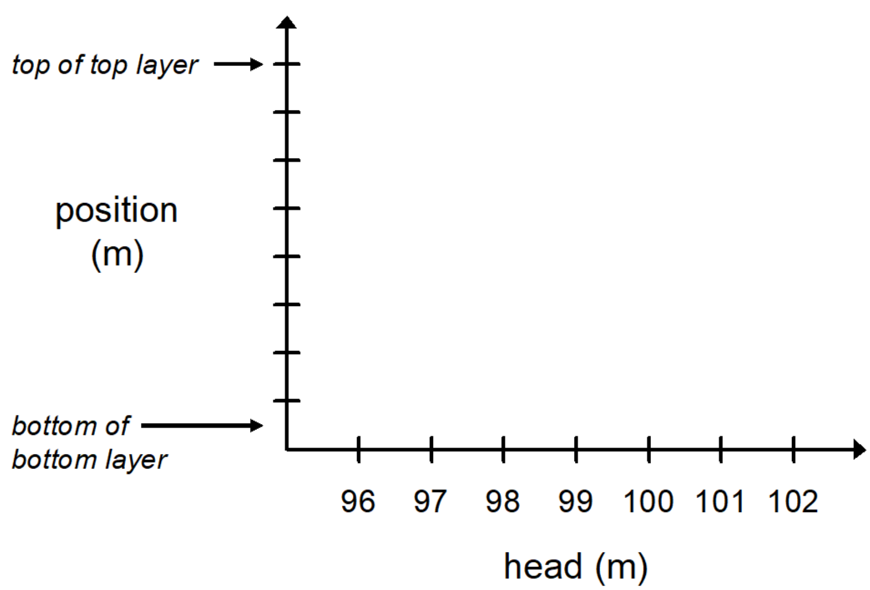
(b) From the average velocity, find q, the specific discharge (Darcy flux). Also re-write your expression in terms of Δh instead of ΔP. Hint: how is Darcy flux q related to average velocity v? ...and how is h related to P?

(c) Use your expression for q in conjunction with Darcy's Law to find the hydraulic conductivity, K, and the intrinsic permeability, k, for the bundle of tubes. How does the permeability depend upon the pore radius?

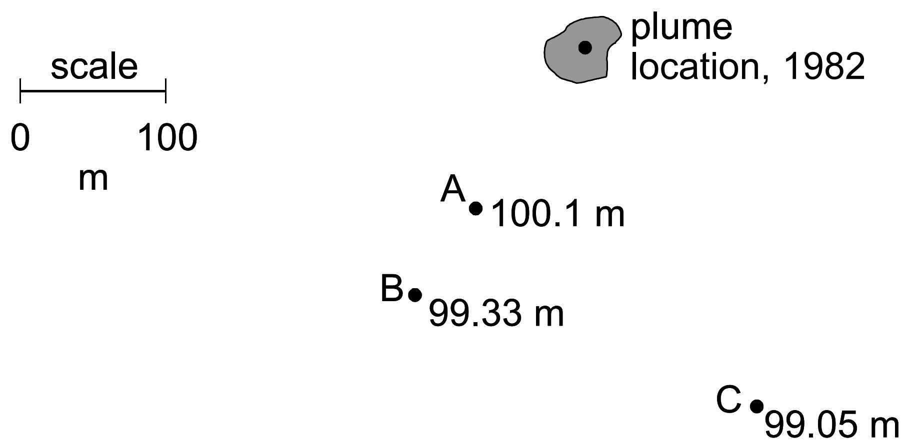
(d) Use your results to explain why fine-grained media have lower conductivities than coarse-grained media.

(2) (20 pts) Look up the *Kozeny-Carman equation* on page 69 of your text book. This equation relates the hydraulic conductivity of an unconsolidated porous medium to the medium's grain size and porosity. Use the equation to estimate the range of hydraulic conductivities that we might expect for clay, silt, sand, and gravel. Then compare your estimated values to the actual ranges of values encountered for these types of media. How closely do the values agree? What is your opinion about using this equation as a predictor of hydraulic conductivity? Would you trust the predicted values to within an order of magnitude? to within a factor of 2 or 3? to within 10–20%? (To complete this problem, you will need to estimate the grain size, porosity, and hydraulic conductivity for clay, silt, sand, and gravel. If you look these up somewhere, be sure to **indicate the sources that you used**. Please include full bibliographic information, so I could verify your information if I wanted to.)

(3) (20 pts) An aquifer consists of three layers. The top layer is 1.4 m thick and has a hydraulic conductivity (in the vertical direction) $K_z = 0.1$ m/day. The middle layer is 3.7 m thick and has a hydraulic conductivity (in the vertical direction) $K_z = 0.02$ m/day. The bottom layer is 2.5 m thick and has a hydraulic conductivity (in the vertical direction) $K_z = 0.05$ m/day. The head at the top of the top layer is 101.3 m. The head at the bottom of the bottom layer is 96.2 m. Graph the head versus depth. Put head on the x-axis and depth on the y-axis, as shown below.



(4) (20 pts) Below is a map showing the location of three piezometers installed in a silty-sand aquifer. Also shown are the measured heads in those piezometers and the location of a contaminant plume in 1982. Estimate the location of the contaminant plume today. Assume that the hydraulic conductivity is isotropic. You will have to make additional assumptions about the hydraulic conductivity and the porosity; state these clearly. Also, show your calculations and explain your reasoning. How far did the plume travel from 1982 until today? What does this tell you about the rate at which groundwater flows? How confident are you in your estimate? Do you think your estimate is correct to within a few cm? within a few m? within tens or hundreds of meters? kilometers?

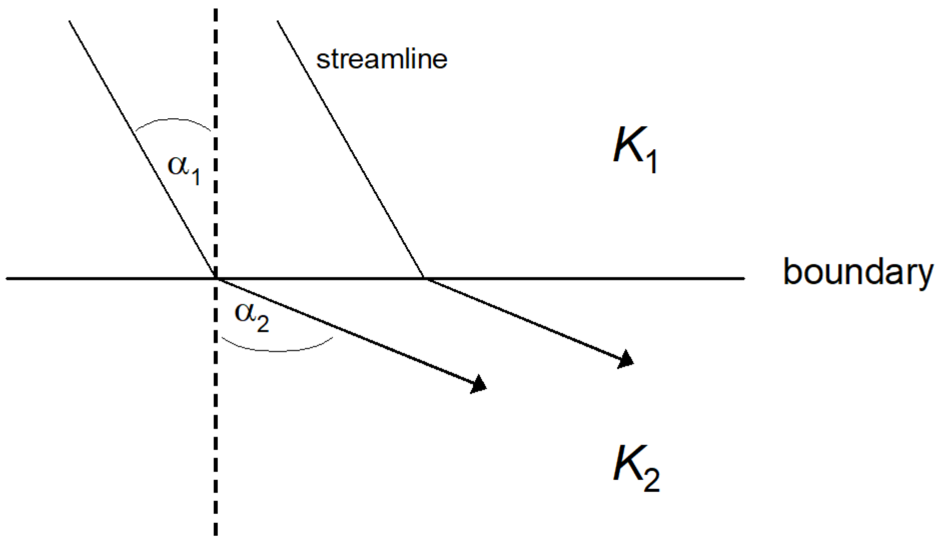


(5) (20 pts) Complete problem 6 in chapter 3 of your text book (p 92). Show all your work.

(6) **This problem will not be graded in 2019.**
Suppose that you are drilling a well into a very deep aquifer. Further suppose that the water in the aquifer is at a temperature of 15 ^{O}C everywhere. What is the density of the water at the water table (i.e., at atmospheric pressure)? What is the density of the water 500 m below the water table? On a percentage basis, how much does the density of water change between the water table and the depth of the aquifer? Does this sound like a significant change in density, or do you think it is OK to ignore it? When solving problems of groundwater flow, do you think it is important that we account for density variations with depth?

(7) **This problem will not be graded in 2019.**

Consider the steady flow of groundwater from a homogeneous formation with hydraulic conductivity K_1 to another formation with conductivity K_2. The figure below shows streamlines in two-dimensional flow (i.e., variability is neglected in the direction perpendicular to the paper) with $K_2 > K_1$.



(a) Demonstrate that the magnitude of the specific discharge changes at the boundary according to the expression

$$\frac{q_2}{q_1} = \frac{\cos\alpha_1}{\cos\alpha_2}$$

where α_1 is called the angle of incidence and α_2 is the angle of refraction.

(b) Demonstrate the refraction law

$$\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{K_1}{K_2}$$

(c) For $K_1 = 10^{-6}$ m/sec, $K_2 = 10^{-5}$ m/sec, and $\alpha_1 = 30^0$, find α_2 and the ratio q_2/q_1.

(d) Now for the important part: what do these relationships imply for groundwater flow? In what type of layer is flow predominantly parallel to the layer? In what type of layer is flow predominantly perpendicular to the layer? What happens to the flow in high- and low-conductivity layers? What happens to the groundwater flow as the ratio K_2/K_1 gets larger and larger?