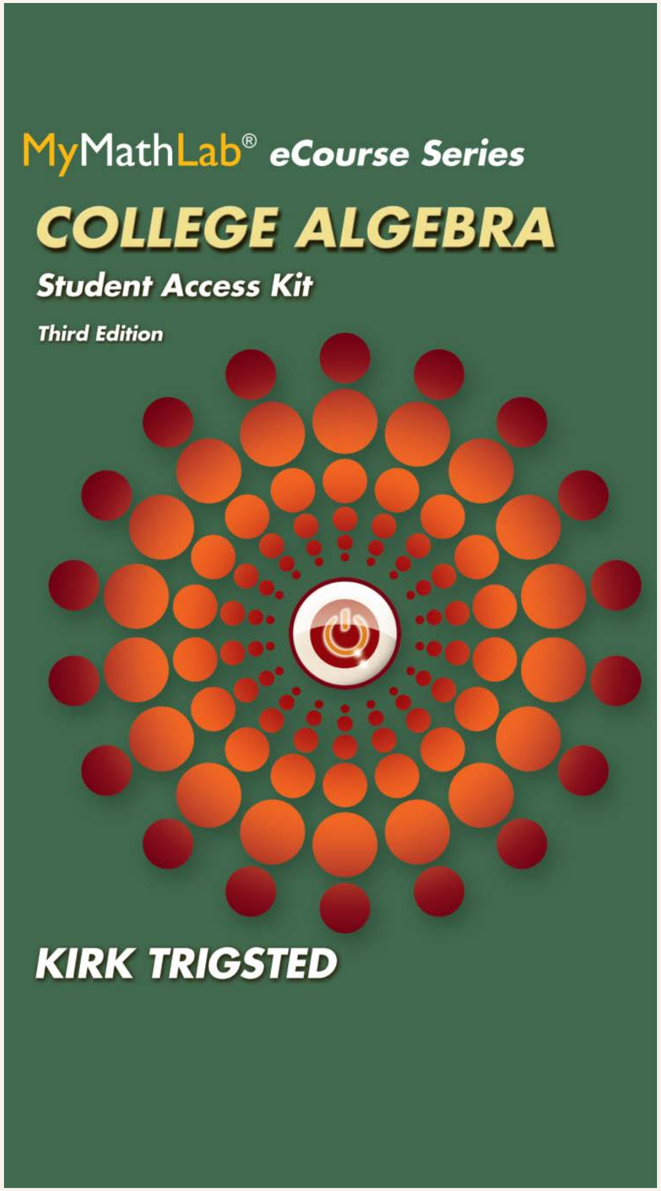
Topic 1.4

Quadratic Equations



OBJECTIVES

1. Solving Quadratic Equations by Factoring and the Zero Product Property
2. Solving Quadratic Equations using the Square Root Property
3. Solving Equations by Completing the Square
4. Solving Equations by Using the Quadratic Formula
5. Using the Discriminant to Determine the Types of Solutions of a Quadratic Equation.

Solving Quadratic Equations by Factoring and the Zero Product Property

Definition: Quadratic Equation in One Variable

A **quadratic equation in one variable** is an equation that can be written in the form

$$ax^2 + bx + c = 0 \ , a \neq 0.$$

- Quadratic equations in this form are said to be in *standard form*.

Solving Quadratic Equations by Factoring and the Zero Product Property

Zero Product Property

If $AB = 0$, then $A = 0$ or $B = 0$.

Solving Quadratic Equations by Factoring and the Zero Product Property

Example Solve $6x^2 - 17x = -12.$

$$6x^2 - 17x + 12 = 0$$

$$(3x - 4)(2x - 3) = 0$$

$$3x - 4 = 0 \quad \text{or} \quad 2x - 3 = 0$$

$$3x = 4 \quad \text{or} \quad 2x = 3$$

$$x = \frac{4}{3} \quad \text{or} \quad x = \frac{3}{2}$$

The solution is $\left\{\frac{4}{3}, \frac{3}{2}\right\}$.

Solve Quadratic Equations Using the Square Root Property

Square Root Property

The solution to the quadratic equation

$x^2 - c = 0$, or equivalently $x^2 = c$,

is $x = \pm\sqrt{c}$.

Solve Quadratic Equations Using the Square Root Property

EXAMPLE

Solve using the square root property:

a. $x^2 - 16 = 0$

$$x^2 = 16$$

$$x = \pm 4$$

b. $2x^2 + 72 = 0$

$$2x^2 = -72$$

$$x^2 = -36$$

$$x = \pm\sqrt{-36}$$

$$x = \pm 6i$$

c. $(x-1)^2 = 7$

$$x - 1 = \pm\sqrt{7}$$

$$x = 1 \pm \sqrt{7}$$

Solving Quadratic Equations by Completing the Square

EXAMPLE

What number must be added to each binomial to make it a perfect square trinomial?

a. $x^2 - 12x$

$$\left(\frac{1}{2} \cdot -12\right)^2$$

$$= (-6)^2 = 36$$

b. $x^2 + 5x$

$$\left(\frac{1}{2} \cdot 5\right)^2$$

$$= \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

c. $x^2 - \frac{3}{2}x$

$$\left(\frac{1}{2} \cdot -\frac{3}{2}\right)^2$$

$$= \left(-\frac{3}{4}\right)^2 = \frac{9}{16}$$

36 must be added to complete the square.

$\frac{25}{4}$ must be added to complete the square.

$\frac{9}{16}$ must be added to complete the square.

Solving Quadratic Equations by Completing the Square

Every quadratic equation can be written in the form $(x-h)^2 = k$, using a method known as completing the square

Steps for Solving $ax^2 + bx + c = 0,\ a \neq 0$, by Completing the Square

1. If $a \neq 1$, divide the equation by a
2. Move all constants to the right hand side.
3. Take half the coefficient of the x-term, square it, and add it to both sides of the equation
4. The left-hand side is now a perfect square. Rewrite it as a binomial squared.
5. Use the square root property to solve for x.

Solving Quadratic Equations by Completing the Square

EXAMPLE Solve by completing the square.

$$3x^2 - 18x + 19 = 0$$

$$\frac{3x^2}{3} - \frac{18x}{3} + \frac{19}{3} = \frac{0}{3}$$

Divide both sides by 3

$$x^2 - 6x + \frac{19}{3} = 0$$

Simplify

$$x^2 - 6x = -\frac{19}{3}$$

Subtract 19/3 from both sides

$$x^2 - 6x + 9 = -\frac{19}{3} + 9$$

Complete the square:

$$\left(\frac{1}{2}\cdot(-6)\right)^2 = (-3)^2 = 9$$

Solving Quadratic Equations by Completing the Square

EXAMPLE Solve by completing the square.

$$x^2 - 6x + 9 = \frac{8}{3}$$ Simplify

$$(x-3)^2 = \frac{8}{3}$$ Factor

$$x - 3 = \pm\sqrt{\frac{8}{3}}$$ Use the Square Root property

$$x - 3 = \pm\frac{\sqrt{8}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$ Rationalize the denominator

$$x - 3 = \pm\frac{\sqrt{24}}{3}$$ Simplify

$$x - 3 = \pm\frac{2\sqrt{6}}{3}$$

Add 3 to both sides

$$x = 3 \pm \frac{2\sqrt{6}}{3} \text{ or } \frac{9 \pm 2\sqrt{6}}{3}$$

Solving Quadratic Equations Using the Quadratic Formula

Quadratic Formula

The solution to the equation $ax^2 + bx + c = 0,\ a \neq 0,$ is given by the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving Quadratic Equations Using the Quadratic Formula

EXAMPLE Solve by using the quadratic formula.

$3x^2 + 2x - 2 = 0$ $\quad a = 3,\ b = 2,$ and $c = -2.$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{-2 \pm \sqrt{28}}{6}$$

$$x = \frac{-2 \pm 2\sqrt{7}}{6} = \frac{2\left(-1 \pm \sqrt{7}\right)}{6} = \frac{-1 \pm \sqrt{7}}{3}$$

Using the Discriminant to Determine the Type of Solutions of a Quadratic Equation

Discriminant

Given a quadratic equation, $ax^2 + bx + c = 0,\ a \neq 0,$ the expression $D = b^2 - 4ac$ is called the **discriminant**.

If $D > 0$, then the quadratic equation has two real solutions

If $D < 0$, then the quadratic equation has two nonreal solutions

If $D = 0$, then the quadratic equation has exactly one real solution.

Using the Discriminant to Determine the Type of Solutions of a Quadratic Equation

Use the discriminant to determine the number and nature of the solutions to each of the following quadratic equations:

a. $3x^2 + 2x + 2 = 0$

$a = 3,\ b = 2,$ and $c = 2.$

$$\begin{aligned} D &= 2^2 - 4(3)(2) \\ &= 4 - 24 \\ &= -20 \end{aligned}$$

2 nonreal solutions

b. $4x^2 + 1 = 4x$

$$4x^2 - 4x + 1 = 0$$

$a = 4,\ b = -4,$ and $c = 1.$

$$\begin{aligned} D &= (-4)^2 - 4(4)(1) \\ &= 16 - 16 \\ &= 0 \end{aligned}$$

Exactly 1 real solution