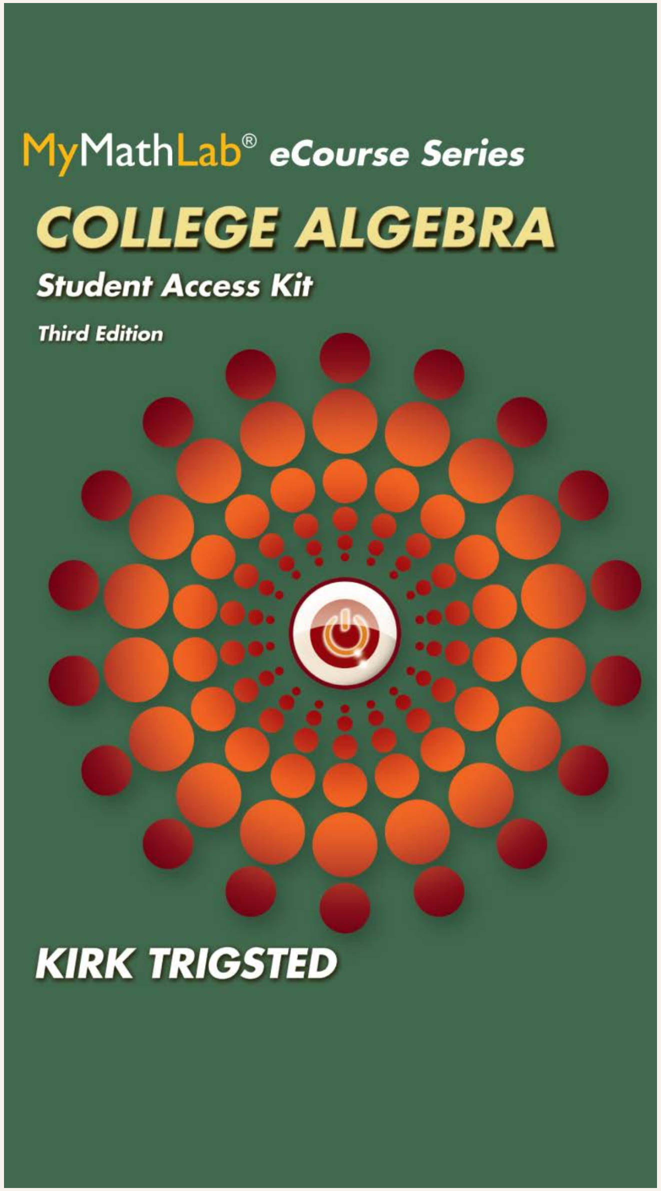
Topic 1.5

Applications of Quadratic Equations



OBJECTIVES

1. Solving Applications Involving Unknown Numeric Quantities
2. Using the Projectile Motion Model
3. Solving Geometric Applications
4. Solving Applications Involving Distance, Rate, and Time
5. Solving Working Together Applications

Solving Applications Involving Unknown Numeric Quantities

Polya's Guidelines for Problem Solving

1. Understand the Problem
   1. Read the problem several times until you have an understanding of what is being asked
2. Devise a Plan
   1. Pick a variable that describes the unknow quantity that is to be found.
   2. Write all other quantities in terms of that variable
   3. Write an equation
3. Carry out the plan
   1. Solve the equation
4. Look Back
   1. Be sure you have answered the question
   2. Check all answers to make sure they make sense

Solving Applications Involving Unknown Numeric Quantities

EXAMPLE

The product of a number and 1 more than twice the number is 36. Find the two numbers.

Step 1. We are looking for two numbers.

Step 2. Let x = the first number,
$2x + 1$ = the other number.

Their product is 36, so we get the equation

$$x(2x+1) = 36 \quad \text{or} \quad 2x^2 + x - 36 = 0$$

Using the Projectile Motion Model

EXAMPLE

A toy rocket is launched at an initial velocity of 14.7 m/s from a 49-m-tall platform. The height h of the object at any time t seconds after launch is given by the equation $h = -4.9t^2 + 14.7h + 49.$ When will the rocket hit the ground?

Step 1. We are looking for the time when h is equal to zero.

Step 2. Set h equal to 0, and solve for t: $0 = -4.9t^2 + 14.7h + 49.$

Using the Projectile Motion Model

EXAMPLE

Step 3. Solve: $0 = -4.9t^2 + 14.7h + 49$

$$0 = t^2 - 3h - 10$$

$$0 = (t - 5)(t + 2)$$

$$t - 5 = 0 \quad \text{or} \quad t + 2 = 0$$

$$t = 5 \quad \text{or} \quad t = -2$$

Step 4. Because *t* represents the time (in seconds) after launch, the value t = -2 seconds does not make sense. Therefore, the rocket will hit the ground 5 seconds after launch.

Solving Geometric Applications

Example

The length of a rectangle is 6 in. less than four times the width. Find the dimensions of the rectangle if the area of the rectangle is 54 in^2.

Step 1. We are looking for the length and the width of the rectangle. If we let the width = w, then the length = $4w - 6$.

Step 2. The area is 54, so the equation is

$$w(4w-6)=54 \quad \text{or} \quad 4w^2-6w-54=0$$

Solving Geometric Applications

Example

Step 3. Solve: $4w^2 - 6w - 54 = 0$

$$2w^2 - 3w - 27 = 0$$

$$(2w - 9)(w + 3) = 0$$

$$2w - 9 = 0 \quad \text{or} \quad w + 3 = 0$$

$$w = \frac{9}{2} \quad \text{or} \quad w = -3$$

Step 4. Because w represents the width, the value w = -3 in. does not make sense. Therefore, the width is 4.5 in. and the length is $4w - 6 = 4(4.5) - 6 = 18 - 6 = 12$ in.

Solving Applications Involving Distance, Rate, and Time

EXAMPLE

Kevin flew his new Cessna O-2A airplane from Jonesburg to Mountainview, a distance of 2,560 miles. The average speed for the return trip was 64 mph faster than the average outbound speed. If the total flying time for the round trip was 18 hours, what was the plane's average speed on the outbound trip from Jonesburg to Mountainview?

Step 1. We are asked to find the average speed of the plane from Jonesburg to Mountainview.

Step 2. Let r = speed of plane on the outbound trip from Jonesburg to Mountainview, and then $r + 64$ = speed of plane on the return trip from Mountainview to Jonesburg.

Solving Applications Involving Distance, Rate, and Time

EXAMPLE

Step 3. Because distance $=$ rate $\times$ time, we know that time $= \dfrac{\text{distance}}{\text{time}}$.

We can organize our data in the following table.

	Distance	Rate	Time = Distance/Rate
Outbound to Mountainview	2,560	r	$\dfrac{2{,}560}{r}$
Return to Jonesburg	2,560	$r + 64$	$\dfrac{2{,}560}{r + 64}$

The total flying time was 18 hours, so we can set up the following equation:

$$\underbrace{\frac{2{,}560}{r}}_{\substack{\text{Outbound}\\ \text{Time}}} + \underbrace{\frac{2{,}560}{r + 64}}_{\substack{\text{Return}\\ \text{Time}}} = \underbrace{18}_{\substack{\text{Total Flight}\\ \text{Time}}}$$

Solving Applications Involving Distance, Rate, and Time

EXAMPLE

Step 3 continued Solve for r: $\dfrac{2,560}{r} + \dfrac{2,560}{r+64} = 18$

$$(r)(r+64)\frac{2,560}{r} + (r)(r+64)\frac{2,560}{r+64} = (r)(r+64)18$$

$$2,560(r+64) + 2,560(r) = 18(r)(r+64)$$

$$2,560r + 163,840 + 2,560r = 18r^2 + 1152$$

$$0 = 18r^2 - 5,120r - 162,688$$

$$r = 256 \text{ or } r = -\frac{320}{9} \approx -35.56$$

Step 4. Because the rate cannot be negative, the average outbound speed to Mountainview was 256 mph.

Solving Working Together Problems

EXAMPLE

Dawn can finish the monthly sales reports in 2 hours less time than it takes Adam. Working together, they were able to finish the sales reports in 8 hours. How long does it take each person to finish the monthly sales reports alone? (Round to the nearest minute.)

Step 1. We are asked to find the time that it takes each person to finish the monthly sales reports.

Step 2. Let t = Adam's time to do the job alone and $t - 2$ = Dawn's time to do the job alone.

Solving Working Together Problems

EXAMPLE

Step 2 continued:

We can now set up a table with the given date.

	Time needed to complete the job, in hours	Portion of job completed in 1 hour (rate)
Adam	t	$\frac{1}{t}$
Dawn	$t-2$	$\frac{1}{t-2}$
Together	8	$\frac{1}{8}$

Which gives us the following equation:

$$\underbrace{\frac{1}{t}}_{\text{Adam's Rate}} + \underbrace{\frac{1}{t-2}}_{\text{Dawn's Rate}} = \underbrace{\frac{1}{8}}_{\text{Rate Together}}$$

Solving Working Together Problems

EXAMPLE

Step 3. Solve for t: $\frac{1}{t}+\frac{1}{t-2}=\frac{1}{8}$

$$(8t)(t-2)\frac{1}{t}+(8t)(t-2)\frac{1}{t-2}=(8t)(t-2)\frac{1}{8}$$

$$(8)(t-2)+(8t)=(t)(t-2)$$

$$8t-16+8t=t^2-2t$$

$$0=t^2-18t+16$$

$$t=9-\sqrt{65}\approx 0.93774 \quad \text{or} \quad t=9+\sqrt{65}\approx 17.062$$

Step 4. Because the time it takes Adam to do the job must be larger than 2 hours larger than Dawn's time, then Adam can do the job by himself in approximately 17 hours and 4 minutes, while it would take Dawn approximately 15 hours and 4 minutes to do the same job.