$$T(n) = \sum_{i=1}^{n/2}\sum_{j=i}^{n-i}\sum_{k=1}^{j} 1$$

$$= \sum_{i=1}^{n/2}\sum_{j=i}^{n-i} j$$

▷ Using the definition of summation we can show each element of the inner summation:

$$= \sum_{i=1}^{n/2} \left((i) + (i+1) + \ldots + (n-i-1) + (n-i)\right)$$

▷ If I add elements to an equation and then subtract the same elements from the equation

▷ then I do not change the resulting value of the equation.

$$= \sum_{i=1}^{n/2} \left[\underline{(1+2+\ldots+(i-1)) + (i) + (i+1) + \ldots + (n-i-1) + (n-i)} - (1+2+\ldots+(i-1))\right]$$

▷ Now we can rewrite the underlined portion above in summation notation where the summation starts at 1

$$= \sum_{i=1}^{n/2} \left[\left(\sum_{j=1}^{n-i} j\right) - (1+2+\ldots+(i-1))\right]$$

▷ Now we can rewrite the numbers we are subtracting off in summation notation

$$= \sum_{i=1}^{n/2} \left(\sum_{j=1}^{n-i} j - \sum_{j=1}^{i-1} j\right)$$

$$= \sum_{i=1}^{n/2} \left(\frac{(n-i)(n-i+1)}{2} - \frac{(i-1)i}{2}\right)$$

$$= \sum_{i=1}^{n/2} \frac{(n-i-i+1)(n-i+i)}{2}$$

$$= \frac{1}{2}\sum_{i=1}^{n/2} (n-2i+1)(n)$$

$$= \frac{n}{2}\sum_{i=1}^{n/2} (n-2i+1)$$

$$= \frac{n}{2}\left[\sum_{i=1}^{n/2} n - \sum_{i=1}^{n/2} 2i + \sum_{i=1}^{n/2} 1\right]$$

$$= \frac{n}{2}\left[\frac{n^2}{2} - 2\left(\frac{\frac{n}{2}\left(\frac{n}{2}+1\right)}{2}\right) + \frac{n}{2}\right]$$

$$= \frac{n}{2}\left[\frac{n^2}{2} - \frac{n}{2}\left(\frac{n}{2}+1\right) + \frac{n}{2}\right]$$

$$= \frac{n}{2}\left[\frac{n^2}{2} - \frac{n^2}{4} - \frac{n}{2} + \frac{n}{2}\right]$$

$$= \frac{n}{2}\left(\frac{n^2}{4}\right)$$

$$= \frac{n^3}{8}$$