CGN 6933 Groundwater Engineering

Homework #6 | Fall 2019

"Due" Thurs., Oct. 24, 2019 | Prof. J. A. Cunningham

This assignment will not be collected or graded.
However, diligent completion of this assignment will help prepare you for the examinations.

(1) In class I told you that the hydraulic potential for a circular lake (of radius R) in regional flow is given by the formula

$$\Phi(x,y) = -Q_{x0}\ x \ + \ Q_{x0}\ \frac{x\ R^2}{x^2+y^2} \ + \ \Phi_L$$

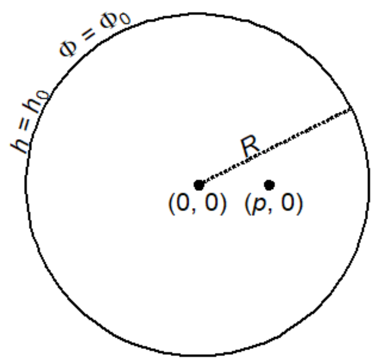
if the lake is neither a net source or a net sink. Now let's see if we can convince ourselves that this formula is correct. To be correct, it must satisfy the partial differential equation and it must satisfy the necessary boundary conditions.

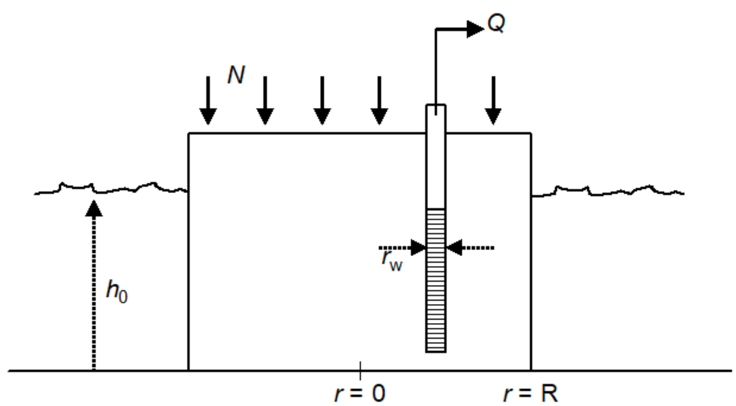
(a) Verify that $\Phi(x,y)$ satisfies the Laplace equation.
(b) Verify that $\Phi(x,y)$ honors the proper boundary condition on the edge of the lake.
(c) Verify that, far away from the lake, the function for $\Phi(x,y)$ looks like regional flow. Hint: what does "far away from the lake" mean mathematically?

If parts (a)–(c) all work out, then $\Phi(x,y)$ should be valid!

(2) A circular island is centered at $(x=0, y=0)$. An extraction well is located at $(x=p, y=0)$. The well pumps at a volumetric flow rate Q. The aquifer is unconfined and is recharged by rainfall with infiltration rate N. The hydraulic head on the boundary of the island is h_0, and the discharge potential on the boundary is Φ_0. The radius of the island is R. The radius of the pumping well bore is r_w. See the figure on the next page.

(2) continued





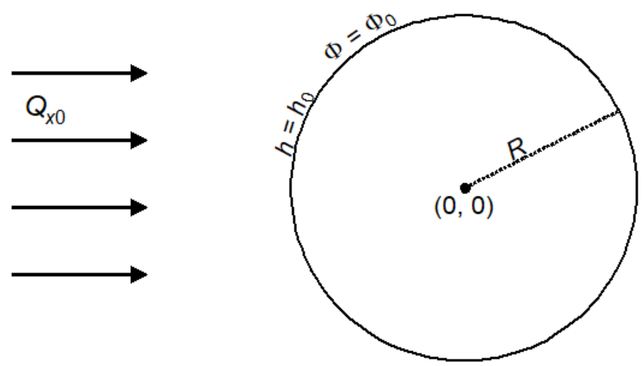
(a) What is $\Phi(x, y)$? Hint: solve the Poisson equation for rainfall on a circular island with *no* well. Then think of the discharge potential for the well on the island with *no* recharge. Then use superposition to figure out Φ that accounts for both the rainfall and the well, while maintaining the necessary boundary condition.

(b) Find the discharge in the x-direction, Q_x, at the point $(x = R, y = 0)$. What is the maximum allowable pumping rate Q if we don't want the well to pull any water in from outside the island?

(c) Suppose $R = 1000$ m, $p = 300$ m, $N = 1 \times 10^{-9}$ m/sec, $Q = 2 \times 10^{-4}$ m^3/sec, $h_0 = 20$ m, $r_w = 0.10$ m, and $K = 1 \times 10^{-6}$ m/sec. Draw the water table along the transsect $y = 0$. For what value of x is the water table highest along this transect? What is the value of h at that location?

(d) Graph the equipotentials (e.g., using MatLab) in the aquifer for the conditions specified in part (c).

(3) A circular lake is located in an aquifer with regional flow. The regional flow is in the $+x$-direction, with discharge rate Q_{x0}. The lake has a radius R. On the boundary of the lake, the hydraulic head is h_0 and the discharge potential is Φ_0. The lake is centered at $(x = 0, y = 0)$. Suppose the lake is a net hydraulic sink. The net volumetric flow rate of water into the lake is Q_L.



(a) Write the expression for the discharge potential, $\Phi(x, y)$, and the streamfunction, $\Psi(x, y)$.

(b) Find the radial discharge rate, $Q_r(\theta)$, on the border of the lake. Define Q_r as positive for flow out of the lake.

(c) Suppose $Q_L = 2\pi R Q_{x0}$. For what values of θ is the groundwater flow into the lake? For what values of θ is the flow out of the lake?

(d) Find the value of Q_L for which the groundwater flow is into the lake at all values of θ, i.e., nowhere is there flow out of the lake.

(e) Consider a dimensionless discharge potential $\Phi/(Q_{x0}R)$, and a dimensionless streamfunction $\Psi/(Q_{x0}R)$. Write these as functions of the dimensionless variables x/R and y/R. Then plot the flownet (using all dimensionless quantities) for the case considered in part (c).

(4) Consider a confined aquifer with thickness b, regional hydraulic gradient J, and background hydraulic conductivity K. Regional groundwater flow is in the $+x$-direction. In the middle of the aquifer, centered at $(x = 0, y = 0)$, is a circular cylindrical heterogeneity of radius 1 m. Suppose the heterogeneity is completely impermeable. In this case, what are the discharge potential $\Phi(x, y)$ and the streamfunction $\Psi(x, y)$? Graph the streamlines for this situation.