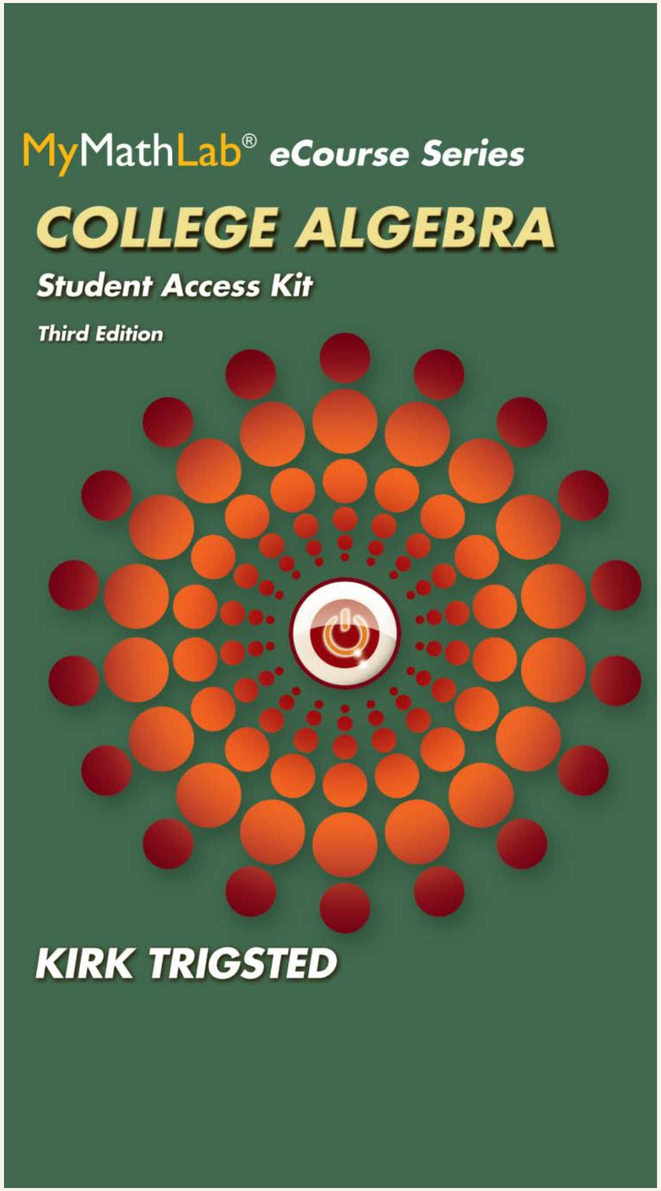
Topic 4.2

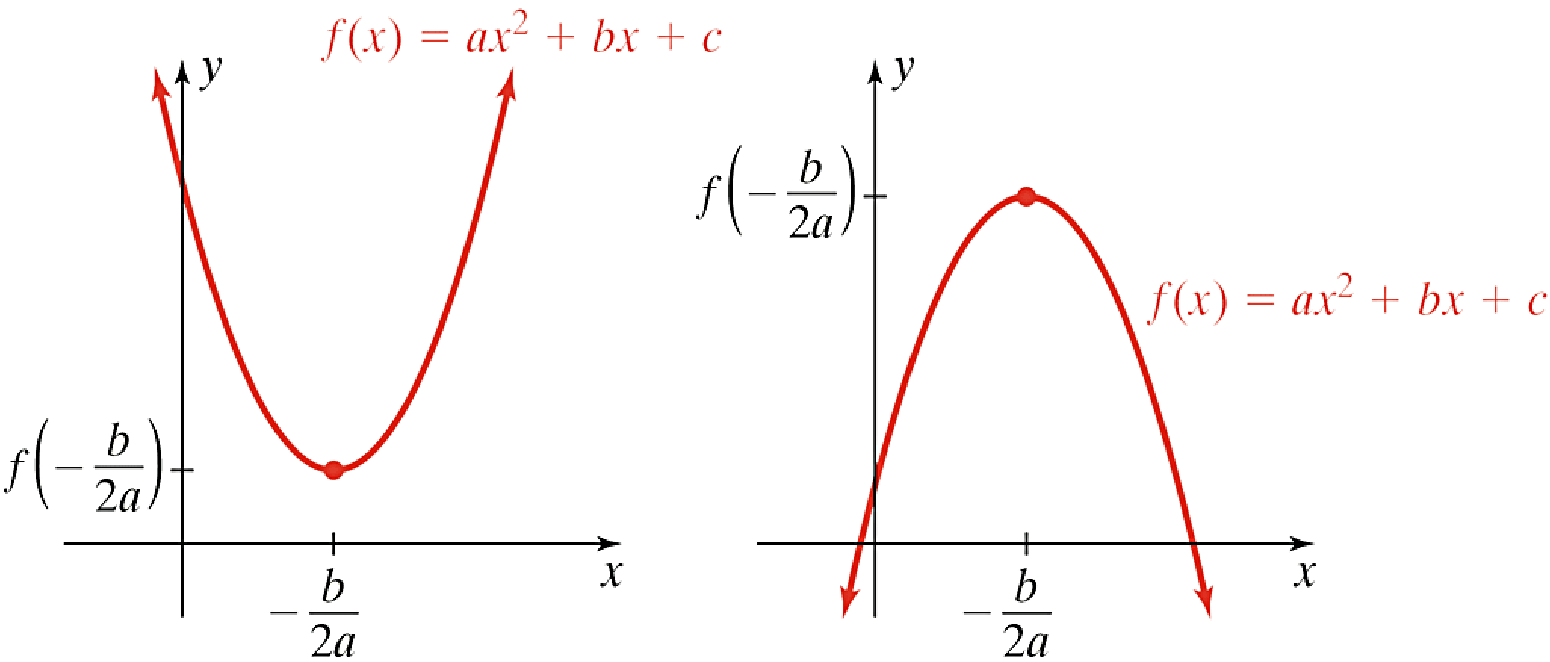
Applications and Modeling of Quadratic Functions



OBJECTIVES

1. Maximizing Projectile Motion Functions
2. Maximizing Functions in Economics
3. Maximizing Area Functions

Minimum and Maximum Values at Vertex



$a > 0$
Minimum value at $f\left(-\frac{b}{2a}\right)$

$a < 0$
Minimum value at $f\left(-\frac{b}{2a}\right)$

Maximizing Projectile Motion Functions

EXAMPLE

A toy rocket is launched with an initial velocity of 44.1 meters per second from a 1-meter-tall platform. The height h of the object at any time t seconds after launch is given by the function $h(t) = -4.9t^2 + 44.1t + 1$.

How long after launch did it take the rocket to reach its maximum height? What is the maximum height obtained by the toy rocket?

Because a = -4.9 < 0, then the parabola opens down and has a maximum value at the vertex.

$$t = -\frac{b}{2a} = -\frac{44.1}{2(-4.9)} = 4.5 \text{ seconds}$$

$$\begin{aligned} h(4.5) &= -4.9(4.5)^2 + 44.1(4.5) + 1 \\ &= 100.225 \text{ meters} \end{aligned}$$

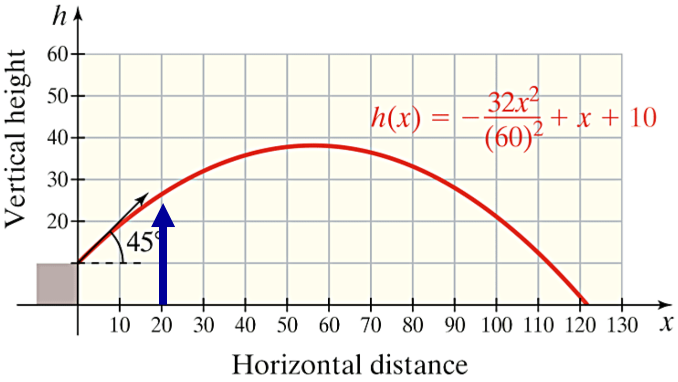
Maximizing Projectile Motion Functions

EXAMPLE

If an object is launched at an angle of 45 degrees from a 10-foot platform at 60 feet per second, it can be shown that the height of the object in feet is given by the quadratic function

$$h(t) = -\frac{32x^2}{(60)^2} + x + 10,$$

where x is the horizontal distance of the object from the platform.



a. What is the height of the object when its horizontal distance from the platform is 20 feet? Round to two decimal places.

$$h(20) = -\frac{32(20)^2}{(60)^2} + 20 + 10 \approx 26.44 \text{ feet}$$

Maximizing Projectile Motion Functions

EXAMPLE continued

If an object is launched at an angle of 45 degrees from a 10-foot platform at 60 feet per second, it can be shown that the height of the object in feet is given by the quadratic function

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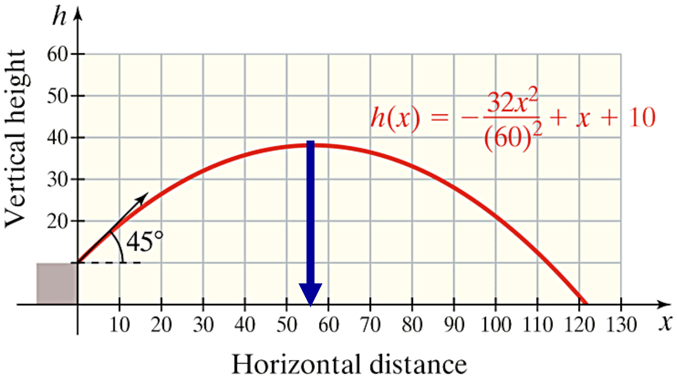


b. What is the horizontal distance from the platform when the object is at its maximum height?

$$x = -\frac{1}{2\left(\frac{-32}{60^2}\right)} = 56.25 \text{ feet}$$

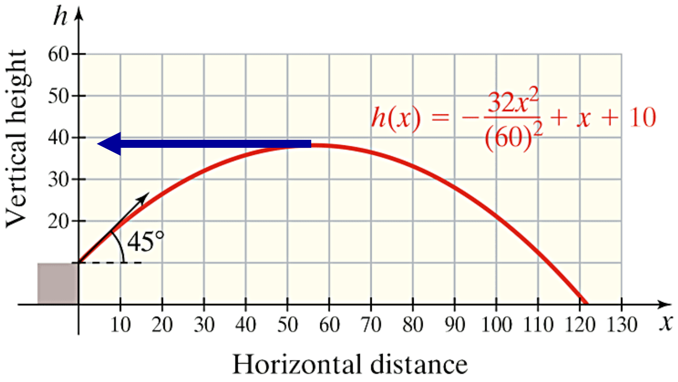
Maximizing Projectile Motion Functions

EXAMPLE continued

If an object is launched at an angle of 45 degrees from a 10-foot platform at 60 feet per second, it can be shown that the height of the object in feet is given by the quadratic function

$$h(t) = -\frac{32x^2}{(60)^2} + x + 10,$$

where x is the horizontal distance of the object from the platform.



c. What is the maximum height of the object?

$$h(56.25) = -\frac{32(56.25)^2}{(60)^2} + 56.25 + 10 \approx 38.125 \text{ feet}$$

Maximizing Functions in Economics

Revenue: Dollar amount received by selling x items at a price of p dollars per item: $R = xp$

As the quantity, x, increases, the price, p, tends to decrease.
As the quantity, x, decreases, the price, p, tends to increase.

Profit = Revenue - Cost

Maximizing Profits in Economics

EXAMPLE

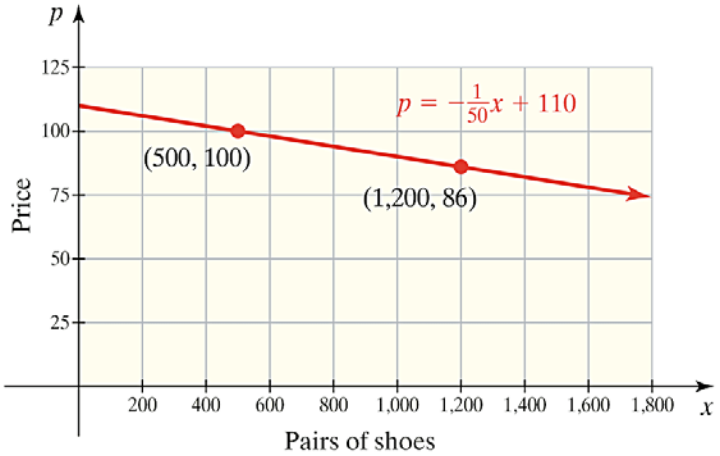
Records can be kept on the price of shoes and the number of pairs sold in order to gather enough data to reasonably model shopping trends for a particular type of shoe. Demand functions of this type are often linear and can be developed using knowledge of the slope and equations of lines. Suppose that the marketing and research department of a shoe company determined that the price of a certain type of basketball shoe obeys the demand equation

$$p = -\frac{1}{50}x + 110.$$

a. According to the demand equation, how much should the shoes sell for if 500 pairs of shoes are sold? 1,200 pairs of shoes?

$$p(500) = -\frac{1}{50}(500) + 110 = \$100$$

$$p(1200) = -\frac{1}{50}(1200) + 110 = \$86$$



Maximizing Profits in Economics

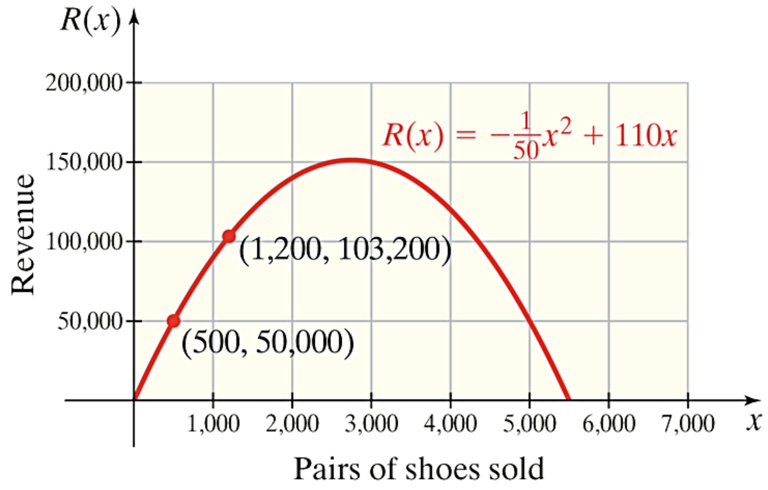
EXAMPLE continued

b. What is the revenue if 500 pairs of shoes are sold? 1,200 pairs of shoes?

$$R = xp$$

$$R(x) = x\left(-\frac{1}{50}x + 110\right)$$

$$R(x) = -\frac{1}{50}x^2 + 110x$$



$$R(500) = -\frac{1}{50}(500)^2 + 110(500) = \$50{,}000$$

$$R(1200) = -\frac{1}{50}(1200)^2 + 110(1200) = \$103{,}000$$

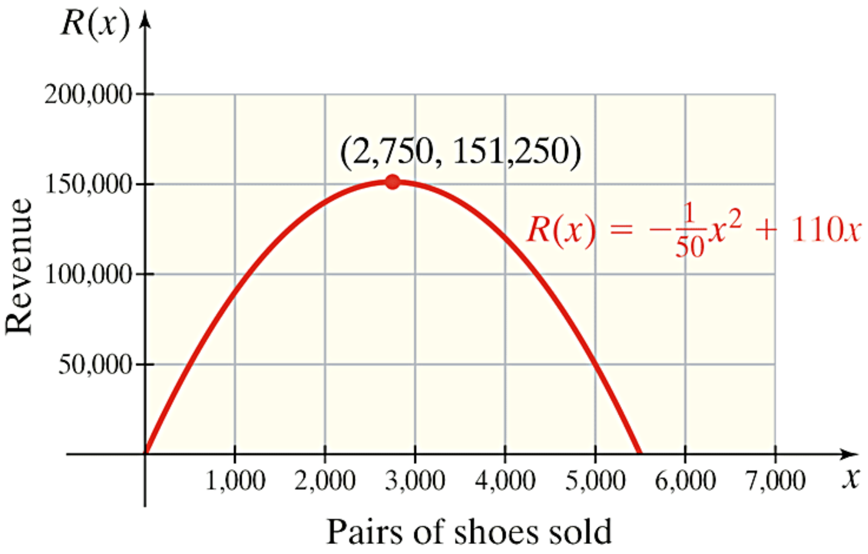
Maximizing Profits in Economics

EXAMPLE continued

c. How many pairs of shows should be sold in order to maximize revenue? What is the maximum revenue?

$$R(x) = -\frac{1}{50}x^2 + 110x$$

$$x = -\frac{b}{2a}$$
$$= -\frac{110}{2\left(-\frac{1}{50}\right)}$$
$$= 2,750 \text{ pairs of shoes}$$



$$R(2750) = -\frac{1}{50}(2750)^2 + 110(2750)$$
$$= \$151,205.$$

Maximizing Profits in Economics

EXAMPLE continued

d. What price should be charged in order to maximize revenue?

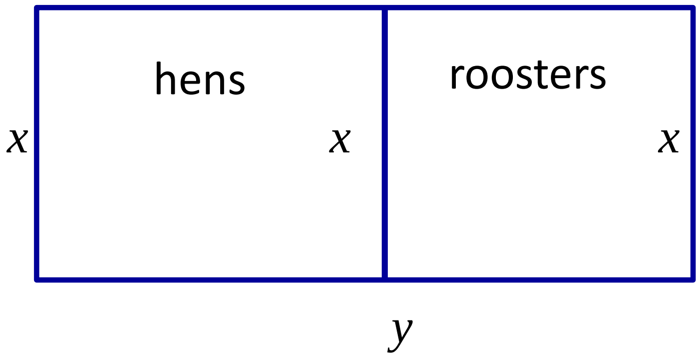
$$p = -\frac{1}{50}x + 110 \qquad x = 2{,}750 \text{ pairs of shoes}$$

$$p(2750) = -\frac{1}{50}(2750) + 110$$
$$= \$55$$

Maximizing Area Functions

EXAMPLE

Mark has 100 feet of fencing available to build a rectangular pen for his hens and roosters. He wants to separate the hens and roosters by dividing the pen into two equal areas. What should the length of the center partition be in order to maximize the area? What is the maximum area?



PERIMETER

$$3x + 2y = 100$$
$$2y = 100 - 3x$$
$$y = 50 - 1.5x$$

AREA

$$A = xy$$
$$A = x(50 - 1.5x)$$
$$A = -1.5x^2 + 50x$$

TO MAXIMIZE AREA

$$x = -\frac{b}{2a} = -\frac{50}{2(-1.5)} = \frac{50}{3} = 16\frac{2}{3} \text{ feet}$$

$$A\left(\frac{50}{3}\right) = -1.5\left(\frac{50}{3}\right)^2 + 50\left(\frac{50}{3}\right) = 416\frac{2}{3} \text{ feet}$$