Topic 1.1

Linear Equations
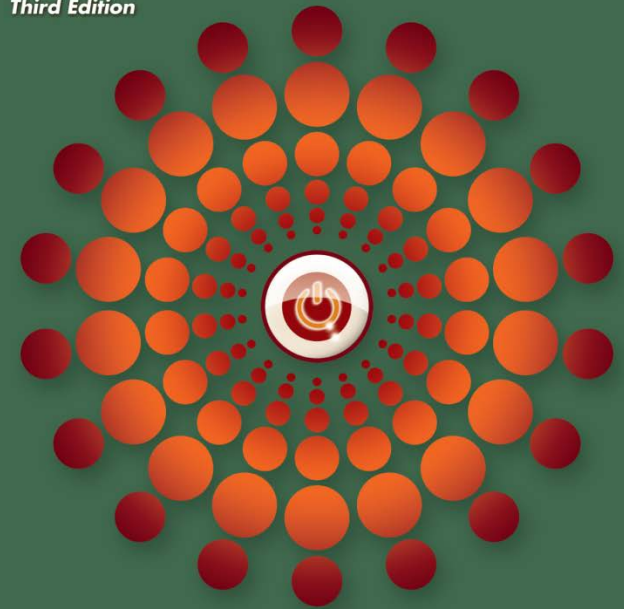
OBJECTIVES

1. Recognizing Linear Equations
2. Solving Linear Equations with Integer Coefficients
3. Solving Linear Equations Involving fractions
4. Solving Linear Equations Involving Decimals
5. Solving Equations that Lead to Linear Equations

Recognizing Linear Equations

DEFINITION: Equation

> Two algebraic expressions are equal.

DEFINITION: Linear Equation in One Variable

> A linear equation in one variable is an equation that can be written in the form $ax + b = c$, where a, b, and c are real numbers and $a \neq 0$.

$$7x - 4 = x - 5, \quad \frac{1}{3}y - \sqrt{2} = 11, \text{ and } 0.4a - 10$$

Solving Linear Equations with Integer Coefficients

EXAMPLE

Solve the equation $5(x-6)-2x = 3-(x+1).$

$5x - 30 - 2x = 3 - x - 1$	Original equation
$3x - 30 = 2 - x$	Distributive property
$3x - 30 + x = 2 - x + x$	Add x to both sides
$4x - 30 = 2$	Simplify
$4x - 30 + 30 = 2 + 30$	Add 30 to both sides
$4x = 32$	Simplify
$\dfrac{4x}{4} = \dfrac{32}{4}$	Divide both sides by 4
$x = 8$	Simplify (be sure to check your answer)

Solving Linear Equations Involving Fractions
EXAMPLE

Solve the equation $\dfrac{1}{3}(1-x) - \dfrac{x+1}{2} = -2.$

$$\dfrac{1}{3}(1-x) - \dfrac{x+1}{2} = -2 \qquad \text{Original equation}$$

$$6\left(\dfrac{1}{3}(1-x) - \dfrac{x+1}{2}\right) = 6(-2) \qquad \text{Multiply both sides by 6}$$

$$6\left(\dfrac{1}{3}(1-x)\right) - 6\left(\dfrac{x+1}{2}\right) = 6(-2) \qquad \text{Use the distributive property}$$

$$2(1-x) - 3(x+1) = -12 \qquad \text{Multiply}$$

$$2 - 2x - 3x - 3 = -12 \qquad \text{Use the distributive property}$$

Solving Linear Equations Involving Fractions
EXAMPLE

Solve the equation

$$\frac{1}{3}(1-x) - \frac{x+1}{2} = -2.$$

$$2 - 2x - 3x - 3 = -12$$ From last slide

$$-5x - 1 = -12$$ Simplify

$$-5x - 1 + 1 = -12 + 1$$ Add -1 to both sides

$$-5x = -11$$ Simplify

$$\frac{-5x}{-5} = \frac{-11}{-5}$$ Divide both sides by -5

$$x = \frac{11}{5}$$ Simplify (check your answer)

Solving Linear Equations Involving Decimals
EXAMPLE

Solve the equation $.1(y-2)+.03(y-4)=.02(10).$

$.1(y-2)+.03(y-4)=.02(10)$	Original equation
$.1y-.2+.03y-.12=.2$	Distributive property
$.13y-.32=.2$	Simplify
$.13y-.32+.32=.2+.32$	Add .32 to both sides
$.13y=.52$	Simplify
$\dfrac{.13y}{.13}=\dfrac{.52}{.13}$	Divide both sides by .13
$y=4$	Simplify (be sure to check your answer)

Solving Equations That Lead to Linear Equations

Solve the equation $3a^2 - 1 = (a+1)(3a+2).$

$$3a^2 - 1 = (a+1)(3a+2)$$ Original equation

$$3a^2 - 1 = 3a^2 + 5a + 2$$ Multiply

$$3a^2 - 3a^2 - 1 = 3a^2 - 3a^2 + 5a + 2$$ Subtract $3a^2$ from both sides

$$-1 = 5a + 2$$ Simplify

$$-1 - 2 = 5a + 2 - 2$$ Subtract -2 from both sides

$$-3 = 5a$$ Simplify

$$\frac{-3}{5} = \frac{5a}{5}$$ Divide both sides by 5

$$-\frac{3}{5} = a$$ Simplify (be sure to check your answer)