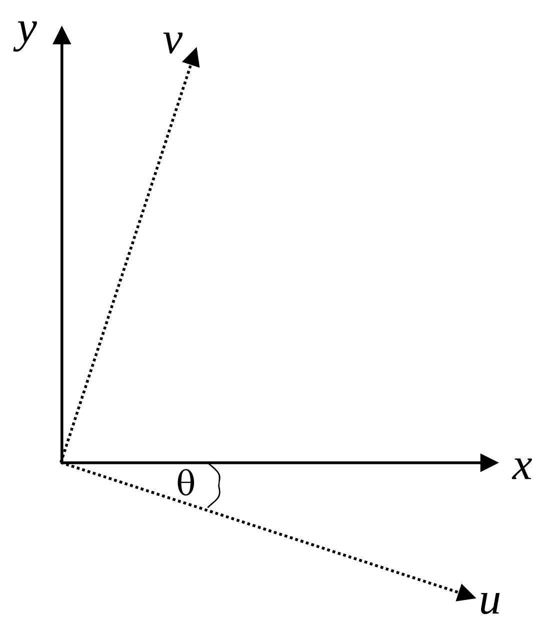
CGN 6933 Groundwater Engineering

Homework #7 | Fall 2019

"Due" Thurs., Oct. 31, 2019 | Prof. J. A. Cunningham

This assignment will not be collected or graded.
However, diligent completion of this assignment will help prepare you for the examinations.

(1) Consider two-dimensional flow in a homogeneous but anisotropic aquifer. Suppose that you know the hydraulic conductivity tensor, $\mathbf{K}$, in the (x, y) coordinate system. However, the (x, y) coordinate system must be rotated by an angle θ to find the principal directions of the aquifer. The principal directions are noted as the u and v directions. See the diagram below.



(a) What is θ in terms of K_{xx}, K_{xy}, and K_{yy}? Recall that $K_{xy} = K_{yx}$ for the hydraulic conductivity tensor. Hint: you may want to use Mohr's circle. Another hint: for this problem, the angle θ is based on a clockwise (not counter-clockwise) rotation – different from how we did it in class.

(b) For such an anisotropic system, under what conditions is the specific discharge vector, $\vec{q}$, parallel to the hydraulic head gradient, $\vec{\nabla} h$?

Problem 1 continues on next page →

(1) continued

(c) Suppose that $K_{xx} = 1$ m/day, $K_{xy} = K_{yx} = -0.2$ m/day, and $K_{yy} = 0.2$ m/day. Estimate the conductivities in the principal directions. Find θ.

(d) Plot the principal directions relative to the (x, y) coordinate system. Make sure you indicate which of the two principal directions is maximum, and which is minimum. First find the principal directions by hand, then check your answer using the `eig` function in MatLab.

(e) If $\vec{\nabla} h = (1, 2)$ in (x, y) coordinates, what is $\vec{q}$ in (x, y) coordinates? What is $\vec{q}$ in (u, v) coordinates?

(f) For the conditions given in part (c), what axis stretching would be needed to make this sytem isotropic? What is the conductivity of the system in the new isotropic coordinate system?

(2) Consider an elliptical island underlain by a homogeneous, isotropic, unconfined aquifer. Define the coordinate system such that the center of the island is located at $(x = 0, y = 0)$ and the edge of the island is given by the ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

as shown in the figure on the following page.

The aquifer has a hydraulic conductivity K, and is recharged at a rate N by rainfall on the island. The water level at the edge of the island is h_0.

(a) Verify that the discharge potential on the island is:

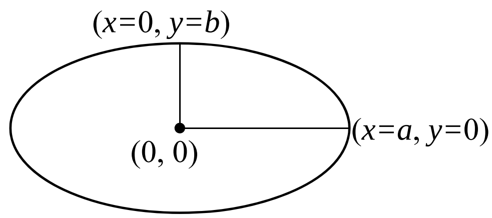
$$\Phi(x, y) = -\frac{N}{2} \frac{1}{a^2 + b^2} \left(b^2 x^2 + a^2 y^2 - a^2 b^2\right) + \Phi_0$$

where $\Phi_0 = \frac{1}{2} K {h_0}^2 + C_u$. You do not need to derive this expression; you just need to show that it satisfies the appropriate equation(s). (What must the expression for $\Phi(x, y)$ satisfy in order for you to be convinced that it is correct?)

(b) Suppose $a = 1000$ m, $b = 500$ m, $N = 1 \times 10^{-4}$ m/day, $K = 0.1$ m/day, and $h_0 = 10$ m. Where is the water table highest? What is the height of the water table at that location?

(c) For the conditions given in part (b), plot the height of the water table along the transsect $y = 0$.

top view:



side view: