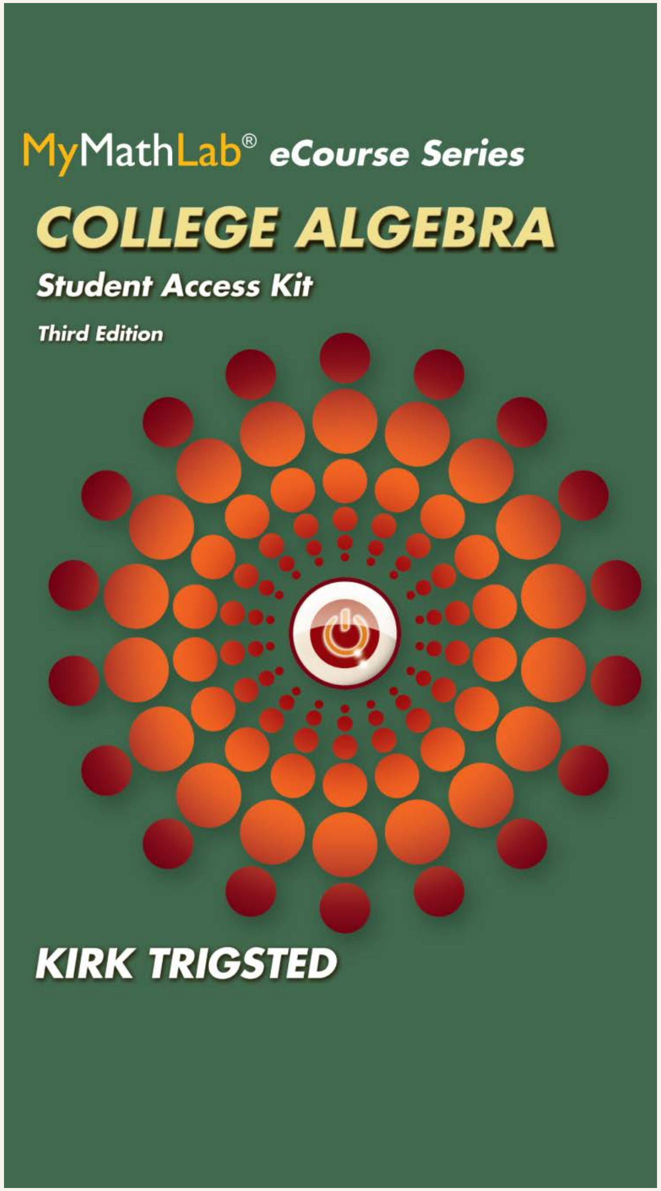
Topic 7.3

Solving Systems of Linear Equations in Three Variables Using Gaussian Elimination and Gauss-Jordan Elimination



OBJECTIVES

1. Solving a System of Linear Equations Using Gaussian Elimination
2. Using an Augmented Matrix to Solve a System of Linear Equations
3. Solving Consistent, Dependent Systems of Linear Equations in Three Variables
4. Solving Inconsistent Systems of Linear Equations in Three Variables
5. Determining Whether a System Has No Solution or Infinitely Many Solutions
6. Solving Linear Systems Having Fewer Equations Than Variables
7. Solving Applied Problems Using a System of Linear Equations Involving Three Variables

Solving a System of Linear Equations Using Gaussian Elimination

When equation **(3)** has only one variable, equation **(2)** contains two variables, and equation **(1)** has three variables, then a system of this form is said to be in **triangular form.**

The process of writing a system of three linear equations in three variables into an **equivalent system** that is in **triangular form** and then using back substitution to solve for each variable is called **Gaussian elimination,**

Solving a System of Linear Equations Using Gaussian Elimination

Elementary Row Operations

The following algebraic operations will result in an equivalent system of linear equations:

1. Interchange any two equations.

2. Multiply any equation by a nonzero constant.

3. Add a multiple of one equation to another equation.

Solving a System of Linear Equations Using Gaussian Elimination

Notation Used to Describe Elementary Row Operations

Notation	Meaning
$R_i \Leftrightarrow R_j$	Interchange Rows i and j.
$kR_i \rightarrow \text{New } R_i$	k times Row i becomes New Row i.
$kR_i + R_j \rightarrow \text{New } R_j$	k times Row i plus Row j becomes New Row j.

Solving a System of Linear Equations Using Gaussian Elimination

EXAMPLE

For the following system, use elementary row operations to find an equivalent system in triangular form and then use back substitution to solve the system:

$$\begin{aligned} 2x+3y+4z&=12\\ x-2y+3z&=0\\ -x+y-2z&=-1 \end{aligned}$$

$$\begin{array}{rcl} 2x+3y+4x=12 & & x-2y+3z=0 \\ x-2y+3z=0 & \xrightarrow{R_1 \Leftrightarrow R_2} & 2x+3y+4z=12 \\ -x+y-2z=-1 & & -x+y-2z=-1 \end{array}$$

$$\xrightarrow{-2R_1+R_2 \to \text{New } R_2} \begin{array}{r} x-2y+3z=0 \\ 7y-2z=12 \\ -x+y-2z=-1 \end{array}$$

$$\xrightarrow{R_1+R_3 \to \text{New } R_3} \begin{array}{r} x-2y+3z=0 \\ 7y-2z=12 \\ -y+z=-1 \end{array} \xrightarrow{R_2 \Leftrightarrow R_3} \begin{array}{r} x-2y+3z=0 \\ -y+z=-1 \\ 7y-2z=12 \end{array} \xrightarrow{7R_2+R_3 \to \text{New } R_3} \begin{array}{r} x-2y+3z=0 \\ -y+z=-1 \\ 5z=5 \end{array}$$

Solving a System of Linear Equations Using Gaussian Elimination

EXAMPLE continued

For the following system, use elementary row operations to find an equivalent system in triangular form and then use back substitution to solve the system:

$$\begin{aligned} 2x+3y+4z&=12 \\ x-2y+3z&=0 \\ -x+y-2z&=-1 \end{aligned}$$

$$\begin{aligned} x-2y+3z&=0 \\ -y+z&=-1 \\ 5z&=5 \end{aligned}$$

$$5z=5 \to z=1$$

$$\begin{aligned} -y+z&=-1 \\ -y+1&=-1 \\ -y&=-2 \\ y&=2 \end{aligned}$$

$$\begin{aligned} x-2y+3z&=0 \\ x-2(2)+3(1)&=0 \\ x-4+3&=0 \\ x-1&=0 \\ x&=1 \end{aligned}$$

The solution to this system is the ordered triple (1, 2, 1).

Using An Augmented Matrix to Solve a System of Linear Equations

System of Equations | **Corresponding Augmented Matrix**

$$\begin{aligned} 2x + 3y + 4z &= 12 \\ x - 2y + 3z &= 0 \\ -x + y - 2z &= -1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 12 \\ 1 & -2 & 3 & 0 \\ -1 & 1 & -2 & -1 \end{array}\right]$$

Using An Augmented Matrix to Solve a System of Linear Equations

We can perform the exact same elementary row operations as in the last example by writing an augmented matrix at each step. We now illustrate this process.

$$\left[\begin{array}{rrr|r} 2 & 3 & 4 & 12 \\ 1 & -2 & 3 & 0 \\ -1 & 1 & -2 & -1 \end{array}\right] \xrightarrow{R_1 \Leftrightarrow R_2} \left[\begin{array}{rrr|r} 1 & -2 & 3 & 0 \\ 2 & 3 & 4 & 12 \\ -1 & 1 & -2 & -1 \end{array}\right] \xrightarrow{-2R_1 + R_2 \to \text{New } R_2} \left[\begin{array}{rrr|r} 1 & -2 & 3 & 0 \\ 0 & 7 & -2 & 12 \\ -1 & 1 & -2 & -1 \end{array}\right] \xrightarrow{R_1 + R_3 \to \text{New } R_3}$$

$$\begin{aligned} 2x + 3y + 4z &= 12 \\ x - 2y + 3z &= 0 \\ -x + y - 2z &= -1 \end{aligned} \qquad \begin{aligned} x - 2y + 3z &= 0 \\ 2x + 3y + 4z &= 12 \\ -x + y - 2z &= -1 \end{aligned} \qquad \begin{aligned} x - 2y + 3z &= 0 \\ 7y - 2z &= 12 \\ -x + y - 2z &= -1 \end{aligned}$$

$$\left[\begin{array}{rrr|r} 1 & -2 & 3 & 0 \\ 0 & 7 & -2 & 12 \\ 0 & -1 & 1 & -1 \end{array}\right] \xrightarrow{R_2 \Leftrightarrow R_3} \left[\begin{array}{rrr|r} 1 & -2 & 3 & 0 \\ 0 & -1 & 1 & -1 \\ 0 & 7 & -2 & 12 \end{array}\right] \xrightarrow{7R_2 + R_3 \to \text{New } R_3} \left[\begin{array}{rrr|r} 1 & -2 & 3 & 0 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 5 & 5 \end{array}\right]$$

This matrix is now in triangular form. (All 0's below the diagonal.)

$$\begin{aligned} x - 2y + 3z &= 0 \\ 7y - 2z &= 12 \\ -y + z &= -1 \end{aligned} \qquad \begin{aligned} x - 2y + 3z &= 0 \\ -y + z &= -1 \\ 7y - 2z &= 12 \end{aligned} \qquad \begin{aligned} x - 2y + 3z &= 0 \\ -y + z &= -1 \\ 5z &= 5 \end{aligned}$$

Once we have a matrix written in triangular form, we can use the last row to solve for z and then use back substitution as before to solve for the remaining variables.

Using An Augmented Matrix to Solve a System of Linear Equations

EXAMPLE

Create an augmented matrix and solve the following linear system using Gaussian elimination by writing an equivalent system in triangular form:

$$\begin{aligned} x+2y-z&=3 \\ x-3y-2z&=11 \\ -x-2y+2z&=-6 \end{aligned}$$

The augmented matrix that corresponds to the linear system is

$$\left[\begin{array}{rrr|r} 1 & 2 & -1 & 3 \\ 1 & -3 & -2 & 11 \\ -1 & -2 & 2 & -6 \end{array}\right]$$

$$\left[\begin{array}{rrr|r} 1 & 2 & -1 & 3 \\ 1 & -3 & -2 & 11 \\ -1 & -2 & 2 & -6 \end{array}\right] \xrightarrow{(-1)R_1 + R_2 \to \text{New } R_2} \left[\begin{array}{rrr|r} 1 & 2 & -1 & 3 \\ 0 & -5 & -1 & 8 \\ -1 & -2 & 2 & -6 \end{array}\right] \xrightarrow{R_1 + R_3 \to \text{New } R_3} \left[\begin{array}{rrr|r} 1 & 2 & -1 & 3 \\ 0 & -5 & -1 & 8 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

Using An Augmented Matrix to Solve a System of Linear Equations

EXAMPLE continued

Create an augmented matrix and solve the following linear system using Gaussian elimination by writing an equivalent system in triangular form:

$$\begin{aligned} x+2y-z&=3 \\ x-3y-2z&=11 \\ -x-2y+2z&=-6 \end{aligned}$$

The augmented matrix is now in **triangular form**. Looking at the last row, we see that z = -3. The second row corresponds to the equation $-5y - z = 8$. Therefore,

$$\begin{aligned} -5y-z&=8 \\ -5y-(-3)&=8 \\ -5y+3&=8 \\ -5y&=5 \\ y&=-1 \end{aligned}$$

$$\begin{aligned} x+2y-z&=3 \\ x+2(-1)-(-3)&=3 \\ x-2+3&=3 \\ x+1&=3 \\ x&=2 \end{aligned}$$

Therefore, the solution to this linear system is the ordered triple (2, -1 -3).

Using An Augmented Matrix to Solve a System of Linear Equations

Triangular Form

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -5 & -1 & 8 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

Triangular form only requires zeros below the diagonal.

To reduce this matrix into **row-echelon form** requires that all zeros remain below the diagonal and that all coefficients along the diagonal are 1.

Row-Echelon Form

Row-echelon form requires zeros below the diagonal . . .

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & \frac{1}{5} & -\frac{8}{5} \\ 0 & 0 & 1 & -3 \end{array}\right]$$

. . . and 1's down the diagonal.

Using An Augmented Matrix to Solve a System of Linear Equations

Reduced row-echelon form requires zeros below *and* above the diagonal and 1's down the diagonal. We can reduce the previous matrix into reduced row-echelon form by performing the following three elementary row operations.

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & \frac{1}{5} & -\frac{8}{5} \\ 0 & 0 & 1 & -3 \end{array}\right] \xrightarrow{(-2)R_2 + R_1 \to \text{New } R_1} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{7}{5} & \frac{31}{5} \\ 0 & 1 & \frac{1}{5} & -\frac{8}{5} \\ 0 & 0 & 1 & -3 \end{array}\right] \xrightarrow{\left(-\frac{1}{5}\right)R_3 + R_2 \to \text{New } R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{7}{5} & \frac{31}{5} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \end{array}\right] \xrightarrow{\left(\frac{7}{5}\right)R_3 + R_1 \to \text{New } R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \end{array}\right] \begin{array}{l} \Rightarrow x = 2 \\ \Rightarrow y = -1 \\ \Rightarrow z = -3 \end{array}$$

The process of reducing a system into **reduced row-echelon form** is called **Gauss-Jordan elimination**.

Using An Augmented Matrix to Solve a System of Linear Equations

EXAMPLE

Solve the following system using Gauss-Jordan elimination:

$$\begin{aligned} x_1 + x_2 + x_3 &= -1 \\ x_1 + 2x_2 + 4x_3 &= 3 \\ x_1 + 3x_2 + 9x_3 &= 3 \end{aligned}$$

The augmented matrix that corresponds to the linear system is:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 9 & 3 \end{array}\right]$$

We need to reduce this matrix into reduced row-echelon form. We must have 1s down the diagonal and zeros everywhere else to the left of the vertical bar. We can accomplish this by performing the following seven elementary row operations.

Using An Augmented Matrix to Solve a System of Linear Equations

EXAMPLE continued

Solve the following system using Gauss-Jordan elimination:

$$\begin{aligned} x_1 + x_2 + x_3 &= -1 \\ x_1 + 2x_2 + 4x_3 &= 3 \\ x_1 + 3x_2 + 9x_3 &= 3 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 9 & 3 \end{array}\right] \xrightarrow{(-1)R_1 + R_2 \to \text{New } R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 0 & 1 & 3 & 4 \\ 1 & 3 & 9 & 3 \end{array}\right] \xrightarrow{(-1)R_1 + R_3 \to \text{New } R_3}$$

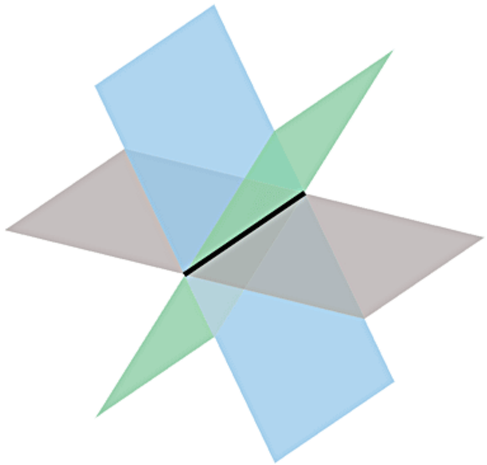
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 0 & 1 & 3 & 4 \\ 0 & 2 & 8 & 4 \end{array}\right] \xrightarrow{(-1)R_2 + R_1 \to \text{New } R_1} \left[\begin{array}{ccc|c} 1 & 0 & -2 & -5 \\ 0 & 1 & 3 & 4 \\ 0 & 2 & 8 & 4 \end{array}\right] \xrightarrow{(-2)R_2 + R_3 \to \text{New } R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & -5 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 2 & -4 \end{array}\right] \xrightarrow{\left(\frac{1}{2}\right)R_3 \to \text{New } R_3} \left[\begin{array}{ccc|c} 1 & 0 & -2 & -5 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{(-3)R_3 + R_2 \to \text{New } R_2}$$

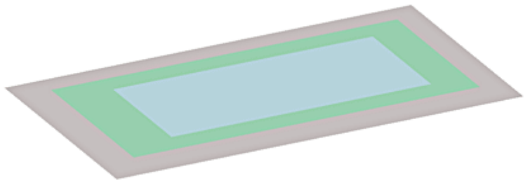
$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & -5 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{2R_3 + R_1 \to \text{New } R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & -2 \end{array}\right] \begin{array}{l} \Rightarrow x_1 = -9 \\ \Rightarrow x_2 = 10 \\ \Rightarrow x_3 = -2 \end{array}$$

Thus, the solution is the ordered triple (-9, 10, -2).

Solving Consistent, Dependent Systems of Linear Equations in Three Variables



Consistent, Dependent
Infinitely many solutions
(Planes intersect at a line.)



Consistent, Dependent
Infinitely many solutions
(Three equations describe the same plane.)

When using Gauss-Jordan elimination, we can determine that a system of three linear equations is a dependent system if one of the rows of the augmented matrix reduces to a row consisting of all zeros.

Solving Consistent, Dependent Systems of Linear Equations in Three Variables

EXAMPLE

Use Gauss-Jordan elimination to solve the system:

$$x+2y+3z=10$$
$$x+y+z=7$$
$$3x+2y+z=18$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 1 & 1 & 1 & 7 \\ 3 & 2 & 1 & 18 \end{array}\right] \xrightarrow{(-1)R_1 + R_2 \to \text{New } R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & -1 & -2 & -3 \\ 3 & 2 & 1 & 18 \end{array}\right] \xrightarrow{(-3)R_1 + R_3 \to \text{New } R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & -1 & -2 & -3 \\ 0 & -4 & -8 & -12 \end{array}\right] \xrightarrow{(-1)R_2 \to \text{New } R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & 1 & 2 & 3 \\ 0 & -4 & -8 & -12 \end{array}\right] \xrightarrow{(-2)R_2 + R_1 \to \text{New } R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & -4 & -8 & -12 \end{array}\right] \xrightarrow{4R_2 + R_3 \to \text{New } R_3} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Solving Consistent, Dependent Systems of Linear Equations in Three Variables

EXAMPLE continued

Use Gauss-Jordan elimination to solve the system:

$$\begin{aligned} x+2y+3z&=10 \\ x+y+z&=7 \\ 3x+2y+z&=18 \end{aligned}$$

Notice that the last row of the final augmented matrix consists entirely of zeros. This corresponds to the **identity** 0 = 0, which is true for every value of *x*, *y*, and *z*. The first two rows correspond to the equations

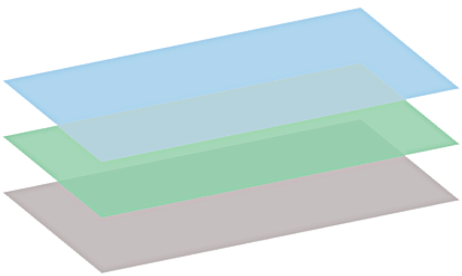
$$\begin{aligned} x \quad - \quad z&=4 \\ y+2z&=3 \end{aligned}$$

We can solve the first equation for *x* and the second equation for *y* to obtain the equations

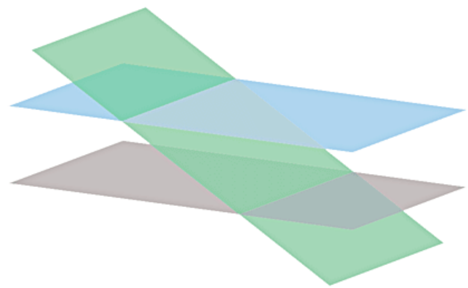
$$\begin{aligned} x&=4+z \\ y&=3-2z \end{aligned}$$

The solutions of this system are all ordered triples of the form $(4+z, 3-2z, z)$.

Solving Inconsistent Systems of Linear Equations in Three Variables



Inconsistent
No solution
(Three planes are parallel.)



Inconsistent
No solution
(Two planes are parallel.)

Inconsistent
No solution
(Planes intersect two at a time.)

Recall that when solving an inconsistent system of two equations we will eventually obtain a **contradiction**. This is also true in the three-variable case.

Solving Inconsistent Systems of Linear Equations in Three Variables

EXAMPLE

Use Gauss-Jordan elimination to solve the system:

$$\begin{aligned} x - y + 2z &= 4 \\ -x + 3y + z &= -6 \\ x + y + 5z &= 3 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ -1 & 3 & 1 & -6 \\ 1 & 1 & 5 & 3 \end{array}\right] \xrightarrow{R_1 + R_2 \to \text{New } R_2} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 2 & 3 & -2 \\ 1 & 1 & 5 & 3 \end{array}\right] \xrightarrow{(-1)R_1 + R_3 \to \text{New } R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 2 & 3 & -2 \\ 0 & 2 & 3 & -1 \end{array}\right] \xrightarrow{(-1)R_2 + R_3 \to \text{New } R_3} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 2 & 3 & -2 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

Looking at the final augmented matrix, we see that the last row represents the equation $0x + 0y + 0z = 1$ or, simply, 0 = 1. Obviously, this is a **contradiction** (zero can never equal one). Therefore, we say that the system is inconsistent and has no solution.

Determining Whether a System Has No Solution or Infinitely Many Solutions

EXAMPLE

Each augmented matrix in row reduced form is equivalent to the augmented matrix of a system of linear equations in variables *x*, *y*, and *z*. Determine whether the system is a consistent, dependent system or an inconsistent system. If it is a consistent, dependent system, the describe the solution.

a. $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

consistent,
dependent system

$x - 2z = 5 \qquad x = 2z + 5$

$y + 3z = -2 \qquad y = -3z - 2$

$(2z + 5, -3z - 5, z)$

b. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 10 \end{array}\right]$

inconsistent system

c. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$

consistent,
dependent system

$x = 3 \qquad x = 3$

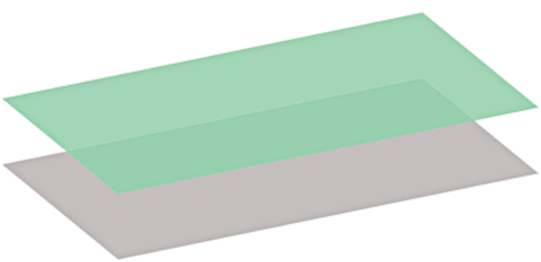
$y - 2z = 4 \qquad y = 2z + 4$

$(3, 2z + 4, z)$

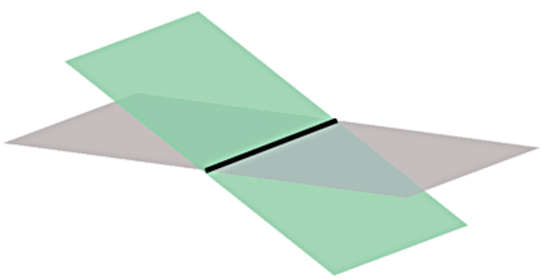
Solving Linear Systems Having Fewer Equations Than Variables

Suppose we encounter a system of linear equations in three variables that has only two equations. We can geometrically illustrate such a system by sketching two planes. There are only three possible scenarios for such a system:

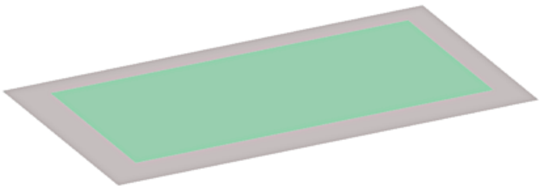
1. The two planes can be distinct parallel planes.
2. The two planes can intersect at a straight line.
3. The two planes can coincide; that is, the two equations describe the same plane.



Two planes are parallel.
No solution



Two planes intersect at a line.
Infinitely many solutions



Two planes coincide.
Infinitely many solutions

Solving Linear Systems Having Fewer Equations Than Variables

EXAMPLE

Solve the linear system using Gauss-Jordan elimination:

$$\begin{aligned} x + y + z &= 1 \\ 2x - 2y + 6z &= 10 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & -2 & 6 & 10 \end{array}\right] \xrightarrow{(-2)R_1 + R_2 \to \text{New } R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -4 & 4 & 8 \end{array}\right] \xrightarrow{\left(-\frac{1}{4}\right)R_2 \to \text{New } R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & -2 \end{array}\right] \xrightarrow{(-1)R_2 + R_1 \to \text{New } R_1} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & -2 \end{array}\right]$$

$$\begin{aligned} x + 2z &= 3 \\ y - z &= -2 \end{aligned} \qquad \begin{aligned} x &= 3 - 2z \\ y &= z - 2 \end{aligned}$$

The solution can be written in the form $(3 - 2z, z - 2, z)$.

Solving Applied Problems Using a System of Linear Equations Involving Three Variables

EXAMPLE

Wendy ordered 30 T-shirts online for her three children. The small T-shirts cost $4 each, the medium T-shirts cost $5 each, and the large T-shirts were $6 each. She spent $40 more purchasing the large T-shirts than the small T-shirts, Wendy's total bill was $154. How many T-shirts of each size did she buy?

Let S, M, and L represent the number of small, medium, and large T-shirts, respectively.
Because a total of 30 T-shirts were purchased, we obtain the first equation:

$$S + M + L = 30$$

Because small T-shirts cost $4, medium T-shirts cost $5, large T-shirts cost $6, and because the total value of the T-shirts was $154, we obtain the second equation:

$$4S + 5M + 6L = 154$$

Solving Applied Problems Using a System of Linear Equations Involving Three Variables

EXAMPLE continued

Wendy ordered 30 T-shirts online for her three children. The small T-shirts cost $4 each, the medium T-shirts cost $5 each, and the large T-shirts were $6 each. She spent $40 more purchasing the large T-shirts than the small T-shirts, Wendy's total bill was $154. How many T-shirts of each size did she buy?

Finally, Wendy spent $40 more buying large T-shirts than small T-shirts, we Have the third equation:

$$4s - 6L = -40$$

$$\begin{aligned} S + M + L &= 30 \\ 4S + 5M + 6L &= 154 \\ 4S \qquad - 6L &= -40 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 30 \\ 4 & 5 & 6 & 154 \\ 4 & 0 & -6 & -40 \end{array}\right]$$

Solving Applied Problems Using a System of Linear Equations Involving Three Variables

EXAMPLE continued

Wendy ordered 30 T-shirts online for her three children. The small T-shirts cost $4 each, the medium T-shirts cost $5 each, and the large T-shirts were $6 each. She spent $40 more purchasing the large T-shirts than the small T-shirts, Wendy's total bill was $154. How many T-shirts of each size did she buy?

Using Gauss-Jordan elimination, we can rewrite the matrix in the following **reduced row-echelon form:**

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 12 \end{array}\right]$$

Wendy purchased 8 small T-shirts, 10 medium T-shirts, and 12 large T-shirts.