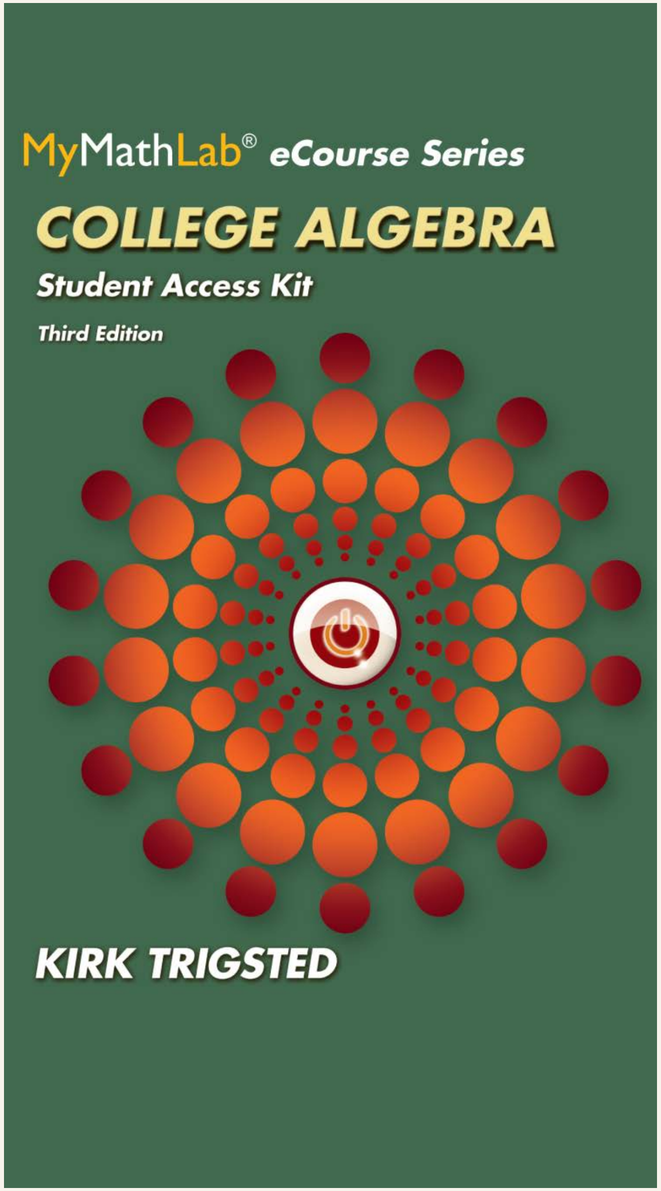
Topic 4.1

Quadratic Functions

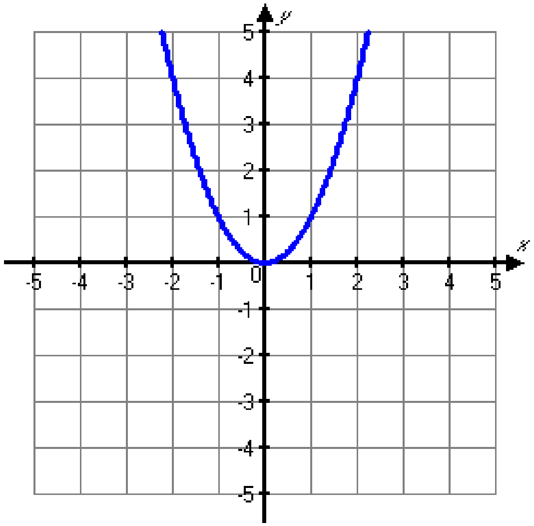


OBJECTIVES

1. Understanding the Definition of a Quadratic Function and Its Graph
2. Graphing Quadratic Functions Written in Standard Form
3. Graphing Quadratic Functions by Completing the Square
4. Graphing Quadratic Functions Using the Vertex Formula
5. Determining the Equation of a Quadratic Function Given Its Graph

Quadratic Function

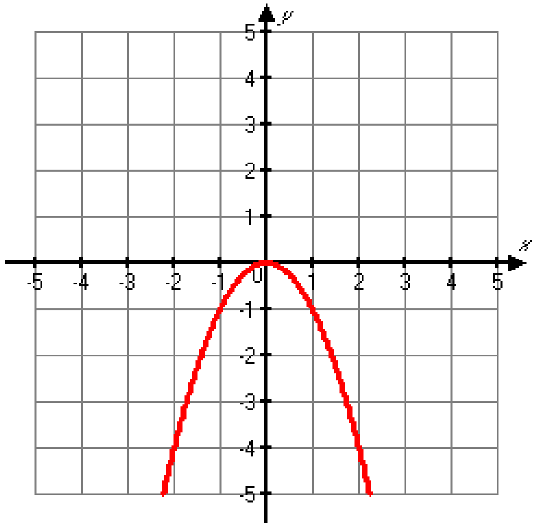
A **quadratic function** is a function of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers with $a \neq 0$. Every quadratic function has a "u-shaped" graph called a parabola.



$f(x) = x^2$

If $a > 0$, the parabola will "open up".

If $a < 0$, the parabola will "open down".



$f(x) = -x^2$

Understanding the Definition of a Quadratic Function and Its Graph

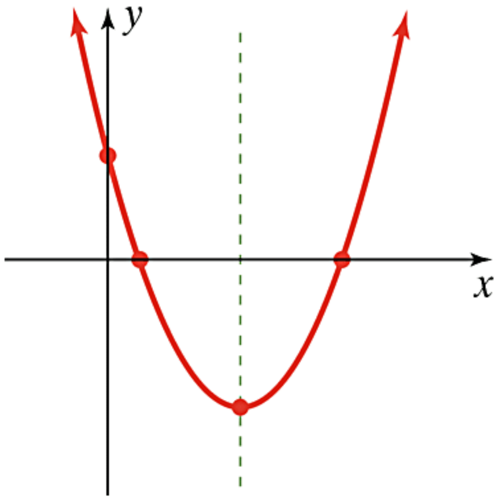
EXAMPLE

Without graphing, determine whether the graph of the quadratic function $f(x) = -3x^2 + 6x + 1$ opens up or down.

Because the leading coefficient is $a = -3 < 0$, the graph of the quadratic Function must open down.

Characteristics of a Parabola

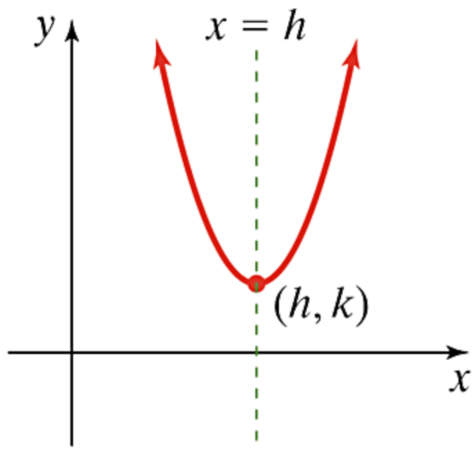
1. Vertex
2. Axis of symmetry
3. *y*-intercept
4. *x*-intercept(s) or real zeros
5. Domain and range



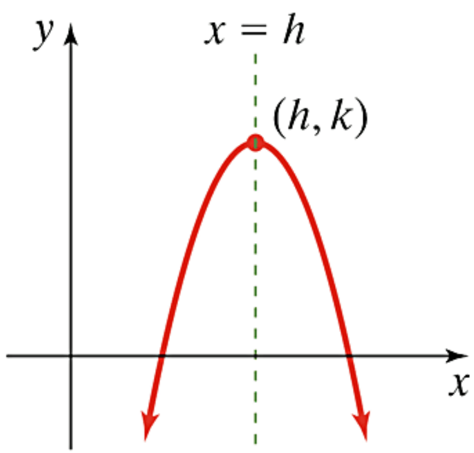
Graphing Quadratics Written In Standard Form

Standard Form of a Quadratic Function

A quadratic function is in **standard form** if it is written as $f(x) = a(x - b)^2 + k$. The graph is a parabola with vertex (h,k). The parabola "opens up" if $a > 0$. The parabola "opens down" if $a < 0$.



$a > 0$
Domain: $(-\infty, \infty)$
Range: $[k, \infty)$



$a < 0$
Domain: $(-\infty, \infty)$
Range: $(-\infty, k]$

Graphing Quadratics Written In Standard Form

EXAMPLE

Given that the quadratic function is in standard form address the following: $f(x) = -(x-2)^2 - 4$

a. Coordinates of vertex

(2,-4)

b. Does the graph open "up" or "down"

a < 0, so it "opens down"

c. Axis of symmetry

The axis of symmetry is $x = 2$

Graphing Quadratics Written In Standard Form

EXAMPLE continued

Given that the quadratic function is in standard form address the following: $f(x) = -(x-2)^2 - 4$

d. x-intercepts

$$
\begin{aligned}
f(x) &= 0 \\
0 &= -(x-2)^2 - 4 \\
4 &= -(x-2)^2 \\
-4 &= (x-2)^2 \\
\pm 2i &= x - 2 \\
2 \pm 2i &= x
\end{aligned}
$$

No x-intercepts

e. y-intercepts

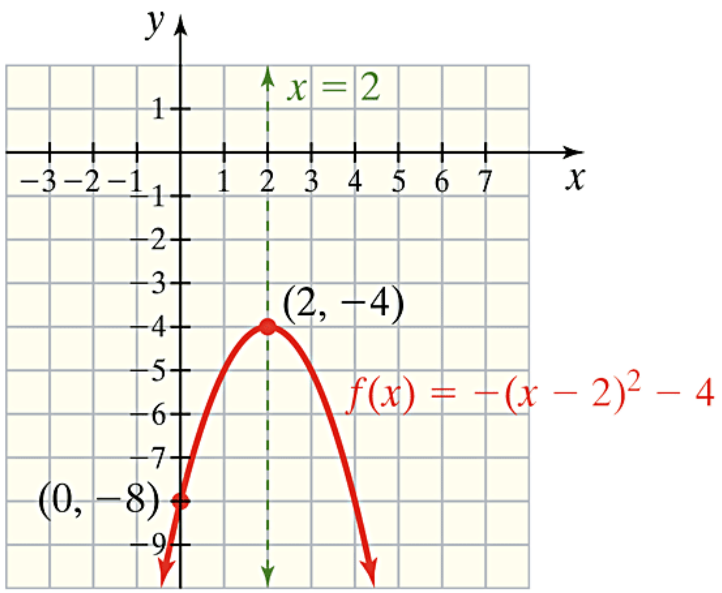
$$
\begin{aligned}
f(0) &= -(0-2)^2 - 4 \\
&= -(-2)^2 - 4 \\
&= -(4) - 4 \\
&= -8
\end{aligned}
$$

Graphing Quadratics Written In Standard Form

EXAMPLE continued

Given that the quadratic function is in standard form address the following: $f(x) = -(x-2)^2 - 4$

f. Sketch the graph



g. State the domain and range in interval notation.

Domain: $(-\infty, \infty)$

Range: $(-\infty, -4]$

Graphing Quadratic Functions by Completing the Square

EXAMPLE

Rewrite the quadratic function in standard form, and then complete a) through g). $f(x) = 2x^2 - 4x - 3$

$f(x) = 2x^2 - 4x - 3$ Original function

$f(x) = (2x^2 - 4x) - 3$ Isolate constant

$f(x) = 2(x^2 - 2x) - 3$ Factor out 2

$f(x) = 2(x^2 - 2x + 1) - 3 - 2$ Complete the square

$f(x) = 2(x - 1)^2 - 5$ Rewrite

Graphing Quadratic Functions by Completing the Square

EXAMPLE continued

Rewrite the quadratic function in standard form, and then complete a) through g). $f(x) = 2(x-1)^2 - 5$

a) The vertex is: (1, -5)

b) The parabola opens: up

c) The equation of the axis of symmetry is: $x=1$

d) The x-intercepts are:

$$0 = 2(x-1)^2 - 5$$

$$5 = 2(x-1)^2$$

$$\frac{5}{2} = (x-1)^2$$

$$\pm\frac{\sqrt{10}}{2} = x - 1$$

$$1 \pm \frac{\sqrt{10}}{2} = x$$

$$x = 1 + \frac{\sqrt{10}}{2} \approx 2.5811 \text{ and}$$

$$x = 1 - \frac{\sqrt{10}}{2} \approx -0.5811$$

Graphing Quadratic Functions by Completing the Square

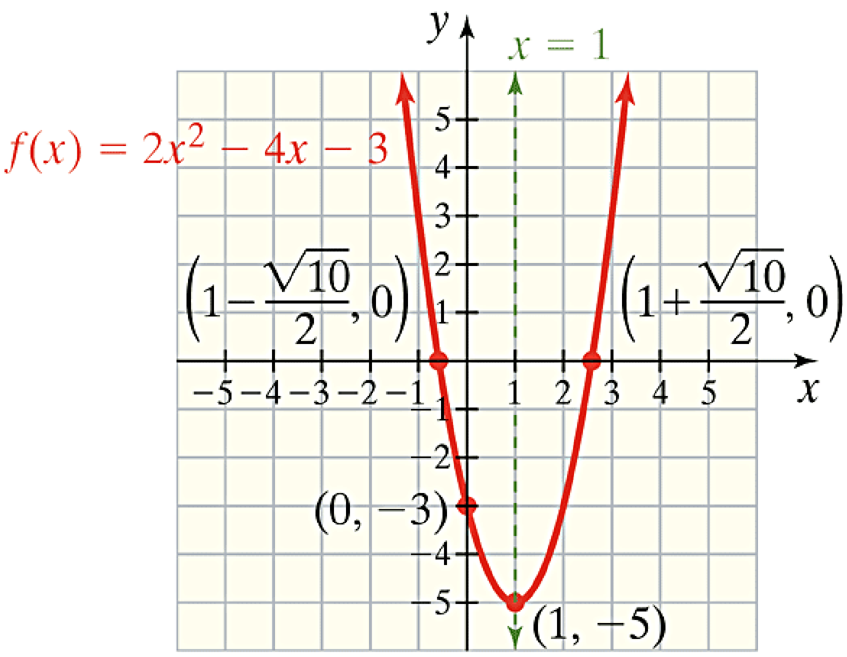
EXAMPLE continued

Rewrite the quadratic function in standard form, and then complete a) through g).

$$f(x) = 2(x-1)^2 - 5$$

e) The y-intercept is: -3

f) The graph is sketched:



g) The domain and range are:

Domain: $(-\infty, \infty)$

Range: $[-5, \infty)$

Graphing Quadratic Functions Using the Vertex Formula

Formula for the Vertex of a Parabola

Given a quadratic function of the form $f(x) = ax^2 + bx + c,\ a \neq 0$, the vertex of the parabola is $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$.

Graphing Quadratic Functions by Using the Vertex Formula

EXAMPLE

Given the quadratic function, address the following:

$$f(x) = -2x^2 - 4x + 5$$

a) Use the vertex formula to determine the vertex.

$$x = -\frac{b}{2a} = -\frac{(-4)}{2(-2)} = -1$$

$$f\left(-\frac{b}{2a}\right) = f(-1) = -2(-1)^2 - 4(-1) + 5 = 7$$

b) Does the graph “open up” or “open down”

down

c) What is the equation of the axis of symmetry?

$x = -1$

Graphing Quadratic Functions by Using the Vertex Formula

EXAMPLE continued

Given the quadratic function, address the following:

$$f(x) = -2x^2 - 4x + 5$$

d) Find the x-intercepts: $f(x) = -2x^2 - 4x + 5 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-2)(5)}}{2(-2)}$$

$$= \frac{4 \pm \sqrt{56}}{4}$$

$$= \frac{4 \pm 2\sqrt{14}}{4}$$

$$= \frac{2 \pm \sqrt{14}}{2}$$

$x = -1 + \dfrac{\sqrt{14}}{2} \approx 0.8708$ and

$x = -1 - \dfrac{\sqrt{14}}{2} \approx -2.8708$

Graphing Quadratic Functions by Completing the Square

EXAMPLE continued

Given the quadratic function, address the following:

$$f(x) = -2x^2 - 4x + 5$$

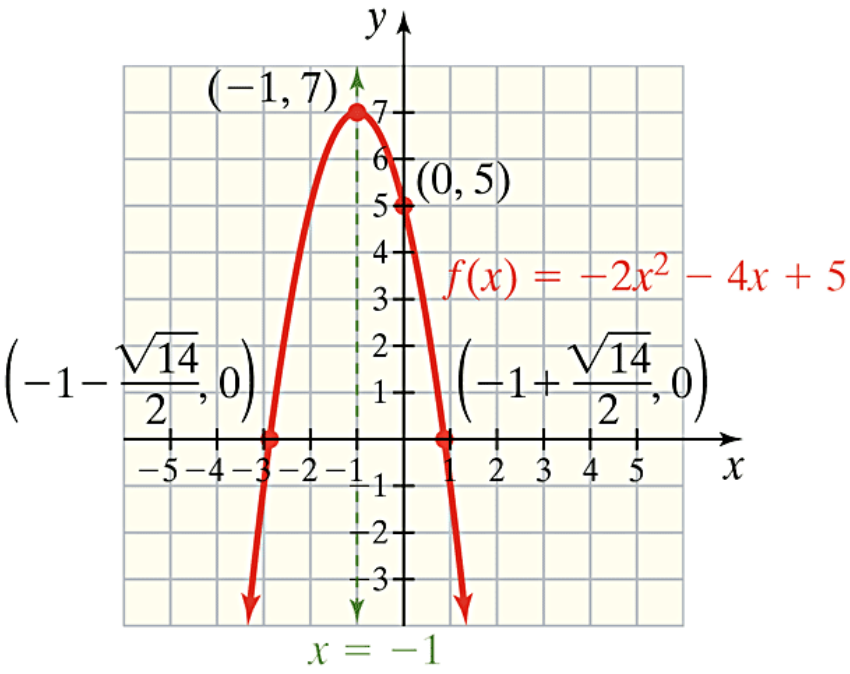
e) Find the y-intercept. 5

f) Sketch the graph.

g) State the domain and range in interval notation

Domain: $(-\infty, \infty)$

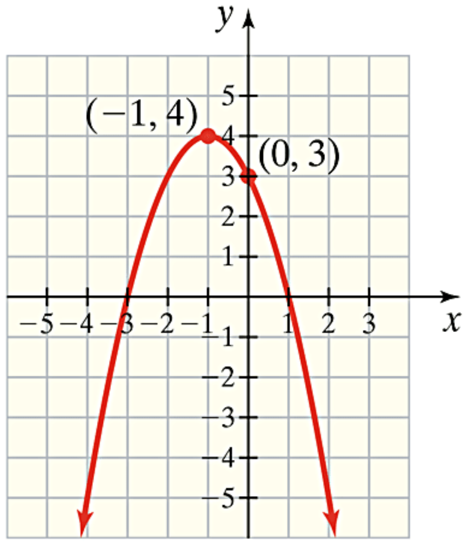
Range: $(-\infty, 7]$



Determine the Equation of a Quadratic Function Given Its Graph.

EXAMPLE

Analyze the graph to address the following about the quadratic function it represents.



a. Is the leading coefficient positive or negative?

negative

b. What is the value of h? What is the value of k?

$$h = -1; k = 4$$

c. What is the value of the leading coefficient a?

$f(x) = a(x - (-1))^2 + 4$

$f(x) = a(x + 1)^2 + 4$

$f(0) = a(0 + 1)^2 + 4$

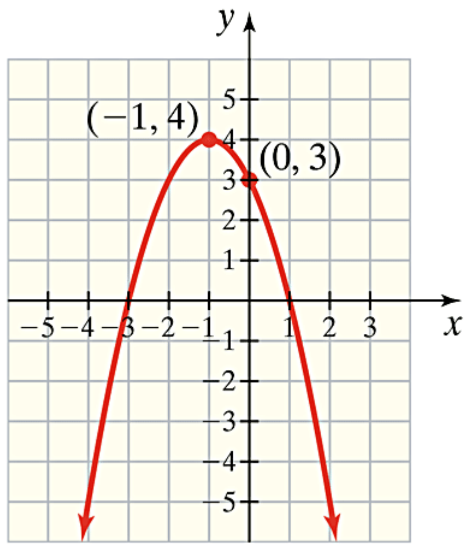
$f(0) = 3$

$a(0 + 1)^2 + 4 = 3$

$a = -1$

Determine the Equation of a Quadratic Function Given Its Graph.

EXAMPLE continued



d. Write the equation of the function in standard form $f(x) = a(x-h)^2 + k.$

$a = -1$

$(h,k) = (-1,4)$

$$f(x) = -(x+1)^2 + 4.$$

e. Write the equation of the function in the form $f(x) = ax^2 + bx + c.$

$$f(x) = -(x^2 + 2x + 1) + 4$$

$$f(x) = -x^2 - 2x + 3$$