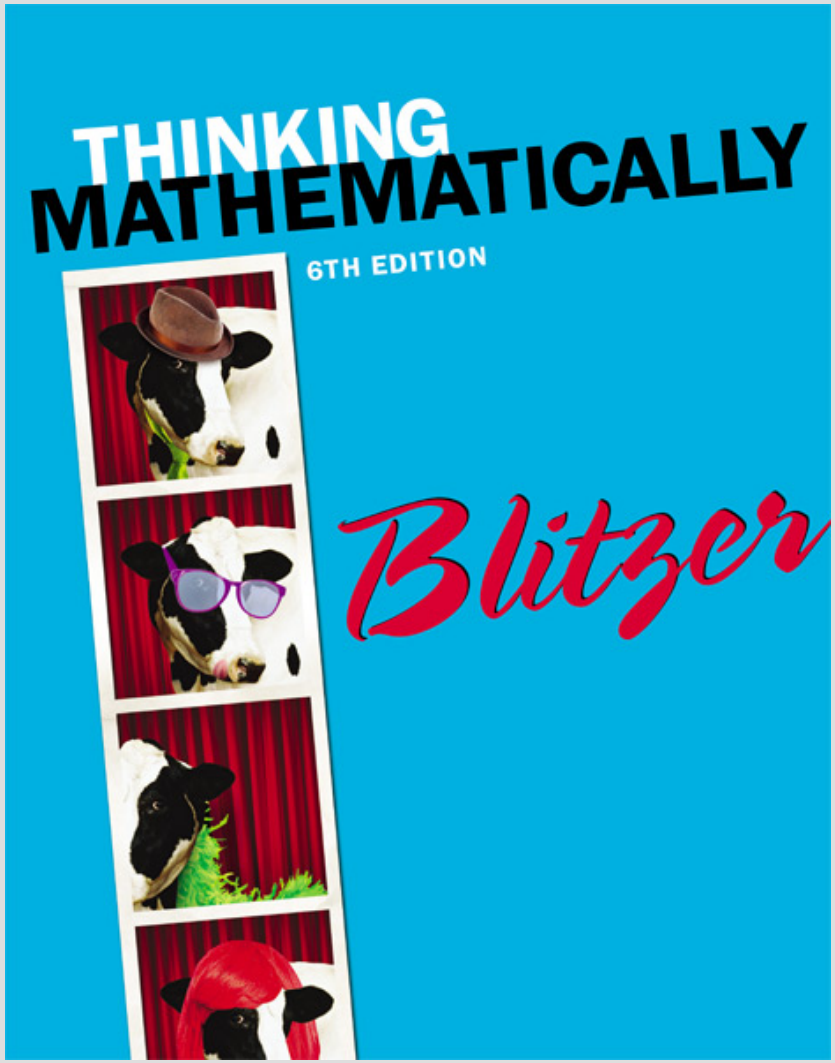
CHAPTER 10

Geometry



10.1

Points, Lines, Planes, and Angles

Objectives

1. Understand points, lines, and planes as the basis of geometry.

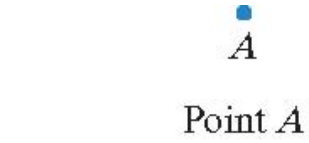
2. Solve problems involving angle measures.

3. Solve problems involving angles formed by parallel lines and transversals.

Defining Points, Lines and Planes

Point

- Represented as a small dot
- Has no length, width or thickness



Point A

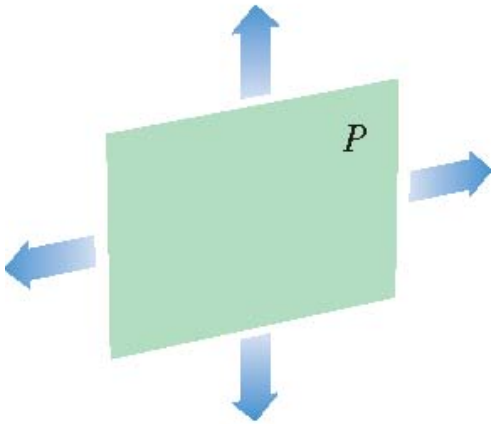
Line

- Connecting two points along the shortest path
- Has no thickness and infinite length

Line AB

Plane

- Flat surface
- Has no thickness and no boundaries



Plane P

Lines

A **line** may be named using any two of its points.

A **half line** is formed when a point divides a line

A **ray** is a half-line with its endpoint included

A **line segment** is a portion of a line joining two points.

Lines

In the diagrams below, a closed circle indicates that the point is included. An open circle indicates that the point is not included.

(a) Line AB
$\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$

(b) Half line AB
$\overrightarrow{AB}$

(c) Ray AB
$\overrightarrow{AB}$

(d) Line Segment AB
$\overline{AB}$ or $\overline{BA}$

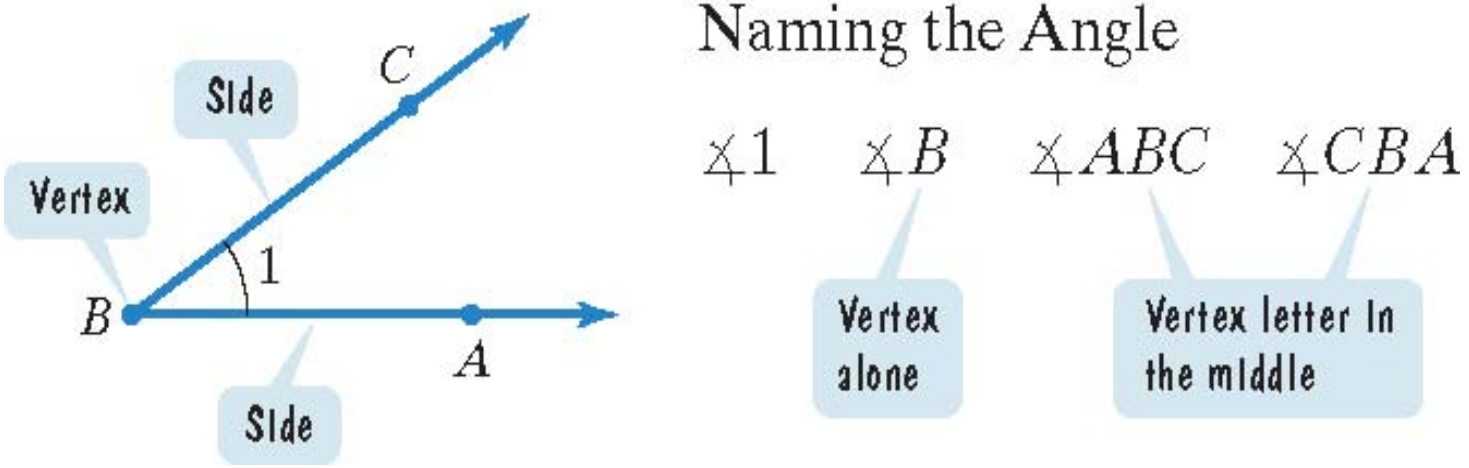
Angles

Angle

Formed by the union of two rays.

One ray is called the initial side.

The other ray is called the terminal side.



Measuring Angles Using Degrees

Angles are measured by the amount of rotation from the initial side to the terminal side.

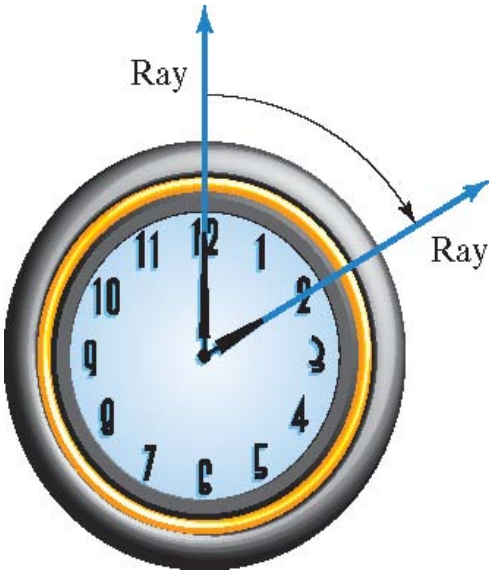
Angles are measured in degrees, symbolized by °.

There are 360° in a full rotation (circle).

1 degree is $\frac{1}{360}$ of a complete rotation.

Example: Using Degree Measure

The hour hand of a clock moves from 12 to 2 o'clock. Through how many degrees does it move?



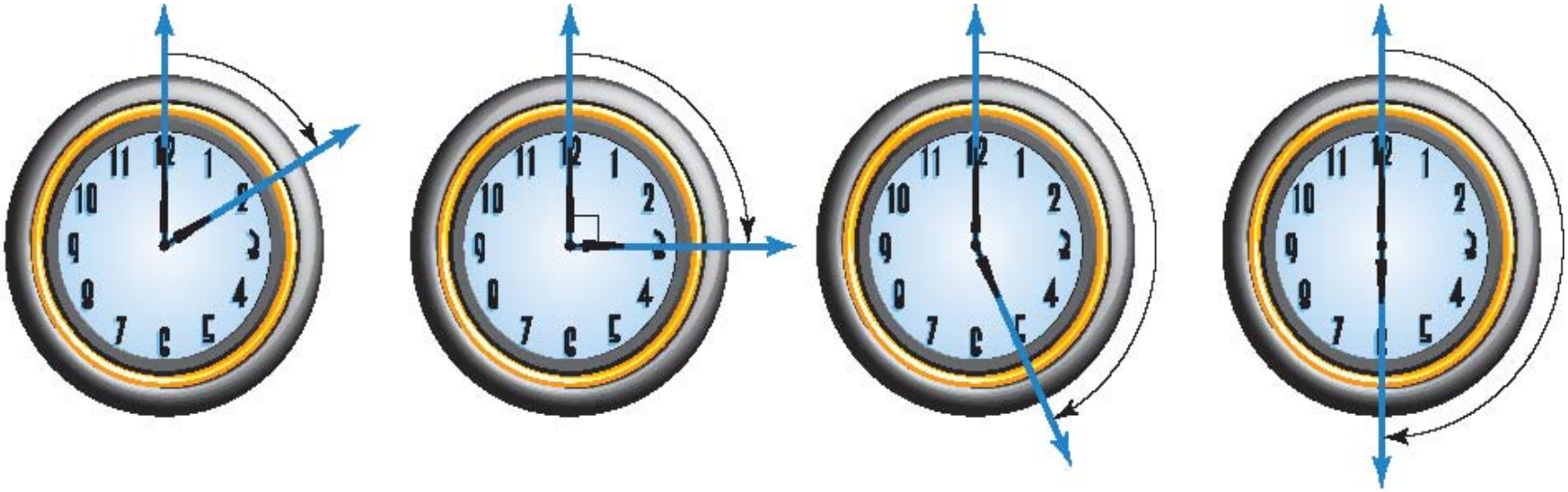
Solution:

Moving from 12 to 2 o'clock is $\frac{2}{12}$ or $\frac{1}{6}$ of a complete revolution.

Thus, the hour hand moves,

$$\frac{1}{6} \times 360^\circ = \frac{360^\circ}{6} = 60^\circ$$

Classifying Angles by Their Degree Measurement



(a) **Acute angle**
Less than 90°

(b) **Right angle**
90°

(c) **Obtuse angle**
More than 90° but less than 180°

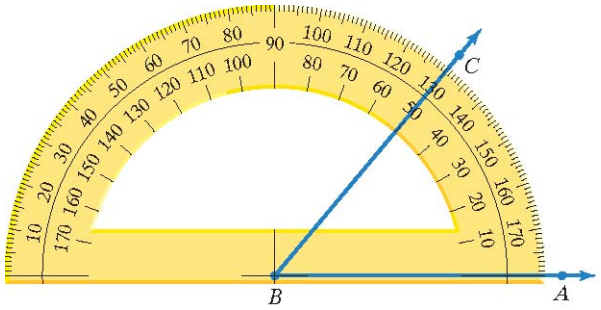
(d) **Straight angle**
180°

Protractor

Protractors are used for finding the degree measure of an angle.

We measure the angle by placing the center point of the protractor on the vertex of the angle and the straight side of the protractor along one side of the angle. We read the degree by reading where the other side of the angle intercepts the edge of the protractor.

Choose the number based on whether the angle is obtuse or acute.



Special Pairs of Angles

Complementary Angles are two angles whose sum is 90°. To find the complement of an angle, subtract it from 90°.

The complement of 70° is:

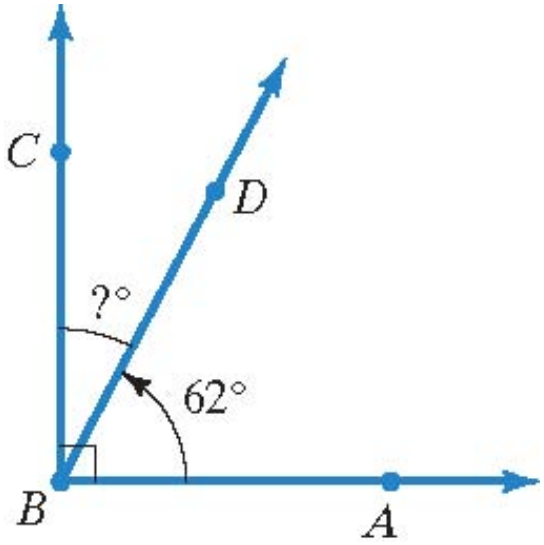
$90° - 70° = 20°.$

Supplementary Angles are two angles whose sum is 180°.

The supplement of 110° is:

$180° - 110° = 70°.$

Example: Angle Measures and Complements



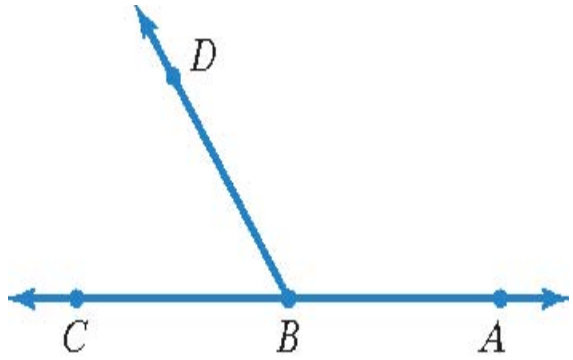
Find $m\angle DBC$.

Solution:

$m \angle DBC = 90° - 62° = 28°$

Example: Angle Measures and Supplements

$m\angle ABD$ is 66° greater than $m\angle DBC$ and they are supplementary angles.
Find the measure of each angle.



Solution:

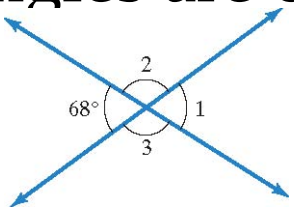
$$m\angle DBC + m\angle ABD = 180°$$
$$x + (x + 66°) = 180°$$
$$2x + 66° = 180°$$
$$2x = 114°$$
$$x = 57°$$

$m \angle DBC = 57°$

$m \angle ABD = 57° + 66° = 123°$

Example: Using Vertical Angles

When two lines intersect, the opposite angles formed are called Vertical Angles. Vertical Angles are equal.



The angle on the left measures 68°.

Find the other angles.

Solution:

From the figure we see that:

$\angle 1 = 68°$ (Vertical angles are equal)

$\angle 1 + \angle 2 = 180°$ (Supplementary angles)

$\angle 2 = 180° - 68° = 112°$

$\angle 3 = \angle 2 = 112°$ (Vertical angles are equal)

Special Line Relationships

Parallel Lines

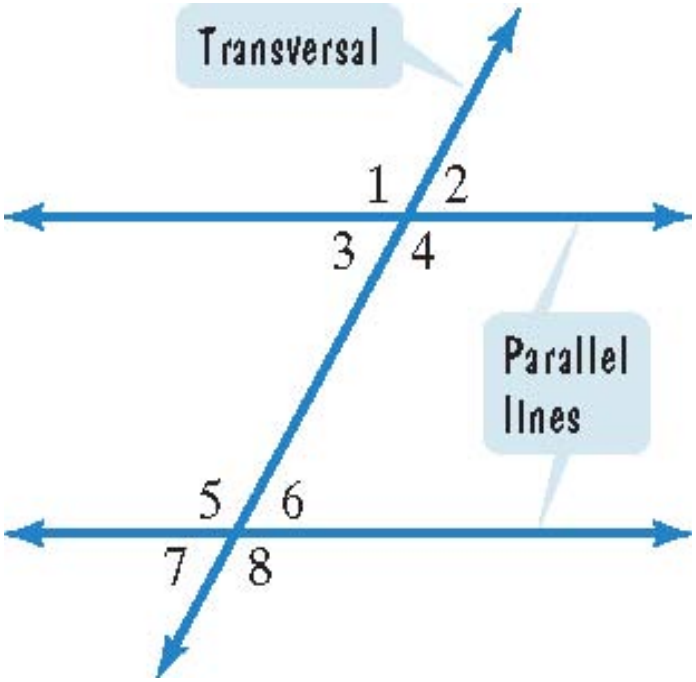
Lines that lie in the same plane and have no points in common.

Intersecting Lines

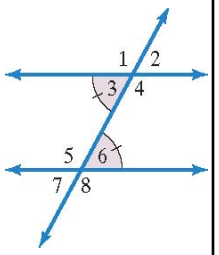
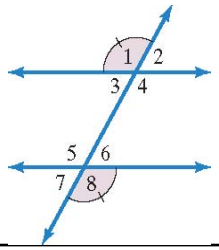
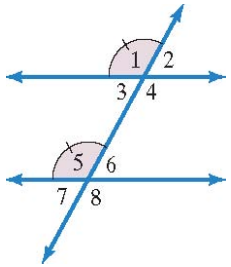
Two lines that are not parallel and have a single point in common.

Transversal

A line that intersects two parallel lines.



Names of Angle Pairs Formed by a Transversal Intersecting Parallel Lines

Name	Description	Sketch	Angle Pairs Described	Property
Alternate interior angles	Interior angles that do not have a common vertex, and are on alternate sides of the transversal		$\angle 3$ and $\angle 6$ $\angle 4$ and $\angle 5$	Alternate interior angles have the same measure. $\angle 3 = \angle 6$ $\angle 4 = \angle 5$
Alternate exterior angles	Exterior angles that do not have a common vertex, and are on alternate sides of the transversal		$\angle 1$ and $\angle 8$ $\angle 2$ and $\angle 7$	Alternate exterior angles have the same measure. $\angle 1 = \angle 8$ $\angle 2 = \angle 7$
Corresponding angles	One interior and one exterior angle on the same side of the transversal		$\angle 1$ and $\angle 5$ $\angle 2$ and $\angle 6$ $\angle 3$ and $\angle 7$ $\angle 4$ and $\angle 8$	Corresponding angles have the same measure. $\angle 1 = \angle 5$ $\angle 2 = \angle 6$ $\angle 3 = \angle 7$ $\angle 4 = \angle 8$

Parallel Lines and Angle Pairs

PARALLEL LINES AND ANGLE PAIRS

If parallel lines are intersected by a transversal,

- alternate interior angles have the same measure,
- alternate exterior angles have the same measure, and
- corresponding angles have the same measure.

Conversely, if two lines are intersected by a third line and an alternate interior angle pair or an alternate exterior angle pair or a pair of corresponding angles have the same measure, then the two lines are parallel.

Example: Finding Angle Measure When Parallel Lines are Intersected by a Transversal

Find the measure of all the angles:

Solution:

$m\angle 1 = 35°$

$m\angle 6 = 180° - 35° = 145°$

$m\angle 7 = 145$

$m\angle 2 = 35°$

$m\angle 3 = 145°$

$m\angle 5 = 35°$

$m\angle 4 = 180° - 35° = 145°$