CGN 6933 Groundwater Engineering

Homework #3 — Fall 2019

"Due" Tues., Sept. 24, 2019 — Prof. J. A. Cunningham

This assignment will not be collected or graded.
However, diligent completion of this assignment will help prepare you for the examinations.

(1) A confined aquifer has an estimated compressibility $\alpha = 2 \times 10^{-9}$ m^2/N, a thickness $b = 20$ m, and a porosity $n = 0.26$. The water temperature in the aquifer is 20 $^{\text{O}}$C.

(a) Calculate the specific storage, S_s, and the storativity, S.

(b) How much water would be released from storage in this confined aquifer under an area 1 km $\times$ 1 km, if the head in that area were lowered by an average of 2 m?

(2) An unconfined sand aquifer is 20 m thick. Its compressibility is $\alpha = 3 \times 10^{-8}$ m^2/N. Its specific yield is $S_y = 0.13$.

(a) Estimate the elastic storativity $S = S_s b$ of the sand aquifer. Estimate the ratio of water table storage (phreatic storage) to elastic storage in this aquifer. In your opinion, is it acceptable to ignore elastic storage?

(b) How much, on average, would head have to be lowered in an area of 2.5 km^2 to release 560,000 m^3 of water from storage in the aquifer?

(3) Consider the potential function $\Phi(x, y) = x^2 - y^2 + x$.

(a) Verify that $\Phi(x, y)$ satisfies the Laplace equation.

(b) Find the discharge components Q_x and Q_y.

(c) Find the location(s) of the stagnation point(s).

(4) For unconfined flow, we defined the discharge potential Φ according to $\Phi = \frac{1}{2}Kh^2 + C_U$, where C_U is an arbitrary constant.

(a) Verify that $Q_x = -\frac{\partial \Phi}{\partial x}$ and $Q_y = -\frac{\partial \Phi}{\partial y}$. Hint: use Darcy's law, coupled with the relationship between $\vec{q}$ and $\vec{Q}$ for unconfined flow.

(b) Verify that $\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0$. Hint: it is OK to start with the simplest form of the "general flow equation" that we derived in class for unconfined flow.

(c) What are the advantages of working in terms of Φ instead of h for unconfined flow?

(5) Suppose there is a homogeneous isotropic aquifer where the groundwater flow is stagnant under natural conditions. You drill a well in the aquifer at location $(x = 0, y = 0)$, and you start pumping water out of the well at the volumetric flow rate Q. Your instructor has told you that, once the system reaches steady state, the discharge potential $\Phi(x, y)$ in the aquifer is given by $\Phi(x, y) = \frac{Q}{2\pi}\ln(r) + C$, where C is an arbitrary constant, and $r = \sqrt{x^2 + y^2}$. Here, you will try to convince yourself that this expression is correct.

(a) Find Q_x and Q_y. If you can, express Q_x and Q_y in terms of the polar coordinates r and θ. Hint: use the chain rule, i.e., $\frac{d\Phi}{dx} = \frac{d\Phi}{dr}\frac{dr}{dx}$ and $\frac{d\Phi}{dy} = \frac{d\Phi}{dr}\frac{dr}{dy}$.

(b) Verify that the expression for Φ satisfies the Laplace equation.

(c) Consider a circle of any arbitrary radius R with the pumping well located at the center. Verify that the volumetric flow rate of water into the circle from the surrounding aquifer is equal to the pumping rate Q.

(d) Argue that, based on your findings in parts (b) and (c) above, the expression for Φ must be correct.

(6) This problem deals with the principle of "superposition."

(a) Suppose that $\Phi_1(x, y)$ and $\Phi_2(x, y)$ satisfy the Laplace equation. Show that any linear combination $\Phi = C_1\Phi_1(x, y) + C_2\Phi_2(x, y)$, where C_1 and C_2 are arbitrary constants, also satisfies the Laplace equation. This is sometimes called the "superposition principle," and it is also said that the Laplace equation is a "linear" equation.

(b) Two pumping wells are installed in a homogeneous isotropic aquifer. The first well is located at (x_1, y_1) and pumps at a volumetric flow rate Q_1. The second well is located at (x_2, y_2) and pumps at a volumetric flow rate Q_2. What is the discharge potential, $\Phi(x, y)$, within the aquifer?

(c) For the scenario posed in part (b), suppose that $Q_1 = 50$ m^3/day and well 1 is located at $(x_1 = -1 \text{ m}, y_1 = 1 \text{ m})$. Well 2 is located at $(x_2 = 1 \text{ m}, y_2 = -1 \text{ m})$, and pumps at a volumetric flow rate $Q_2 = 10$ m^3/day. Find the location(s) of the stagnation point(s).