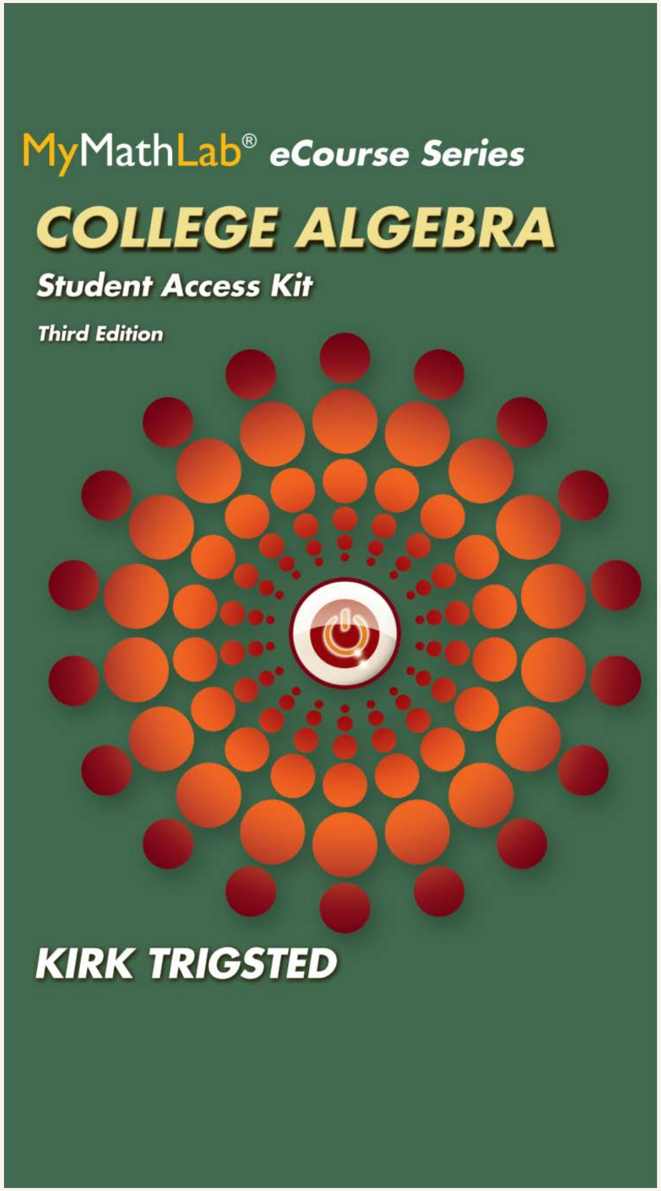
Topic 3.2

Properties of a Function's Graph



OBJECTIVES

1. Determining the Intercepts of a Function
2. Determining the Domain and Range of a Function from its Graph
3. Determining Whether a Function Is Increasing, Decreasing, or Constant
4. Determining Relative Maximum and Relative Minimum Values of a Function
5. Determine Whether a Function Is Even, Odd, or Neither
6. Determining Information about a Function from a Graph

Definition of an Intercept

An **intercept** of a function is a point on the graph of a function where the graph either crosses or touches a coordinate axis. There are two types of intercepts:

1. The y -intercept, which is the y -coordinate of the point where the graph crosses or touches the y –axis

2. The x -intercepts, which are the x -coordinates of the points where the graph crosses or touches the x -axis

y-intercept

A function can have *at most* one y -intercept. The y -intercept exists if $x = 0$ is in the domain of the function. The y -intercept can be found by evaluating $f(0)$.

Determining the Intercepts of a Function

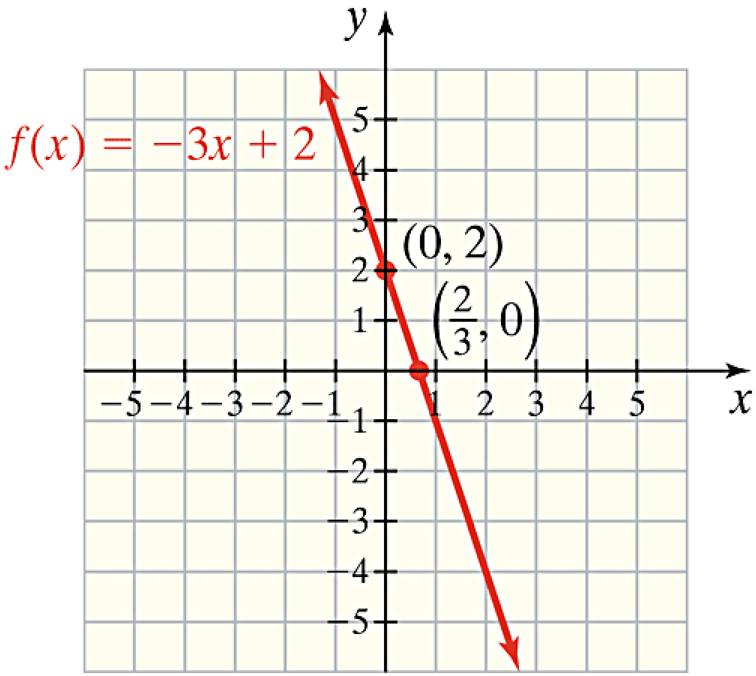
EXAMPLE

Find the y-intercept of the function $f(x) = -3x + 2$.

Because $f(x) = -3x + 2$ is a polynomial function, we know that the domain of f is comprised of all real numbers.

Thus, $x = 0$ is in the domain of f. The y-intercept is $f(0) = -3(0) + 2 = 2$.

The y-intercept is at the point (0,2).



x-intercept

A function may have several (even infinitely many) x-intercepts. The x-intercepts, also called **real zeros**, can be found by finding all *real solutions* to the equation $f(x)=0$.

Although a function may have several zeros, only the real zeros are x-intercepts.

Determining the Intercepts of a Function

EXAMPLE

Find all intercepts of the function $f(x) = x^3 - 2x^2 + x - 2.$

Because $f(0) = -2$, the y-intercept is -2.

To find the x-intercepts, let $f(x) = x^3 - 2x^2 + x - 2 = 0.$

$$x^3 - 2x^2 + x - 2 = 0$$

$$x^2(x-2) + 1(x-2) = 0$$

$$(x-2)(x^2+1) = 1$$

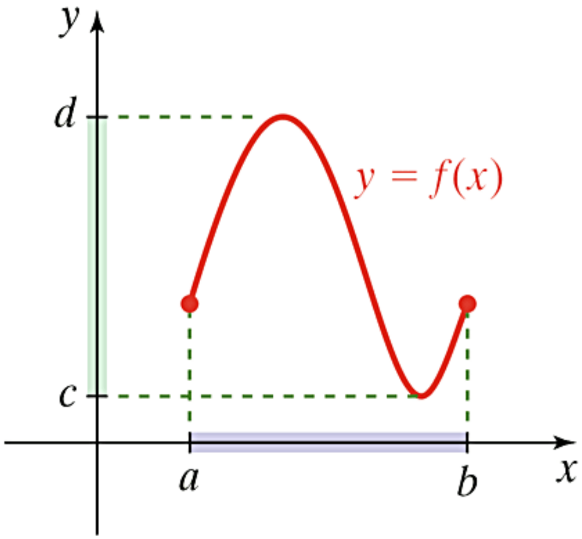
$$x = 2 \text{ or } x = \pm i$$

Because $x = 2$ is the only real solution, then $x = 2$ is the only x-intercept.

Determining the Domain and Range of a Function from Its Graph

The graph illustrates how we can use a function's graph to determine the domain and range. The domain is the set of all input values (x -values), and the range is the set of all output values (y -values).

In the figure, the domain is the interval [*a*, *b*] while the range is the interval [*c*, *d*].

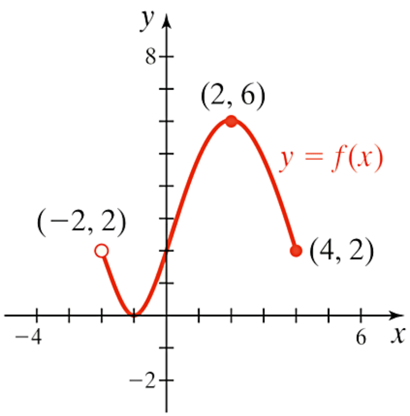


Determining the Domain and Range of a Function from Its Graph

EXAMPLE

Use the graph of the following functions to determine the domain and range.

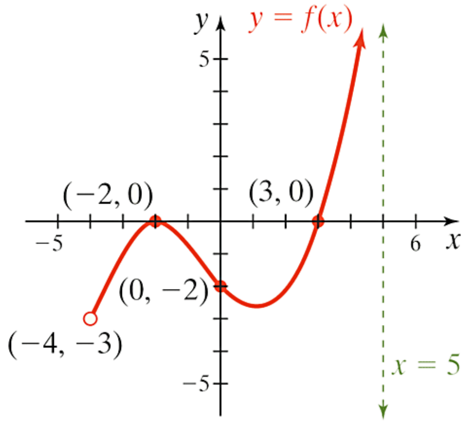
a.



Domain: $(-2, 4]$

Range: $[0, 6]$

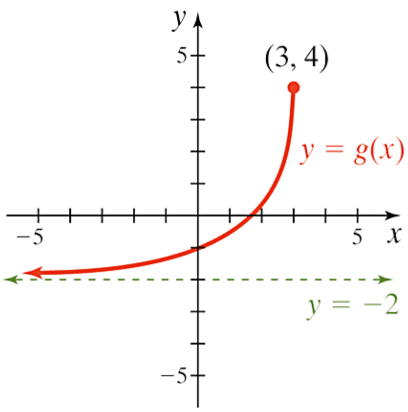
b.



Domain: $(-4, 5)$

Range: $(-3, \infty)$

c.



Domain: $(-\infty, 3]$

Range: $(-2, 4]$

Determining Whether a Function Is Increasing, Decreasing, or Constant

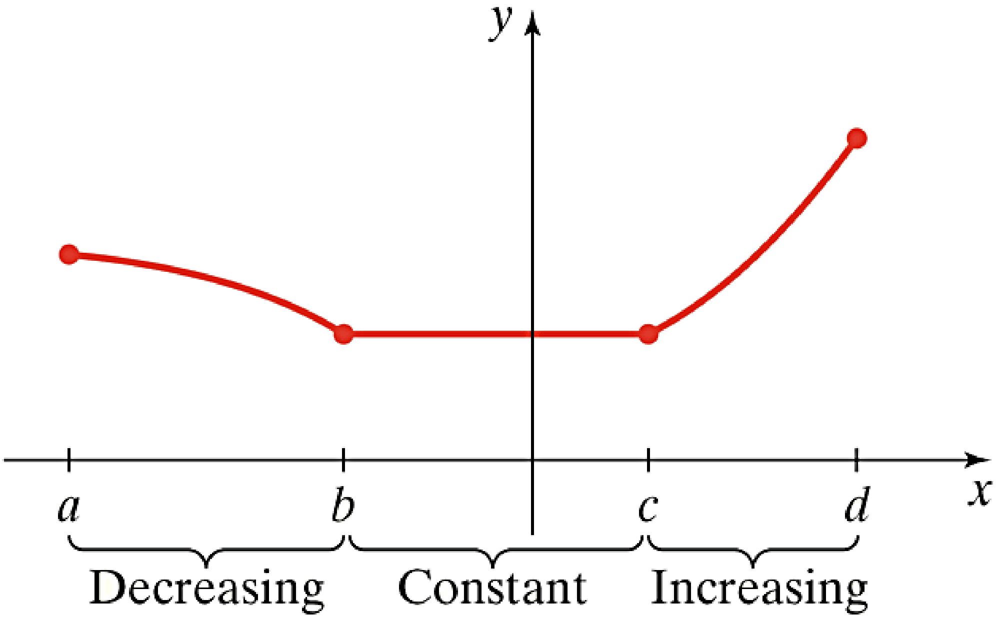
A function f is said to be **increasing** on an open **interval** if the value of $f(x)$ gets larger as x gets larger on the interval. The graph of f rises from left to right on the interval in which f is increasing.

Likewise, a function f is said to be **decreasing** on an open interval if the value of $f(x)$ gets smaller as x gets larger on the interval. The graph of f falls from left to right on the interval in which f is decreasing.

A graph is **constant** on an open interval if the values of $f(x)$ do not change as x gets larger on the interval. In this case, the graph is a horizontal line on the interval.

Determining Whether a Function Is Increasing, Decreasing, or Constant

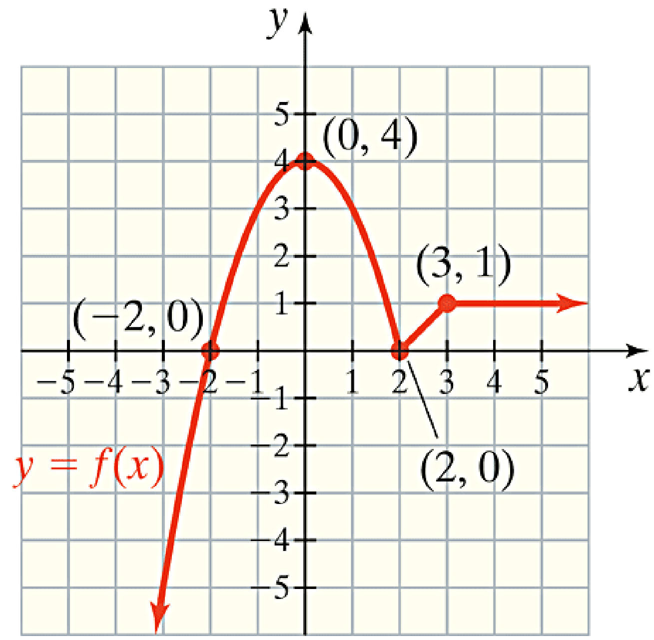
The function is increasing on the interval (c,d).
The function is decreasing on the interval (a,b).
The function is constant on the interval (b,c).



Determining Whether a Function Is Increasing, Decreasing, or Constant

EXAMPLE

Given the graph of $y = f(x)$, determine whether the function is increasing, decreasing or constant.



The function is increasing on the interval $(-\infty, 0)$ and on the interval (2, 3).

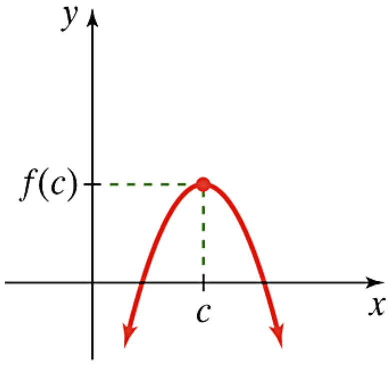
The function is decreasing on the interval (0,2).

The function is constant on the interval $(3, \infty)$.

Determining Relative Maximum and Relative Minimum Values of a Function

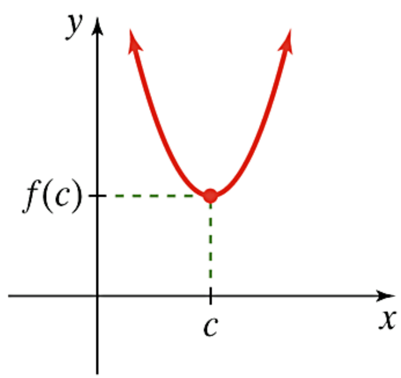
When a function changes from increasing to decreasing at a point $(c, f(c))$, then f is said to have a relative maximum at $x = c$. The relative maximum value is $f(c)$.

(a)



The relative maximum occurs at $x = c$, and the relative maximum value is $f(c)$.

(b)

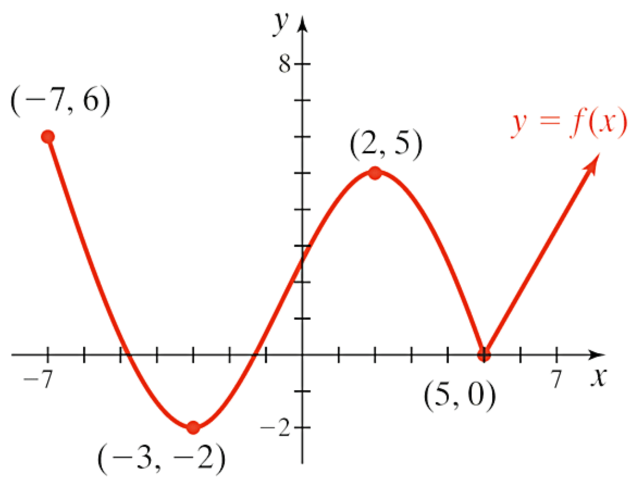


The relative minimum occurs at $x = c$, and the relative minimum value is $f(c)$.

Similarly, when a function changes from decreasing to increasing at a point $(c, f(c))$, then f is said to have a relative minimum at $x = c$. The relative minimum value is $f(c)$.

Determining Relative Maximum and Relative Minimum Values of a Function

EXAMPLE Use the graph of y=$f(x)$ to answer each question.



a. On what interval(s) is f increasing?

$(-3, 2)$ and $(5, \infty)$

b. On what interval(s) is f decreasing?

$(-7, -3)$ and $(2, 5)$

c. For what value(s) of x does is f have a relative minimum?

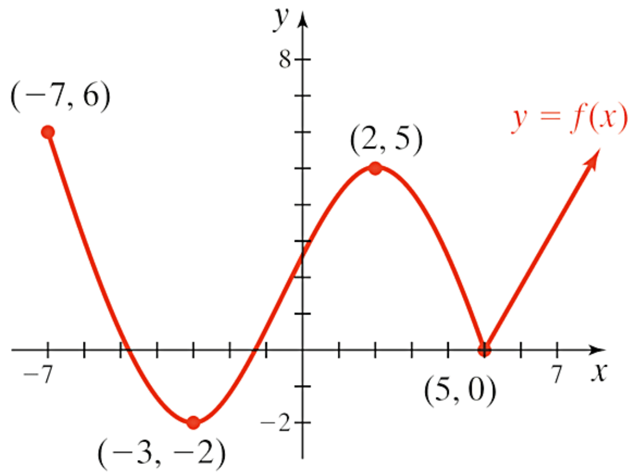
$x = -3$ and $x = 5$

d. For what value(s) of x does is f have a relative maximum?

$x = 2$

Determining Relative Maximum and Relative Minimum Values of a Function

EXAMPLE Use the graph of y=$f(x)$ to answer each question.



e. What are the relative minima?

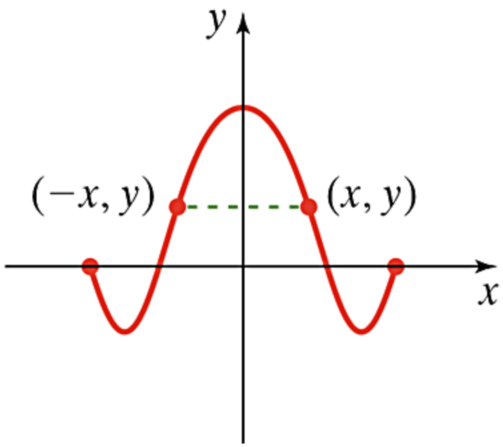
$f(-3) = -2$ and $f(5) = -0$

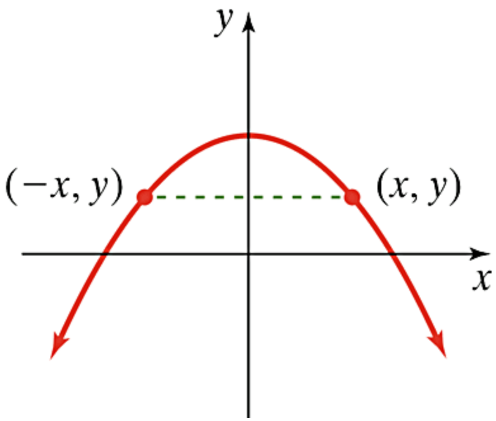
f. What are the relative maxima?

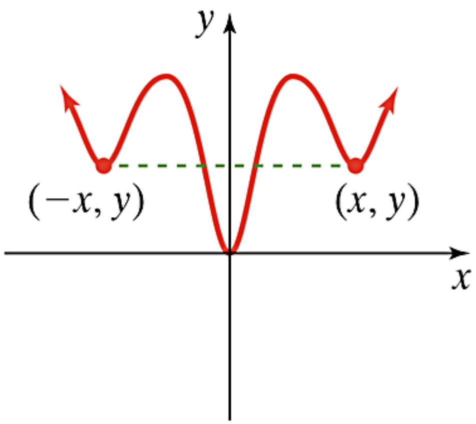
$f(2) = 5$

Determining Whether a Function is Even, Odd, or Neither

Functions whose graphs are symmetric about the y -axis are called **even** functions



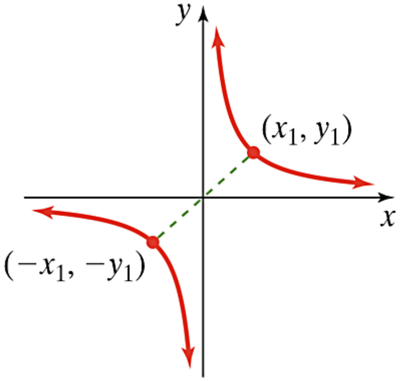


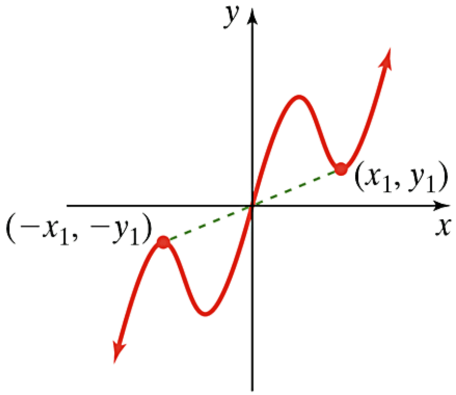


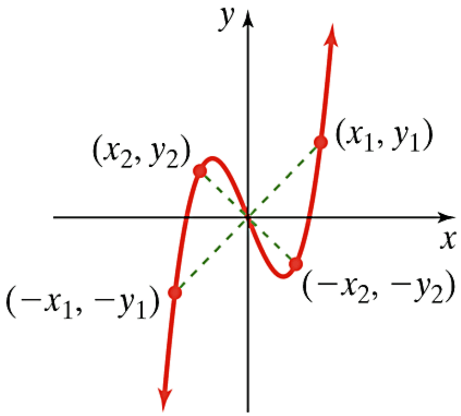
For any point (x, y) on each graph, the point (-x, y) also lies on the graph. Therefore, for any x -value in the domain, $f(x)=f(-x)$.

Determining Whether a Function is Even, Odd, or Neither

Functions whose graphs are symmetric about the origin are called **odd** functions







If a function is an odd function, then for any point (x_1, y_1) on the graph of f in Quadrant I, there is a corresponding point $(-x_1, -y_1)$ on the graph of f in Quadrant III.

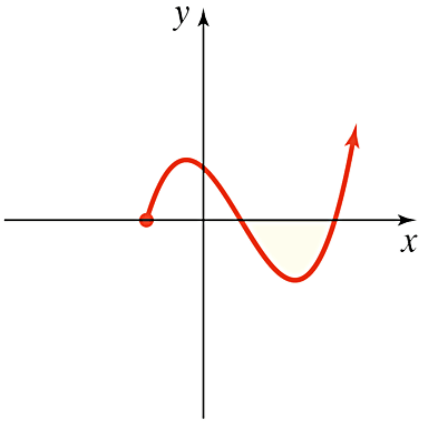
Similarly, for any point (x_2, y_2) on the graph of f in Quadrant II, there is a corresponding point $(-x_2, -y_2)$ on the graph of f in Quadrant IV.

Determining Whether a Function is Even, Odd, or Neither

EXAMPLE

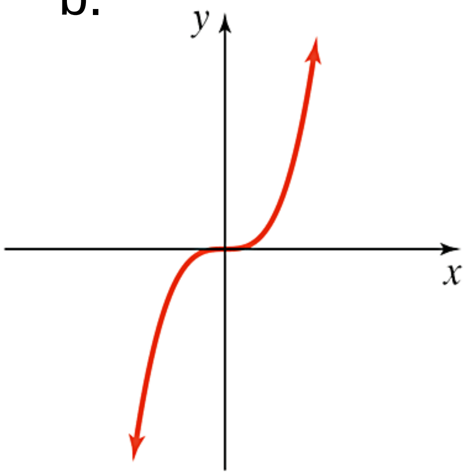
Determine whether each function is even, odd, or neither.

a.



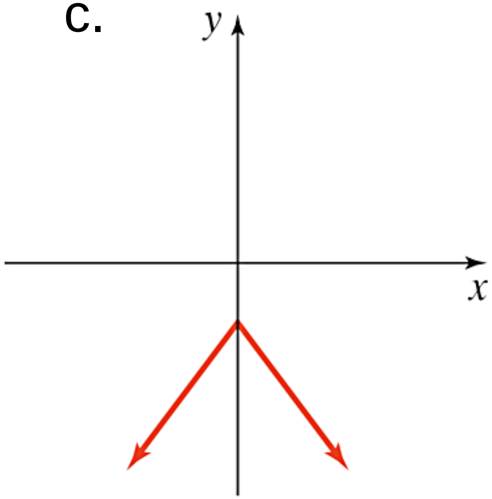
neither

b.



odd

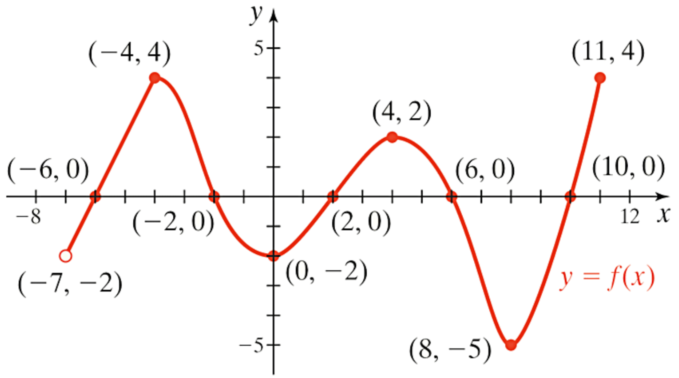
c.



even

Determining Information about a Function from a Graph

EXAMPLE Use the graph of $y=f(x)$ to answer each question.



a. What is the y-intercept?

$(0, -2)$

b. What are the real zeros of f?

$-6, -2, 2, 6,$ and 10

c. Determine the domain and range of f.

Domain: $(-7, 11]$

Range: $[-5, 4]$

d. Determine the interval(s) on which f is increasing, decreasing, and constant.

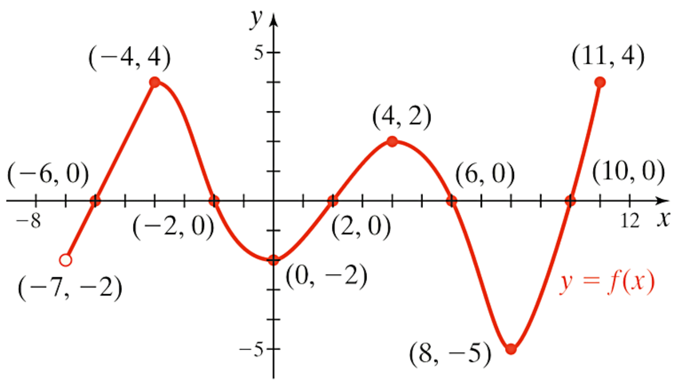
Increasing: $(-7, -4)$ and $(0, 4)$ and $(8, 11)$

Decreasing: $(-4, 0)$ and $(4, 8)$

Constant: never

Determining Information about a Function from a Graph

EXAMPLE Use the graph of $y=f(x)$ to answer each question.



e. For what value(s) of x does f obtain a relative maximum? What are the relative maxima?

$x = -4$ and $x = 4$

$f(-4) = 4$ and $f(4) = 2$

f. For what value(s) of x does f obtain a relative minimum? What are the relative minima?

$x = 0$ and $x = 8$

$f(0) = -2$ and $f(8) = -5$

g. Is f even, odd or neither?

neither