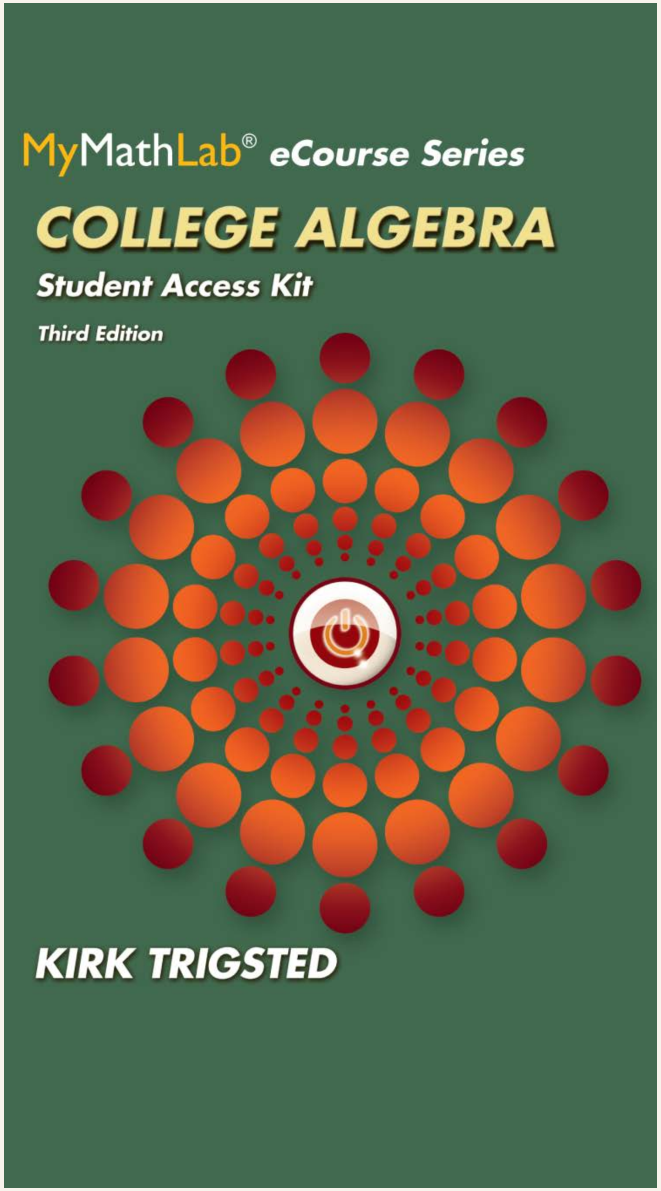
Topic 5.2

Logarithmic Functions



OBJECTIVES

1. Understanding the Definition of a Logarithmic Function
2. Evaluating Logarithmic Expressions
3. Understanding the Properties of Logarithms
4. Using the Common and Natural Logarithms
5. Understanding the Characteristics of Logarithmic Functions
6. Sketching the Graphs of Logarithmic Functions Using Transformations
7. Finding the Domain of Logarithmic Functions

Understanding the Definition of a Logarithmic Function

Definition: Logarithmic Function

For $x>0$, $b>0$ and $b\neq 1$, the logarithmic function with base b is defined by

$$y=\log_b x \text{ if and only if } x=b^y$$

Understanding the Definition of a Logarithmic Function

EXAMPLE

Write each exponential equation as an equation involving a logarithm.

a. $2^3 = 8$

$\log_2 8 = 3$

b. $5^{-2} = \frac{1}{25}$

$\log_5 \frac{1}{25} = -2$

c. $1.1^M = z$

$\log_{1.1} z = M$

Evaluating Logarithmic Expressions

EXAMPLE

Evaluate each logarithm:

a. $\log_5 25$

$$5^x = 25$$

$$x = 2;$$

$$\log_5 25 = 2$$

b. $\log_3 \frac{1}{27}$

$$3^y = \frac{1}{27}$$

$$3^y = 3^{-3}$$

$$y = -3;$$

$$\log_3 \frac{1}{27} = -3$$

c. $\log_{\sqrt{2}} \frac{1}{4}$

$$\sqrt{2}^z = \frac{1}{4}$$

$$\sqrt{2}^z = \sqrt{2}^{-4}$$

$$z = -4;$$

$$\log_{\sqrt{2}} \frac{1}{4} = -4$$

Understanding the Properties of Logarithms

General Properties of Logarithms

For $b>0$ and $b \neq 1$

1. $\log_b b = 1$
2. $\log_b 1 = 0.$

Cancellation Properties of Exponentials and Logarithms

For $b>0$ and $b \neq 1$

1. $b^{\log_b x} = x$
2. $\log_b b^x = x.$

Understanding the Properties of Logarithms

EXAMPLE

Use the properties of logarithms to evaluate each expression.

a. $\log_3 3^4$

$\log_3 3^4 = 4$

b. $\log_{12} 12$

$\log_{12} 12 = 1$

c. $7^{\log_7 13}$

$7^{\log_7 13} = 13$

d. $\log_8 1$

$\log_8 1 = 0$

Using the Common and Natural Logarithms

Definition: Common Logarithmic Function

For $x > 0$, the common logarithmic function is defined by

$$y = \log x \text{ if and only if } x = 10^y$$

Definition: Natural Logarithmic Function

For $x > 0$, the natural logarithmic function is defined by

$$y = \ln x \text{ if and only if } x = e^y$$

Using the Common and Natural Logarithms

EXAMPLE

Write each exponential equation as an equation involving a common logarithm or natural logarithm.

a. $e^0 = 1$

$\ln 1 = 0$

b. $10^{-2} = \frac{1}{100}$

$\log\left(\frac{1}{100}\right) = -2$

c. $e^K = w$

$\ln w = K$

Using the Common and Natural Logarithms

EXAMPLE

Evaluate each expression without the use of a calculator.

a. $\log 100$

$10^{2} = 100$

$\log 100 = \log 10^{2} = 2$

b. $\ln \sqrt{e}$

$\ln \sqrt{e} = \ln e^{\frac{1}{2}}$

$\frac{1}{2}$

c. $e^{\ln 51}$

51

d. $\log 1$

$\log 1 = 0$

Understanding the Characteristics Of Logarithmic Function

Steps for Sketching Logarithmic Functions of the Form $f(x) = \log_b x$

Step 1. Start with the graph of the exponential function $y = b^x$ labeling several ordered pairs.

Step 2. Because $f(x) = \log_b x$ is the inverse of $y = b^x$ we can find several points of the graph of $f(x) = \log_b x$ by reversing the coordinates of the ordered pairs of $y = b^x$.

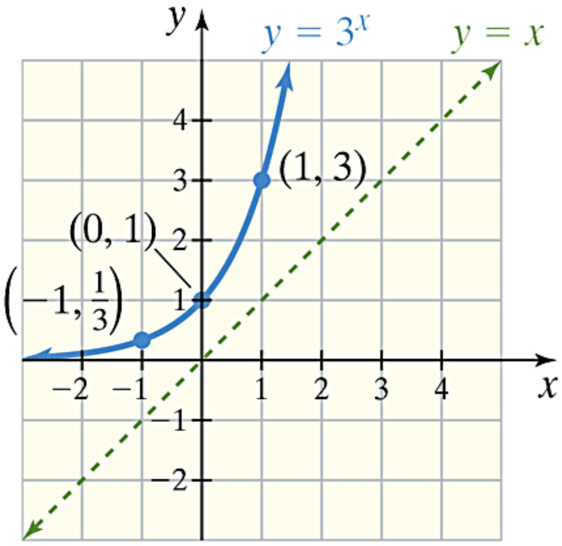
Step 3. Plot the ordered pairs from step 2, and complete the graph of $f(x) = \log_b x$ by connecting the ordered pairs with a smooth curve. The graph of $f(x) = \log_b x$ is a reflection of he graph of $y = b^x$ about the line $y = x$.

Understanding the Characteristics Of Logarithmic Function

EXAMPLE

Sketch the graph of $f(x) = \log_3 x$.

Step 1. $y = 3^x$ passes through $\left(-1, \frac{1}{3}\right), (0,1), (1,3)$



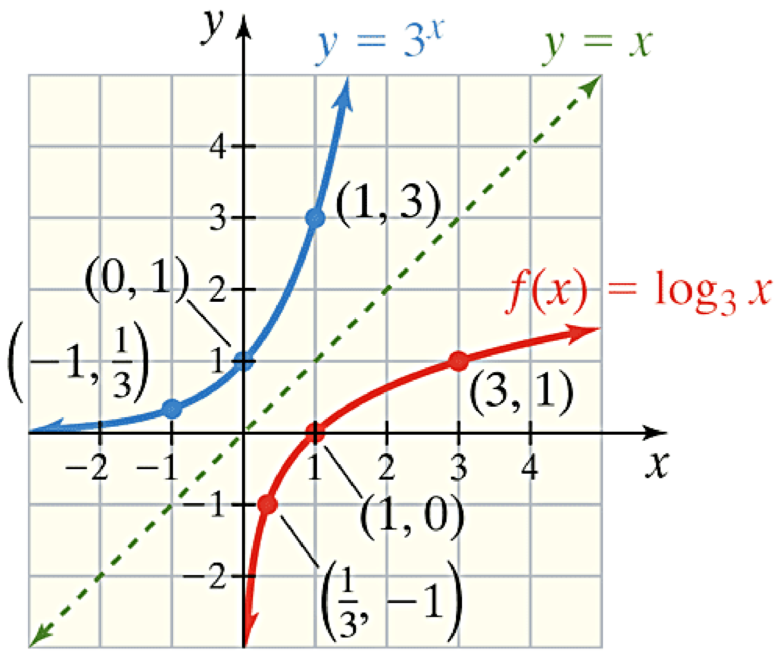
Step 2. reverse ordered pairs: $\left(\frac{1}{3}, -1\right), (1,0), (3,1)$

  PEARSON

Understanding the Characteristics Of Logarithmic Function

EXAMPLE continued

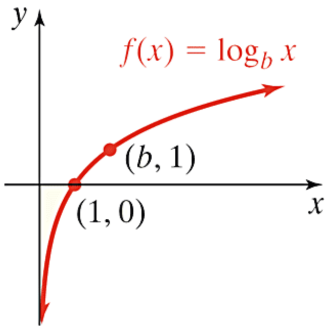
Step 3. plot points and connect with a smooth curve



Understanding the Characteristics Of Logarithmic Function

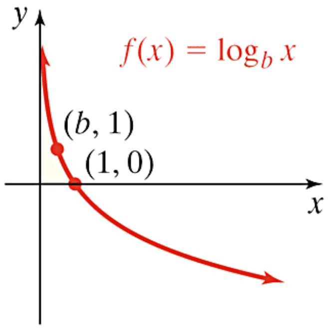
Characteristics of Logarithmic Functions

For $b > 0, b \neq 1$, the logarithmic function with base b is defined by $y = \log_b x$. The domain of $f(x) = \log_b x$ is $(0, \infty)$, and the range is $(-\infty, \infty)$. The graph of $f(x) = \log_b x$ has one of the following two shapes.



$f(x) = \log_b x, b > 1$

- The graph intersects the x-axis at $(1, 0)$.
- The graph contains the point $(b, 1)$.
- The graph is **increasing** on the interval $(0, \infty)$.
- The y-axis $(x = 0)$ is a **vertical asymptote**.
- The function is **one to one**.



$f(x) = \log_b x, 0 < b < 1$

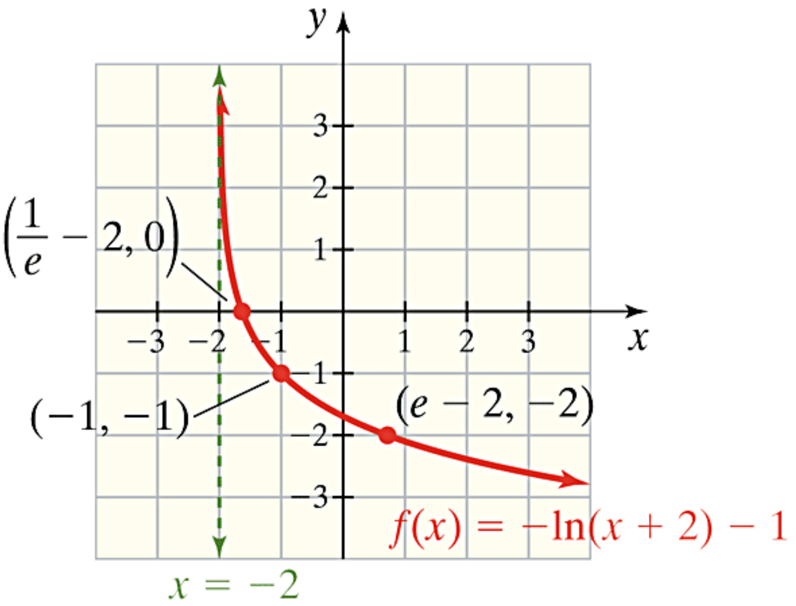
- The graph intersects the x-axis at $(1, 0)$.
- The graph contains the point $(b, 1)$.
- The graph is **decreasing** on the interval $(0, \infty)$.
- The y-axis $(x = 0)$ is a **vertical asymptote**.
- The function is **one to one**.

Sketching the Graphs of Logarithmic Functions Using Transformations

EXAMPLE

Sketch the graph of $f(x) = -\ln(x+2) - 1$.

$y = \ln x$	Base function
$y = \ln(x+2)$	Shift left 2 units
$y = -\ln(x+2)$	Reflect about x-axis
$f(x) = -\ln(x+2) - 1$	Shift down 1 unit



Find the Domain of a Logarithmic Function with a Rational Argument

EXAMPLE

Find the domain of $f(x)=\log_5\left(\frac{2x-1}{x+3}\right)$

$$\left(\frac{2x-1}{x+3}\right)>0$$

$$x<-3 \text{ or } x>\frac{1}{2}$$

$$\left\{x \mid x<-3 \text{ or } x>\frac{1}{2}\right\}$$

$$(-\infty,-3)\cup\left(\frac{1}{2},\infty\right)$$

Interval notation