CGN 6933 Groundwater Engineering

Homework #9 | Fall 2019

"Due" Thurs., Nov. 21, 2019 | Prof. J. A. Cunningham

This assignment will not be collected or graded.
However, diligent completion of this assignment will help prepare you for the examinations.

(1) In class, I presented the Theis solution

$$h(r,t) = h_0 - \frac{Q}{4\,\pi\,T} \int_u^\infty \frac{1}{\omega} \exp(-\omega)\, d\omega,$$

and I claimed that this solution satisfies the partial differential equation

$$S\,\frac{\partial h(r,t)}{\partial t} = T\,\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial h(r,t)}{\partial r}\right).$$

(a) Verify that the Theis solution does, in fact, satisfy the PDE. Hint: Use the second fundamental theorem of calculus. Another hint: remember that the variable u is defined as $u = (r^2\,S)/(4\,T\,t)$. Another hint: don't forget the chain rule for differentiation.

(b) What are the boundary conditions that the Theis solution should obey? Verify that the given solution does, in fact, honor the boundary conditions.

If the proposed solution satisfies the PDE and the boundary conditions, then it is a good solution!

(2) Consider a homogeneous confined aquifer with negligible regional flow. An extraction well is located at $(x = -200, y = 0)$ m. A monitoring well is located at $(x = -100, y = 0)$ m. The radius of the extraction well casing is $r_w = 0.1$ m. At time $t = 0$, the pump in the extraction well is turned on, and water is extracted at a rate $Q = 100$ m^3/day. After 15 hours, the drawdown in the extraction well is 7 m, and the drawdown in the monitoring well is 2 m. Estimate T and S for the aquifer. Hint: you can use Matlab to evaluate the exponential integral function, or you can make use of the Cooper-Jacob approximation; but if you use the Cooper-Jacob approximation, you must verify that it is valid under the conditions of this problem. Another (closely related) hint: Equations 8.39 and 8.40 in your text might be useful, but these equations assume that equations 8.33 and 8.34 are valid, so you have to check to be sure this is OK.

(3) A well with a casing radius $r_w = 0.3$ m produces water from an aquifer with $T = 300$ m^2/day and $S = 0.0006$. The well pumps at a rate of 550 m^3/day for 30 days, and then at a rate of 800 m^3/day for an additional 30 days. The well is then shut off. Estimate/calculate the drawdown in the well after 80 days from the start of pumping (i.e., 20 days after the well was shut off).

(4) Estimate/calculate the drawdown at an observation well due to two pumping wells. The aquifer has $T = 300$ m^2/day and $S = 0.0006$. Well number 1, which is located at a distance $r_1 = 30$ m from the observation well, produces water at a rate $Q_1 = 550$ m^3/day. Well number 2, which is located at a distance $r_2 = 40$ m from the observation well, produces water at a rate $Q_2 = 1100$ m3/day. Both production wells are turned on at $t = 0$ days. Estimate/calculate the drawdown at the observation well at $t = 200$ days. What is the relative contribution of well number 1 and well number 2?

(5) A confined aquifer has a transmissivity $T = 20$ m^2/day and a storativity $S = 0.0002$. The aquifer is overlain by a semi-impermeable aquitard that is 3 m thick with a hydraulic conductivity $K = 0.0016$ m/day = 1.8×10^{-6} cm/s. There is a large source of water above the aquitard. A fully penetrating well begins pumping in the aquifer at a time $t = 0$. The extraction rate is $Q = 50$ m^3/day. Predict the drawdown in an observation well that is 120 m away. Predict the drawdown for the following elapsed times: 10 minutes, 1 hour, 4 hours, 12 hours, and 24 hours. Can you estimate the approximate time at which the observation well reaches steady state? For this problem, neglect storage in the aquitard, i.e., assume that storage in the aquitard is insignificant.

(6) Select any one of the starred problems from pages 354–355 of your text (problem 5, 6, 7, 9, or 11). Complete the problem, and then check your answer in the back of the book.

(7) It has been a while since we solved a problem using hydraulic potential theory / analytic element method. Fear not, the long wait is now over. Consider a homogeneous, isotropic aquifer at steady state. Suppose three wells are all pumping at a rate Q. Two of them are extraction wells, located at $(d_1, 0)$ and $(d_2, 0)$, where d_1 and d_2 are positive real numbers. The third well is an injection well, located at $(0, 0)$. Regional flow is negligible.

(a) Write the potential function, $\Phi(x, y)$, and the streamfunction, $\Psi(x, y)$.

(b) Find the discharge, $Q_x(x, y)$ and $Q_y(x, y)$.

(c) Find the stagnation points and show that they are located on the x-axis.

(d) Find the value of the streamfunction at each of the stagnation points. You should discover something interesting about the relative magnitude of the streamfunction at the two stagnation points. What does this tell you?

(e) Using your answer from part (d), find the equation of the streamline that hydraulically isolates part of the domain.

(f) Plot a nice-looking flownet for the case that $d_2 = 2\,d_1$. Work in terms of dimensionless variables x/d_1 and y/d_1. Identify the important features of the flownet. Be sure to include the streamline(s) the pass(es) through the stagnation points.

(g) What physical situation can be modeled using the configuration of wells described here? Hint: look at the streamlines.