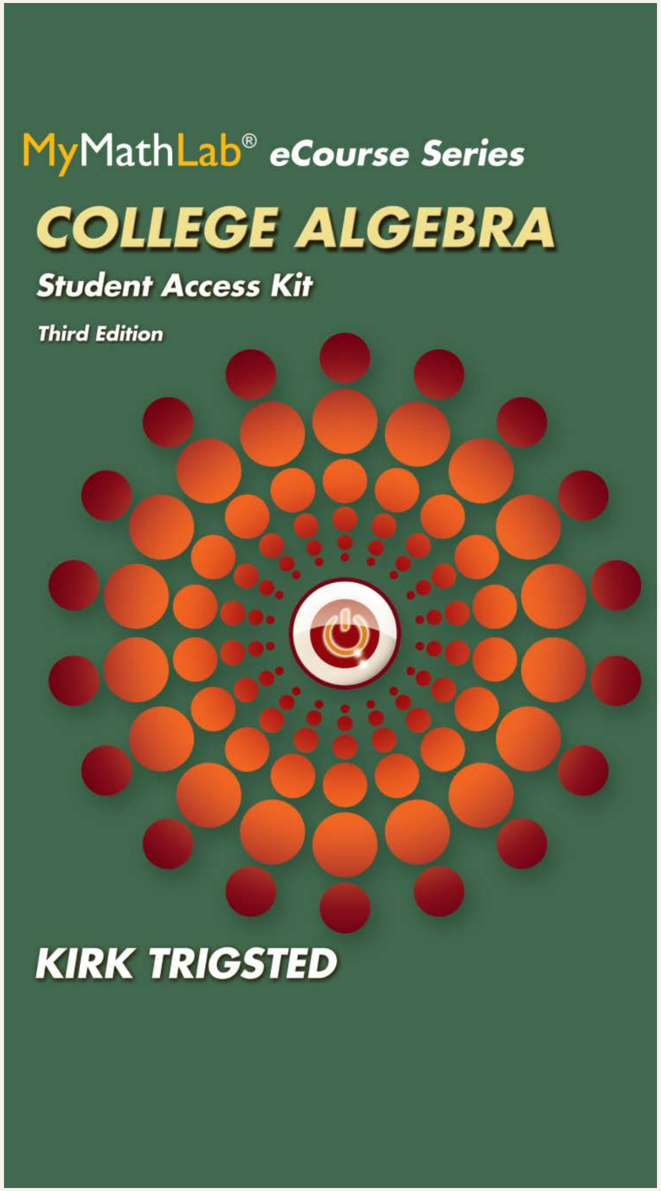
Topic 1.2

Applications of Linear Equations



OBJECTIVES

1. Converting Verbal Statements into Mathematical Statements
2. Solving Applications Involving Unknown Numeric Quantities
3. Solving Applications Involving Decimal Equations (Money, Mixture, Interest)
4. Solving Applied Problems Involving Distance, Rate, and Time
5. Solving Applied Working Together Problems

Converting Verbal Statements into Mathematical Statements

Recognize key words

- Addition
  - Sum More than Total
- Subtraction
  - Difference Less than Minus
- Multiplication
  - Product Times
- Division
  - Quotient Divided by Per
- Is: Often translates into an equal sign

Converting Verbal Statements into Mathematical Statements

Example

Rewrite each statement as an algebraic expression or equation:

a. 7 more than three times a number $3x+7$

b. 5 less than twice a number $2x-5$

c. Three times the quotient of a number and 11

$3(x \div 11)$ or $3\left(\frac{x}{11}\right)$

d. The sum of a number and 9 is 1 less than half of the number $x+9=\frac{1}{2}x-1$

e. The product of a number and 4 is 1 more than 8 times the difference of 10 and the number. $4x=8(10-x)+1$

Converting Verbal Statements into Mathematical Statements

Polya's Guidelines for Problem Solving

1. Understand the Problem
   a. Read the problem several times until you have an understanding of what is being asked
2. Devise a Plan
   a. Pick a variable that describes the unknown quantity that is to be found.
   b. Write all other quantities in terms of that variable
   c. Write an equation
3. Carry out the plan
   a. Solve the equation
4. Look Back
   a. Be sure you have answered the question
   b. Check all answers to make sure they make sense

Solving Applications Involving Unknown Numeric Quantities

Example

Roger Staubach and Terry Bradshaw were both quarterbacks in the National Football League. In 1973, Staubach threw three touchdown passes more than twice the number of touchdown passes thrown by Bradshaw. If the total number of touchdown passes between Staubach and Bradshaw was 33, how many touchdown passes did each player throw?

Step 1. After carefully reading the problem, we see that we are trying to figure out how many touchdown passes were thrown by each player.

Solving Applications Involving Unknown Numeric Quantities

Example

Roger Staubach and Terry Bradshaw were both quarterbacks in the National Football League. In 1973, Staubach threw three touchdown passes more than twice the number of touchdown passes thrown by Bradshaw. If the total number of touchdown passes between Staubach and Bradshaw was 33, how many touchdown passes did each player throw?

Step 2. Let B be the number of touchdown passes thrown by Bradshaw. Because Staubach threw 3 more than twice the number of touchdowns thrown by Bradshaw, then $2B + 3$ represents the number of touchdown passes thrown by Staubach.

$$\underbrace{\text{Bradshaw's touchdown passes}}_{B} + \underbrace{\text{Staubach's touchdown passes}}_{(2B + 3)} = \underbrace{\text{Total}}_{33}$$

Solving Applications Involving Unknown Numeric Quantities

Example

Step 3. Solve:

$$B+(2B+3)=33 \quad \text{Write the equation}$$
$$3B+3=33 \quad \text{Combine like terms}$$
$$3B=30 \quad \text{Subtract 3 from both sides}$$
$$B=10 \quad \text{Divide both sides by 3}$$

To answer the question, Bradshaw threw 10 touchdown passes in 1973, and Staubach threw $2(B)+3=2(10)=3=23$ touchdowns passes in 1973.

Step 4. Check: We see that the total number of touchdown passes is 10 + 23 = 33. The Number of touchdown passes thrown by Staubach is 3 more than two times the number of touchdown passes thrown by Bradshaw.

Solving Applications Involving Decimal Equations (Money, Mixture, Interest)

Example

Billy has $16.50 in his piggy bank, consisting of nickels, dimes, and quarters. Billy notices that he has 20 fewer quarters than dimes. If the number of nickels is equal to the number of quarters and dimes combined, how many of each coin does Billy have?

Step 1. We must find out how many nickels, dimes, and quarters Billy has.

Step 2. Let d = number of dimes.
The remaining quantities must also be expressed in terms of the variable *d*. Because Billy has 20 fewer quarters than dimes, we can express the number of quarters in terms of the number of dimes, or

$$d - 20 = \text{number of quarters.}$$

Solving Applications Involving Decimal Equations (Money, Mixture, Interest)

Example

Step 2. continued

The number of nickels is equal to the sum of the number of dimes and quarters

$$\underbrace{d}_{\substack{\text{number}\\ \text{of dimes}}} + \underbrace{(d - 20)}_{\substack{\text{number}\\ \text{of quarters}}} = \text{number of nickels}$$

Nickels are worth $.05, dimes are worth $.10, quarters are worth $.25, and the total amount is $16.50. So, we get the following equation:

$$\underbrace{.05(\text{d} + (\text{d} - 20))}_{\substack{\text{value of}\\ \text{the nickels}}} + \underbrace{.10d}_{\substack{\text{value of the}\\ \text{dimes}}} + \underbrace{.25(\text{d} - 20)}_{\substack{\text{value of}\\ \text{the quarters}}} = \underbrace{16.50}_{\text{Total}}$$

Solving Applications Involving Decimal Equations (Money, Mixture, Interest)

Example

Step 3. Solve: $$.05(d+(d-20))+.10d+.25(d-20)=16.50$$
$$.05(2d-20)+.10d+.25(d-20)=16.50$$
$$.1d-1+.10d+.25d-5=16.50$$
$$.45d-6=16.50$$
$$.45d=22.50$$
$$d=50$$

There are 50 dimes, $d-20=50-20=30$ quarters, and $2d-20=2(50)-20=100-20=80$ nickels.

Step 4. Check: 50(.10)+30(.25)+80(.05) = $16.50. The values check.

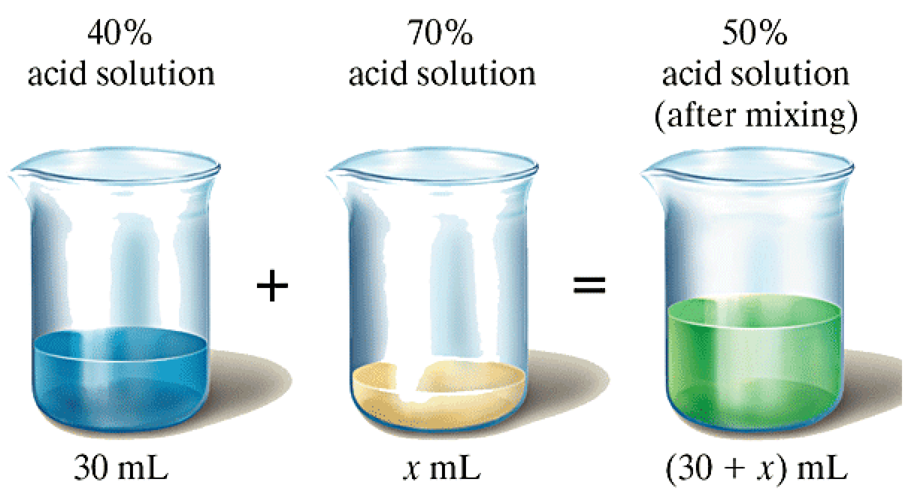
Solving Applications Involving Decimal Equations (Money, Mixture, Interest)

Example

How many milliliters of a 70% acid solution must be mixed with 30 mL of a 40% acid solution to obtain a mixture that is 50% acid?

Step 1. We are mixing two solutions together to obtain a third solution.

Step 2. The unknown quantity in this problem is the amount (in mL) of a 70% acid solution. So, let x = amount of a 70% acid solution. A diagram may help set up the required equation.



Solving Applications Involving Decimal Equations (Money, Mixture, Interest)

Example

Step 3. Solve

$$.40(30)+.70x=.50(30+x)$$
$$12+.7x=15+.5x$$
$$.2x=3$$
$$x=15$$

Therefore, we must mix 15 mL of 70% solution to reach the desired mixture.

Step 4. Check:

$$.40(30)+.70(15)=.50(45)$$
$$12+10.5=22.5$$
$$22.5=22.5$$

Solving Applied Problems Involving Distance, Rate, and Time

Example

An airplane that can maintain an average velocity of 320 mph in still air is transporting smokejumpers to a forest fire. On takeoff from the airport, it encounters a headwind and takes 34 minutes to reach the jump site. The return trip from the jump site takes 30 minutes. What is the speed of the wind? How far is it from the airport to the fire?

Step 1. We are asked to find the speed of the wind, and then to find the distance from the airport to the forest fire.

Solving Applied Problems Involving Distance, Rate, and Time

Example

Step 2. Let w = speed of the wind.
The plane flies 320 mph in still air (with no wind). The net rate (speed) of the plane into a headwind can be described as 320 - w. Similarly, the net speed of the plane flying with the tailwind is 320 + w.
We must change the time from minutes to hours by dividing by 60.

	Rate	Time (hours)	Distance
Headwind	$320 - w$	$\frac{17}{30}$	$\frac{17}{30}(320 - w)$
Tailwind	$320 + w$	$\frac{1}{2}$	$\frac{1}{2}(320 + w)$

Solving Applied Problems Involving Distance, Rate, and Time

Example

Step 2. continued

Because both distances are the same, we can equate the two distances:

$$\underbrace{\frac{17}{30}(320 - w)}_{\substack{\text{distance traveled} \\ \text{by the plane into} \\ \text{the headwind}}} = \underbrace{\frac{1}{2}(320 + w)}_{\substack{\text{distance traveled} \\ \text{by the plane with} \\ \text{the tailwind}}}$$

Step 3. Solve:

$$\frac{17}{30}(320 - w) = \frac{1}{2}(320 + w)$$

$$17(320 - w) = 15(320 + w)$$ Multiply both sides by 30

$$5440 - 17w = 4800 + 15w$$

$$640 = 32w$$

$$20 = w$$

Solving Applied Problems Involving Distance, Rate, and Time

Example

Step 4. To answer the question, the wind is blowing at a speed of 20 mph. We are also asked to find the distance from the airport to the fire. We can substitute $w = 20$ into either distance expression. Substituting $w = 20$ into the distance traveled with the tailwind expression, we see that the distance from the airport to the fire is as follows:

$$\frac{1}{2}(320+20)=\frac{1}{2}(340)=170 \text{ mi}$$

Solving Applied Working Together Problems

Example

Brad and Michelle decide to paint the entire upstairs of their new house. Brad can do the job by himself in 8 hours. If it took them 3 hours to paint the upstairs together, how long would it have taken Michelle to paint it by herself?

Step 1. We are asked to find the time that it would take Michelle to do the job by herself.

Step 2. Let t = time it takes for Michelle to complete the job; thus, she can complete $\frac{1}{t}$ of the job in an hour. Brad can do this job in 8 hours by himself, so he can complete $\frac{1}{8}$ of the job per hour. Working together it took them 3 hours to complete the job.

Solving Applied Working Together Problems

Example

Step 2. continued

We can complete the following table:

	Time needed to complete the job in hours	Portion of job completed in 1 hour (rate)
Brad	8	$\frac{1}{8}$
Michelle	t	$\frac{1}{t}$
Together	3	$\frac{1}{3}$

Solving Applied Working Together Problems

Example

Step 3. From the table we can see that 1/3 = the portion completed in 1 hour together. Adding Brad and Michelle's rate together, we get another expression describing the portion of the job completed together in 1 hour, Thus equating these rates we obtain:

$$\left(\frac{1}{8}+\frac{1}{t}\right)=\frac{1}{3}$$

$$24t\left(\frac{1}{8}+\frac{1}{t}\right)=24t\left(\frac{1}{3}\right)$$

$$3t+24=8t$$

$$24=5t$$

$$\frac{24}{5}=t$$

S0, $t=\frac{24}{5}=4\frac{4}{5}$ hours

or 4 hours and 48 minutes.