Topic 3.4

Transformation of Functions

MyMathLab[®] eCourse Series **COLLEGE ALGEBRA** Student Access Kit Third Edition KIRK TRIGSTED

OBJECTIVES

- 1. Using Vertical Shifts to Graph Functions
- 2. Using Horizontal Shifts to Graph Functions
- 3. Using Reflections to Graph Functions
- 4. Using Vertical Stretches and Compressions to Graph Functions
- Using Horizontal Stretches and Compressions to Graph Functions
- 6. Using Combinations of Transformations to Graph Functions

Vertical Shifts of Functions

If c is a positive real number

•The graph of y = f(x) + c is obtained by shifting the graph of y = f(x) vertically upward c units.

•The graph of y = f(x) - c is obtained by shifting the graph of

y = f(x) vertically downward c units.



Vertical Shifts of Functions EXAMPLE

Sketch the graphs of f(x) = |x| and f(x) = |x|+2.

x	f(x) = x	g(x) = x + 2
-3	3	5
-2	2	4
-1	1	3
0	0	2
1	1	3
2	2	4
3	3	5



Horizontal Shifts of Functions

If c is a positive real number

•The graph of y = f(x + c) is obtained by shifting the graph of y = f(x) horizontally to the left c units.

•The graph of y = f(x - c) is obtained by shifting the graph of y = f(x) horizontally to the right c units.



Horizontal Shifts of Functions

Sketch the graphs of $f(x) = x^2$ and $f(x) = (x+2)^2$.



-5 - 4 - 3 - 2 - 2

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3

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Reflections of Functions about the *x*-Axis



The graph of y = -f(x) is obtained by reflecting the graph of y = f(x) about the *x*-axis.



Reflections of Functions about the *y***-Axis**

The graph of y = f(-x) is obtained by reflecting the graph of y = f(x) about the *y*-axis.



EXAMPLE

Use the graph of the basic function $y = \sqrt[3]{x}$ to sketch each graph.

a.
$$g(x) = -\sqrt[3]{x} - 2$$





EXAMPLE

Use the graph of the basic function $y = \sqrt[3]{x}$ to sketch each graph.

b.
$$h(x) = \sqrt[3]{1-x}$$



Start with the graph of the basic function $y = \sqrt[3]{x}$.







Using Vertical Stretches and Compressions to Graph Functions

Suppose *a* is a positive real number:

- •The graph of y = af(x) is obtained by multiplying each y-coordinate of y = f(x) by a.
- •If a >1, the graph of y = af(x) is a vertical stretch of y = f(x)
- •If 0 < a < 1, the graph of y = af(x) is a vertical compression of y = f(x)



Using Vertical Stretches and Compressions to Graph Functions

EXAMPLE Use the graph of the graph of $f(x) = x^2$ to sketch the graph of $g(x) = 2x^2$.

x	$f(x) = x^2$	$g(x) = 2x^2$
-2	4	8
-1	1	2
0	0	0
1	1	2
2	4	8



Using Vertical Stretches and Compressions to Graph Functions

Use the graph of the graph of $f(x) = x^2$ to

sketch the graph of $h(x) = \frac{1}{2}x^2$.

x	$f(x) = x^2$	$h(x) = \frac{1}{2}x^2$
-2	4	2
-1	1	$\frac{1}{2}$
0	0	0
0	1	$\frac{1}{2}$
2	4	2



Use Horizontal Stretches and Compressions to Graph Functions

Suppose *a* is a positive real number:

- •The graph of y = f(ax) is obtained by dividing each x-coordinate of y = f(x) by a.
- •If a >1, the graph of y = f(ax) is a horizontal compression of y = f(x)
- •If 0 < a < 1, the graph of y = f(ax) is a horizontal stretch of y = f(x)



Use Horizontal Stretches and Compressions to Graph Functions EXAMPLE

Use the graph of the graph of $f(x) = \sqrt{x}$ to

sketch the graph of $g(x) = \sqrt{4x}$ and $h(x) = \sqrt{\frac{1}{4}x}$.



"Order of Operations" for Combining Transformations

- 1. Horizontal Shifts
- 2. Horizontal stretches/compressions
- 3. Reflection about y-axis
- 4. Vertical stretches/compressions
- 5. Reflection about x-axis
- 6. Vertical shifts

EXAMPLE Use transformations to sketch the graph of

 $f(x) = -2(x+3)^2 - 1.$

- 1. Horizontal Shift: (*x*+3) indicates a shift left 3 units
- 2. Horizontal stretches/compressions: not applicable
- 3. Reflection about *y*-axis: not applicable
- 4. Vertical stretches/compressions: stretches by a factor of 2 units
- 5. Reflection about *x*-axis: -2 indicates a reflection about the *x*-axis
- 6. Vertical shifts: -1 indicates it shifts down 1 unit



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