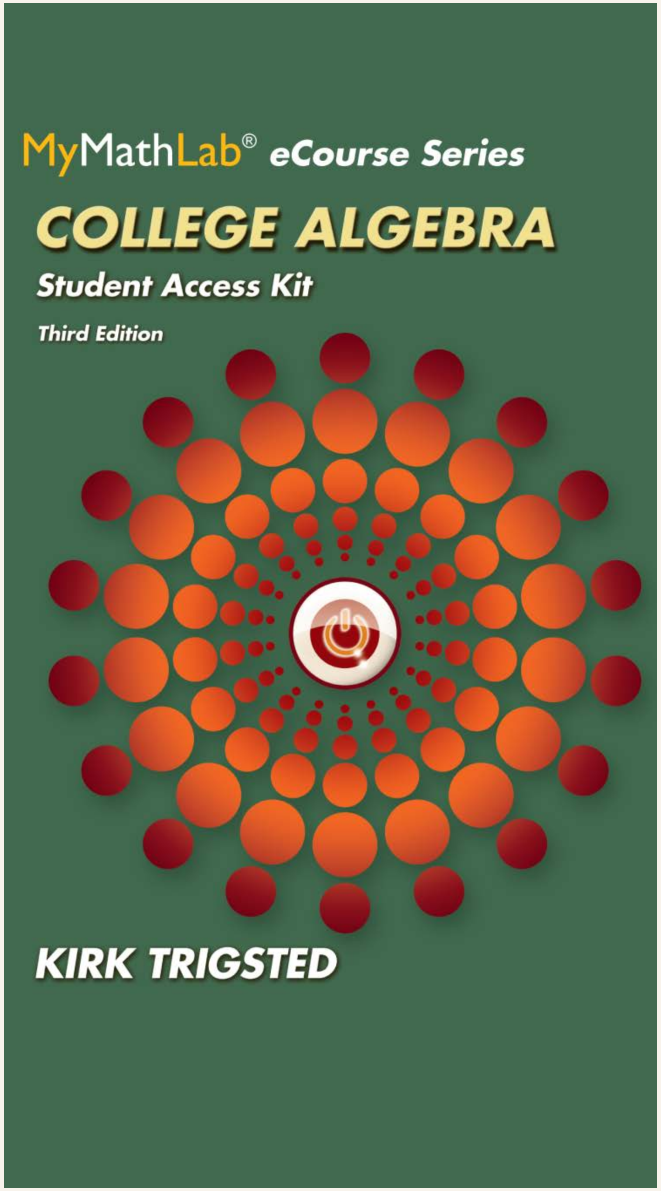
Topic 1.6

Other Types of Equations



OBJECTIVES

1. Solving Higher-Order Polynomial Equations
2. Solving Equations That Are Quadratic in Form (Disguised Quadratics)
3. Solving Equations Involving Radicals

Solving Higher-Order Polynomial Equations

EXAMPLE

Find all solutions of the equation $3x^3 - 2x = -5x^2$.

$3x^3 + 5x^2 - 2x = 0$	Add $5x^2$ to both sides
$x(3x^2 + 5x - 2) = 0$	Factor out x
$x(3x - 1)(x + 2) = 0$	Factor the trinomial
$x = 0$ or $3x - 1 = 0$ or $x + 2 = 0$	Set each factor equal to zero
$x = 0$ or $x = \frac{1}{3}$ or $x = -2$	Solve each equation

The solution set is $\left\{-2, 0, \frac{1}{3}\right\}$.

Solving Higher-Order Polynomial Equations

CAUTION!

In the equation $3x^3 + 5x^2 - 2x = 0$, do not divide both sides by x. This would produce the equation $3x^2 + 5x - 2 = 0$, which has only $\frac{1}{3}$ and -2 as solutions. The solution $x = 0$ would be "lost". In addition, because $x = 0$ is a solution of the original equation, dividing by x would mean dividing by 0, which of course is undefined and produces incorrect results.

Solving Higher-Order Polynomial Equations

EXAMPLE

Find all solutions of the equation $2x^3 - x^2 + 8x - 4 = 0.$

$$\left(2x^3 - x^2\right) + \left(8x - 4\right) = 0$$

Group the terms in pairs

$$x^2\left(2x-1\right) + 4\left(2x-1\right) = 0$$

Factor the GCF from each pair

$$\left(2x-1\right)\left(x^2+4\right) = 0$$

Factor 2x-1 from the left-hand side

$$2x - 1 = 0 \quad \text{or} \quad x^2 + 4 = 0$$

Use the zero product property

$$2x = 1 \quad \text{or} \quad x^2 = -4$$

Solve for x

$$x = \frac{1}{2} \quad \text{or} \quad x = \pm\sqrt{-4} = \pm 2i$$

The solution set is $\left\{-2i, 2\text{i}, \frac{1}{2}\right\}.$

Solving Equations That Are Quadratic in Form (Disguised Quadratics)

EXAMPLE Solve each of the following:

a. $2x^4 - 11x^2 + 12 = 0$

Let $u = x^2$ then $u^2 = x^4$.

$2u^2 - 11u + 12 = 0$

$(2u - 3)(u - 4) = 0$

$2u - 3 = 0$ or $u - 4 = 0$

$u = \frac{3}{2}$ or $u = 4$

$x^2 = \frac{3}{2}$ or $x^2 = 4$

$x = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{6}}{2}$ or $x = \pm 2$

Solving Equations That Are Quadratic in Form (Disguised Quadratics)

EXAMPLE Solve each of the following:

b. $\left(\frac{1}{x-2}\right)^2+\frac{2}{x-2}-15=0$ Let $u=\frac{1}{x-2}$ then $u^2=\left(\frac{1}{x-2}\right)^2$.

$$u^2+2u-15=0$$

$$(u+5)(u-3)=0$$

$$u=-5 \text{ or } u=3$$

$$-5=\frac{1}{x-2} \text{ or } 3=\frac{1}{x-2}$$

$$-5x+10=1 \text{ or } 3x-6=1$$

$$x=\frac{9}{5} \text{ or } x=\frac{7}{3}$$

Solving Equations That Are Quadratic in Form (Disguised Quadratics)

EXAMPLE Solve each of the following:

c. $x^{2/3} - 9x^{1/3} + 8 = 0$

Let $u = x^{1/3}$ then $u^2 = x^{2/3}$.

$$u^2 - 9u + 8 = 0$$

$$(u-8)(u-1) = 0$$

$$u - 8 = 0 \quad \text{or} \quad u - 1 = 0$$

$$u = 8 \quad \text{or} \quad u = 1$$

$$8 = x^{1/3} \quad \text{or} \quad 1 = x^{1/3}$$

$$512 = x \quad \text{or} \quad 1 = x$$

Solving Equations Involving Radicals

EXAMPLE Solve $\sqrt{x-1}-2=x-9$

$$\sqrt{x-1}=x-7$$

$$\left(\sqrt{x-1}\right)^2=(x-7)^2$$

$$x-1=x^2-14x+49$$

$$0=x^2-15x+50$$

$$0=(x-10)(x-5)$$

$$0=x-10 \quad \text{or} \quad 0=x-5$$

$$10=x \quad \text{or} \quad 5=x$$

CHECK:

$$\sqrt{10-1}-2=10-9$$

$$\sqrt{9}-2=1$$

$$3-2=1$$

$$1=1$$

$$\sqrt{5-1}-2=5-9$$

$$\sqrt{4}-2=-4$$

$$2-2=-4$$

$$0\neq-4$$

The only solution that checks is x = 10.

Solving Equations Involving Radicals

CAUTION!

Because the squaring operation can make a false statement true ($-2 \neq 2$, but $(-2)^2 = (2)^2$), it is essential to always check your answers after solving an equation in which this operation was performed. The squaring operation is usually used when solving an equation involving square roots.