

INTERVAL NOTATION

A **set** is a collection of objects whose contents can be clearly determined.
The objects in the set are called the elements of the set.

Sets of real numbers can be represented using one of the following forms:

1. **SET - BUILDER NOTATION.** In this notation the elements are described, but not listed.
Ex: $N = \{x \mid x \text{ is a natural number}\}$
(Note: The small vertical line " \mid " stands for the phrase "such that")
2. **GRAPH.** Visually represents the set of numbers on a number line by placing a dot at the correct location for each number. The value of the numbers increase from left to right.

If we are considering an entire span or interval of numbers rather than just a set of individual values, we use shading to indicate **ALL** the numerical values in that interval.

If the set we are considering is an entire span or interval of values on the number line we can also use a form called Interval Notation.

3. **INTERVAL NOTATION.** Represents a shaded span of numbers on the number line by showing the numbers at the end of the interval separated with a comma.

The numbers are surrounded by symbols that indicate whether or not those endpoints are included.

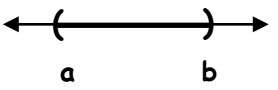
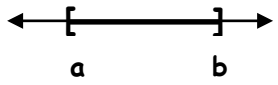
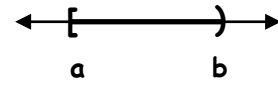
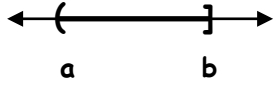
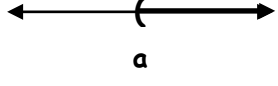
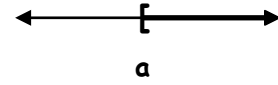
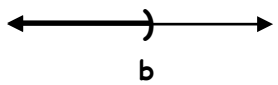
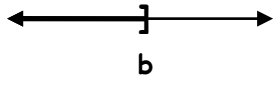
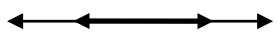
(Parentheses) indicate endpoints that are NOT included in the interval (when graphing you may have used open circles to indicate this.)

[Brackets] indicate endpoints that are included in the interval (when graphing you may have used closed or solid circles to indicate this.)

NOTE: (Parentheses) are ALWAYS used to surround ∞ and $-\infty$.

The table below lists nine possible types of intervals used to describe sets of real numbers.

Suppose a and b are two real numbers such that $a < b$

Type of interval	Interval Notation	Description	Set- Builder Notation	Graph
Open interval	(a, b)	Represents the set of real numbers between a and b , but NOT including the values of a and b themselves.	$\{x \mid a < x < b\}$	
Closed interval	$[a, b]$	Represents the set of real numbers between, and including a and b .	$\{x \mid a \leq x \leq b\}$	
Half closed - half open interval	$[a, b)$	Represents the set of real numbers between a and b , including a but NOT including b .	$\{x \mid a \leq x < b\}$	
Half open - half closed interval	$(a, b]$	Represents the set of real numbers between a and b , NOT including a but including b .	$\{x \mid a < x \leq b\}$	
Infinite Interval	(a, ∞)	Represents the set of real numbers that are greater than a .	$\{x \mid x > a\}$	
Infinite Interval	$[a, \infty)$	Represents the set of real numbers that are greater than or equal to a .	$\{x \mid x \geq a\}$	
Infinite Interval	$(-\infty, b)$	Represents the set of real numbers that are less than b .	$\{x \mid x < b\}$	
Infinite Interval	$(-\infty, b]$	Represents the set of real numbers that are less than or equal to b .	$\{x \mid x \leq b\}$	
Infinite Interval	$(-\infty, \infty)$	The set of all real numbers.	$\{x \mid x \text{ is a real number}\}$	

EXERCISES:

Write the following inequalities in interval notation and graph it.

1. $x \leq 3$

2. $-2 < x \leq 4$

3. $-9 \leq x \leq 0$

4. $x > -4$

5. $x < -3$

6. $x \geq 6$

Express each of the following intervals in set-builder notation.

7. $(2, 8)$

8. $[-5, 0)$

9. $(3, \infty)$

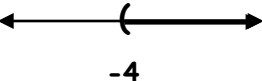
10. $(-\infty, -4]$

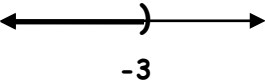
Answers:

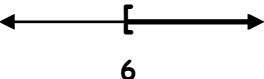
1. $(-\infty, 3]$ 

2. $(-2, 4]$ 

3. $[-9, 0]$ 

4. $(-4, \infty)$ 

5. $(-\infty, -3)$ 

6. $[6, \infty)$ 

7. $\{x \mid 2 < x < 8\}$

8. $\{x \mid -5 \leq x < 0\}$

9. $\{x \mid x > 3\}$

10. $\{x \mid x \leq -4\}$