Section 1.6

Properties of Integral Exponents

Objective #1

Use the product rule.

Properties of Exponents

Exponent Rules	
Product Rule	$b^m \cdot b^n = b^{m+n}$
	When multiplying exponential expressions with the same base, add the exponents.

1a. Multiply using the product rule: $b^6 \cdot b^5$

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$$b^6 \cdot b^5 = b^{6+5}$$
$$= b^{11}$$

1b. Multiply using the product rule: $\left(4x^3y^4\right)\left(10x^2y^6\right)$

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$$\left(4x^3y^4\right)\left(10x^2y^6\right) = 4\cdot 10\cdot x^3\cdot x^2\cdot y^4\cdot y^6$$

$$= 40x^{3+2}y^{4+6}$$

$$= 40x^5y^{10}$$

Objective #2

Use the quotient rule.

Properties of Exponents

Exponent Rules	
Quotient Rule	$\frac{b^m}{b^n} = b^{m-n}, b \neq 0$ When dividing exponential expressions with the same nonzero base, subtract the exponent in the denominator from the exponent in the numerator.

Objective #2: Example

2. Divide using the quotient rule: $\dfrac{27x^{14}y^{8}}{3x^{3}y^{5}}$

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$$\frac{27x^{14}y^{8}}{3x^{3}y^{5}} = \frac{27}{3}x^{14-3}y^{8-5}$$

$$= 9x^{11}y^{3}$$

Objective #3

Use the zero-exponent rule.

Properties of Exponents

The Zero Exponent Rule:

If b is any real number other than 0, then $b^0 = 1$.

3. Use the zero-exponent rule to simplify: -5^0

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$$-5^0 = -(5^0)$$
$$= -1$$

Objective #4

Use the negative-exponent rule.

Properties of Exponents

Negative Exponent Rule: If b is any real number other than 0 and n is a natural number, then

$$b^{-n} = \frac{1}{b^n} \quad \text{and} \quad \frac{1}{b^{-n}} = b^n.$$

4a. Use the negative-exponent rule to write 5^{-2} with a positive exponent. Simplify, if possible.

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$$5^{-2} = \frac{1}{5^2}$$

$$= \frac{1}{25}$$

4b. Use the negative-exponent rule to write $\frac{1}{5x^{-2}}$ with a positive exponent. Simplify, if possible.

4b. Use the negative-exponent rule to write $\dfrac{1}{5x^{-2}}$ with a positive exponent. Simplify, if possible.

$$\frac{1}{5x^{-2}} = \frac{x^2}{5}$$

Objective #5

Use the power rule.

Properties of Exponents

The Power Rule (Powers to Powers)

$$\left(b^m\right)^n = b^{mn}$$

When an exponential expression is raised to a power, multiply the exponents. Place the product of the exponents on the base and remove the parentheses.

5. Simplify $(b^{-3})^{-4}$ using the power rule.

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$$(b^{-3})^{-4} = b^{(-3)(-4)}$$
$$= b^{12}$$

Objective #6

Find the power of a product.

Properties of Exponents

Products to Powers

$$(ab)^n = a^n b^n$$

When a product is raised to a power, raise each factor to that power.

6. Simplify the expression using the products-to-powers rule: $\left(-4x^5y^{-1}\right)^{-2}$

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$$\left(-4x^5y^{-1}\right)^{-2} = (-4)^{-2}\left(x^5\right)^{-2}\left(y^{-1}\right)^{-2}$$

$$= \frac{1}{(-4)^2} \cdot x^{-10} \cdot y^2$$

$$= \frac{y^2}{16x^{10}}$$

Objective #7

Find the power of a quotient.

Properties of Exponents

Quotients to Powers

If b is a nonzero real number, then

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

When a quotient is raised to a power, raise the numerator to that power and divide by the denominator to that power.

7. Simplify the expression using the quotients-to-powers rule: $\left(\frac{2x^{-3}}{y^2}\right)^4$

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$$\left(\frac{2x^{-3}}{y^2}\right)^4 = \frac{2^4 x^{(-3)(4)}}{y^{(2)(4)}}$$

$$= \frac{16x^{-12}}{y^8}$$

$$= \frac{16}{x^{12}y^8}$$

Objective #8

Simplify exponential expressions.

Simplifying Exponential Expressions

Simplification Techniques	Examples
If necessary, remove parentheses by using the Products to Powers Rule or the Quotient to Powers Rule.	$(2ab)^4 = 2^4 a^4 b^4$ $\left(\frac{17}{x^2}\right)^3 = \frac{17^3}{(x^2)^3} = \frac{17^3}{x^{2\cdot 3}} = \frac{4913}{x^6}$
If necessary, simplify powers to powers by using the Power Rule.	 $(W^5)^{10} = W^{5\cdot 10} = W^{50}$

Simplifying Exponential Expressions

Simplification Techniques	Examples
Be sure each base appears only once in the final form by using the Product Rule or Quotient Rule	$H^4 \cdot H^{16} = H^{4+16} = H^{20}$ $\frac{V^{23}}{V^{17}} = V^{23-17} = V^6$
If necessary, rewrite exponential expressions with zero powers as 1. Furthermore, write the answer with *positive* exponents by using the Negative Exponent Rule	$3 + 2\left(45X^3Y^{-4}\right)^0 = 3 + 2 \cdot 1 = 5$ $\frac{31}{K^{-12}} = 31K^{12}$

Objective #8: Example

8a. Simplify: $\left(-3x^{-6}y\right)\left(-2x^{3}y^{4}\right)^{2}$

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$$\begin{aligned}(-3x^{-6}y)(-2x^3y^4)^2 &= (-3x^{-6}y)(-2)^2(x^3)^2(y^4)^2\\ &= -3\cdot x^{-6}\cdot y\cdot 4\cdot x^6\cdot y^8\\ &= -12\cdot x^{-6+6}\cdot y^{1+8}\\ &= -12x^0y^9\\ &= -12y^9\end{aligned}$$

Objective #8: Example

8b. Simplify: $\left(\dfrac{10x^3y^5}{5x^6y^{-2}}\right)^2$

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$$\left(\frac{10x^3y^5}{5x^6y^{-2}}\right)^2 = \left(2x^{3-6}y^{5+2}\right)^2$$

$$= \left(2x^{-3}y^7\right)^2$$

$$= 4x^{-6}y^{14}$$

$$= \frac{4y^{14}}{x^6}$$