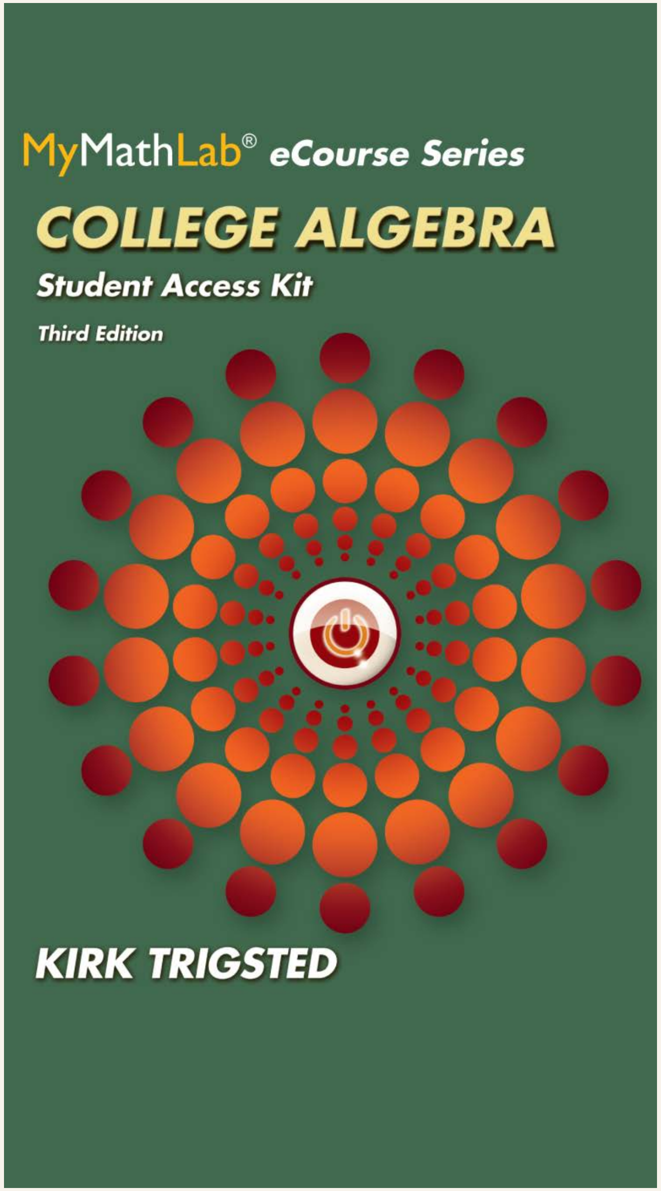
Topic 5.4

Exponential and Logarithmic Equations



OBJECTIVES

1. Solving Exponential Equations
2. Solving Logarithmic Equations

Solving Exponential Equations

Solving Exponential Equations

- If the equation can be written in the form $b^u = b^v$, then solve the equation $u = v$.
- If the equation cannot be easily written in the form $b^u = b^v$,
  1. Use the logarithmic property of equality to “take the log of both sides”
  2. Use the power rule of logarithms to “bring down any exponents.
  3. Solve for the given variable

Solving Exponential Equations

EXAMPLE

Solve each equation. For part b, round to four decimal places.

a. $3^{x-1} = \left(\frac{1}{27}\right)^{2x+1}$

$$3^{x-1} = (3)^{-3(2x+1)}$$

$$x - 1 = -3(2x+1)$$

$$x - 1 = -6x - 3$$

$$7x = -2$$

$$x = -\frac{2}{7}$$

b. $7^{x+3} = 4^{2-x}$

$$\ln 7^{x+3} = \ln 4^{2-x}$$

$$(x+3)\ln 7 = (2-x)\ln 4$$

$$x\ln 7 + 3\ln 7 = 2\ln 4 - x\ln 4$$

$$x(\ln 7 + \ln 4) = 2\ln 4 - 3\ln 7$$

$$x = \frac{2\ln 4 - 3\ln 7}{\ln 7 + \ln 4}$$

$$x = \frac{\ln 16 - \ln 343}{\ln 28}$$

$$x = \frac{\ln\left(\frac{16}{343}\right)}{\ln 28}$$

$$x \approx -.9199$$

Solving Exponential Equations

EXAMPLE

Solve each equation. Round to four decimal places.

a. $25e^{x-5} = 17$

$$e^{x-5} = \frac{17}{25}$$

$$\ln e^{x-5} = \ln \frac{17}{25}$$

$$x - 5 = \ln \frac{17}{25}$$

$$x = \ln \frac{17}{25} + 5$$

$$x \approx 4.6143$$

b. $e^{2x-1} \cdot e^{x+4} = 11$

$$e^{(2x-1)+(x+4)} = 11$$

$$e^{3x+3} = 11$$

$$\ln e^{3x+3} = \ln 11$$

$$3x + 3 = \ln 11$$

$$3x = \ln 11 - 3$$

$$x = \frac{\ln 11 - 3}{3}$$

$$x \approx -.2007$$

Solving Logarithmic Equations

Properties of Logarithms

If $b>0$, $b\neq 0$, u and v, represent positive numbers and r is any real number, then

$$\log_b uv = \log_b u + \log_b v$$ Product rule for logarithms

$$\log_b \frac{u}{v} = \log_b u - \log_b v$$ Quotient rule for logarithms

$$\log_b u^r = r\log_b u$$ Power rule for logarithms

Solving Logarithmic Equations

1. Determine the domain of the variable.
2. Use properties of logarithms to combine all logarithms, and write as a single logarithm, if needed.
3. Eliminate the logarithm by rewriting the equation in exponential form.
4. Solve for the given variable.
5. Check for any extraneous solutions. Verify that each solution is in the domain of the variable.

Solving Logarithmic Equations

EXAMPLE

Solve $2\log_5(x-1)=\log_5 64.$

$$\log_5(x-1)^2=\log_5 64$$

$$(x-1)^2=64$$

$$x-1=\pm 8$$

$$x=9 \text{ or } x=-7$$

Recall that the domain of a logarithmic function must contain only positive numbers; thus, x - 1 must be positive. Therefore, the solution of x = -7 must be discarded. The only solution is x = 9.

Solving Logarithmic Equations

EXAMPLE

Solve $\log_4(2x-1)=2.$

$$4^2=2x-1$$

$$16=2x-1$$

$$17=2x$$

$$x=\frac{17}{2}$$

Solving Logarithmic Equations

EXAMPLE

Solve $\log_2(x+10)+\log_2(x+6)=5.$

$$\log_2(x+10)(x+6)=5$$

$$x^2+16x+60=2^5$$

$$x^2+16x+28=0$$

$$(x+14)(x+2)=0$$

$$x=-14 \text{ or } x=-2$$

Because the domain of the variable is *x* > -6, we must *exclude* the solution *x* = -14. Therefore, the only solution to this logarithmic equation is *x* = -2