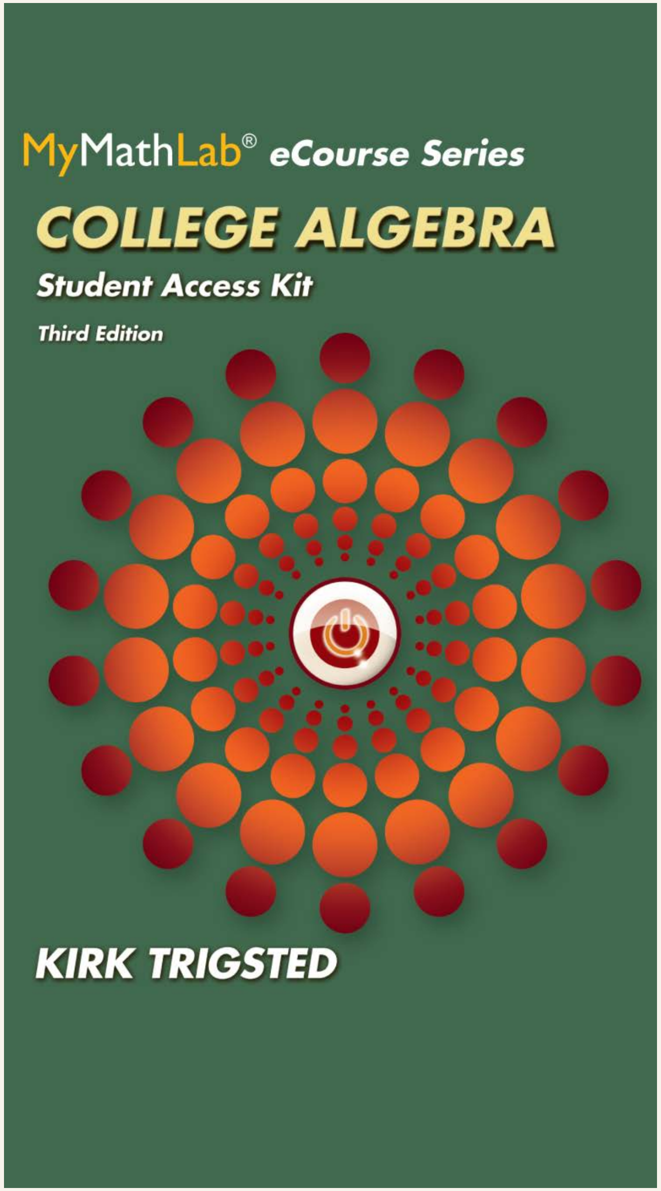
Topic 5.3

Properties of Logarithms



OBJECTIVES

1. Using the Product Rule, Quotient Rule, and Power Rule for Logarithms
2. Expanding and Condensing Logarithmic Expressions
3. Solving Logarithmic Equations Using the Logarithm Property of Equality
4. Using the Change of Base Formula

Using the Product Rule, Quotient Rule, and Power Rule for Logarithms

Properties of Logarithms

If $b>0$, $b\neq 0, u$ and v, represent positive numbers and r is any real number, then

$\log_b uv = \log_b u + \log_b v$	Product rule for logarithms
$\log_b \frac{u}{v} = \log_b u - \log_b v$	Quotient rule for logarithms
$\log_b u^r = r \log_b u$	Power rule for logarithms

Using the Product Rule, Quotient Rule, and Power Rule for Logarithms

EXAMPLE

Use the product rule for logarithms to expand each expression. Assume $x>0$.

a. $\ln(5x)$

$\ln 5 + \ln x$

b. $\log_2(8x)$

$\log_2 8 + \log_2 x$

$3 + \log_2 x$

Using the Product Rule, Quotient Rule, and Power Rule for Logarithms

EXAMPLE

Use the quotient rule for logarithms to expand each expression. Assume $x>0$.

a. $\log_5\left(\frac{12}{x}\right)$

$\log_5 12-\log_5 x$

b. $\log\left(\frac{x}{e^5}\right)$

$=\ln x-\ln e^5$

$\ln x-5$

Using the Product Rule, Quotient Rule, and Power Rule for Logarithms

EXAMPLE

Use the power rule for logarithms to rewrite each expression. Assume $x>0$.

a. $\log 6^3$

$3\log 6$

b. $\log_{1/2} \sqrt[4]{x}$

$= \log_{1/2} x^{1/4}$

$\frac{1}{4}\log_{1/2} x$

Expanding and Condensing Logarithmic Expressions

EXAMPLE

Use properties of logarithms to expand each logarithmic expression as much as possible.

a. $\log_7\left(49x^3\sqrt[5]{y^2}\right)$

$\log_7 49+\log_7 x^3\sqrt[5]{y^2}$ — Product rule for logarithms

$\log_7 49+\log_7 x^3+\log_7\sqrt[5]{y^2}$ — Product rule for logarithms

$\log_7 49+\log_7 x^3+\log_7 y^{2/5}$

$2+3\log_7 x+\frac{2}{5}\log_7 y$ — Power rule for logarithms

Expanding and Condensing Logarithmic Expressions

EXAMPLE continued

Use properties of logarithms to expand each logarithmic expression as much as possible.

b. $$\ln\left(\frac{(x^2-4)}{9e^{x^3}}\right) = \ln\left(\frac{(x-2)(x+2)}{9e^{x^3}}\right)$$

$$\ln(x-2)(x+2) - \ln 9e^{x^3}$$ Quotient rule for logarithms

$$\ln(x-2) + \ln(x+2) - \left[\ln 9 + \ln e^{x^3}\right]$$ Product rule for logarithms (twice)

$$\ln(x-2) + \ln(x+2) - \left[\ln 9 + x^3\right]$$

$$\ln(x-2) + \ln(x+2) - \ln 9 - x^3$$

Expanding and Condensing Logarithmic Expressions

EXAMPLE

Use properties of logarithms to rewrite each expression as a single logarithm.

a. $\frac{1}{2}\log(x-1)-3\log z+\log 5$

$\log(x-1)^{1/2}-\log z^3+\log 5$ Use the power rule twice

Quotient rule for logarithms

$\log\frac{(x-1)^{1/2}}{z^3}+\log 5$ Product rule for logarithms

$\log\frac{5(x-1)^{1/2}}{z^3}$ or $\log\frac{5\sqrt{x-1}}{z^3}$

Expanding and Condensing Logarithmic Expressions

EXAMPLE continued

Use properties of logarithms to rewrite each expression as a single logarithm.

b. $\frac{1}{3}(\log_3 x - 2\log_3 y) + \log_3 10$

$$\frac{1}{3}(\log_3 x - \log_3 y^2) + \log_3 10$$

Use the power rule

$$\frac{1}{3}(\log_3 \frac{x}{y^1}) + \log_3 10$$

Quotient rule for logarithms

$$\left[\log_3 \left(\frac{x}{y^1}\right)^{1/3}\right] + \log_3 10$$

Use the power rule

$$\log_3 \left[10\left(\frac{x}{y^2}\right)^{1/3}\right] \text{ or } \log_3 \left[10\sqrt[3]{\frac{x}{y^2}}\right]$$

Product rule for logarithms

Solving Logarithmic Equations Using the Logarithm Property of Equality

Logarithm Property of Equality

If a logarithmic equation can be written in the form $log_b u = log_b v$, then $u = v$. Furthermore, if $u = v$, then $log_b u = log_b v$.

Solving Logarithmic Equations Using the Logarithm Property of Equality

EXAMPLE

Solve the following equations:

a. $\log_7(x-1) = \log_7 12$

$(x-1) = 12$

$x = 13$

b. $2\ln x = \ln 16$

$\ln x^2 = \ln 16$

$x^2 = 16$

$x = \pm 4$

$*however\ \ x \neq -4$

$so,\ \ x = 4$

Using the Change of Base Formula

Change of Base Formula

For any positive base $b \neq 1$ and any positive real number u, then

$$\log_b u = \frac{\log_a u}{\log_a b},$$

Where a is any positive number such that $a \neq 1$.

Using the Change of Base Formula

EXAMPLE

Approximate the following expressions. Round each to four decimal places.

a. $\log_9 200$

$$\frac{\log 200}{\log 9} \approx 2.4114$$

b. $\log_{\sqrt{3}} \pi$

$$\frac{\ln \pi}{\ln \sqrt{3}} \approx 2.0840$$

Using the Change of Base Formula

EXAMPLE

Use the change of base formula and the properties of logarithms to solve the following equation:

$$2\log_3 x = \log_9 16$$

$$\log_3 x^2 = \log_9 16$$

$$\log_3 x^2 = \frac{\log_3 16}{\log_3 9}$$

$$\log_3 x^2 = \frac{\log_3 16}{2}$$

$$\log_3 x^2 = \log_3 4$$

$$x^2 = 4 \qquad \text{But } x \neq -2,$$

$$x = \pm 2 \qquad \text{so, } x = 2$$