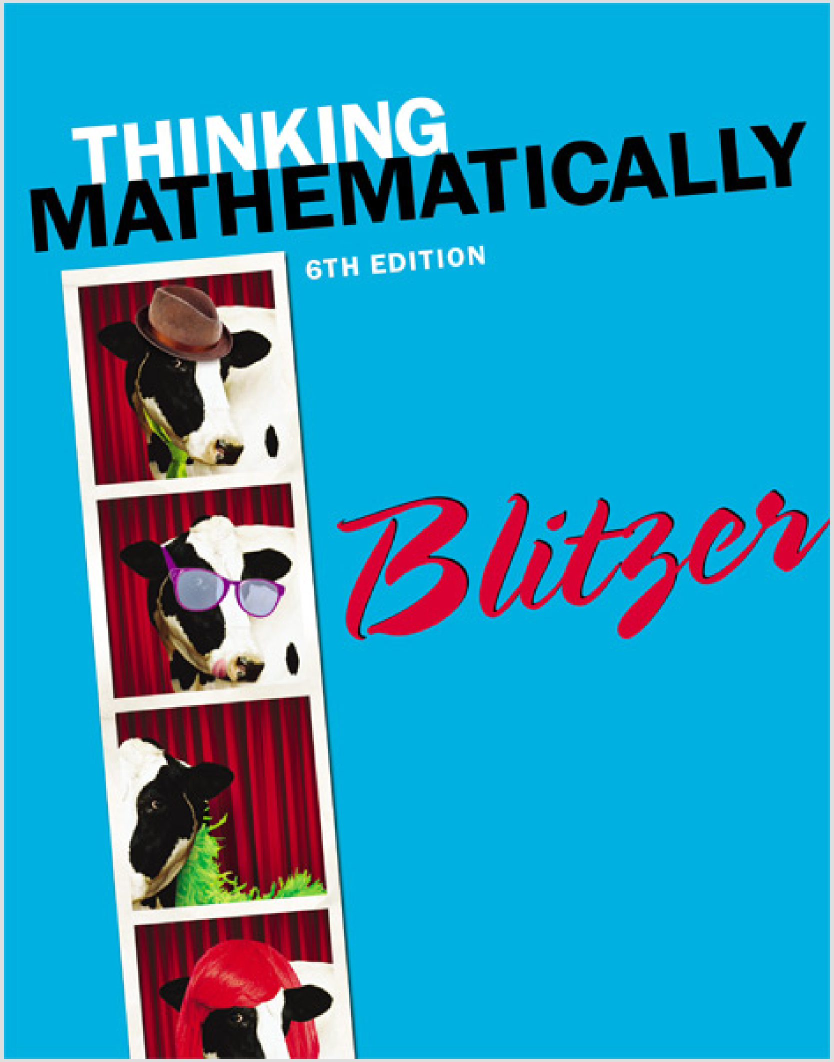
CHAPTER 10

Geometry



10.2

Triangles

Objectives

1. Solve problems involving angle relationships in triangles.
2. Solve problems involving similar triangles.
3. Solve problems using the Pythagorean Theorem.

Triangle

A closed geometric figure that has three sides, all of which lie on a flat surface or plane.

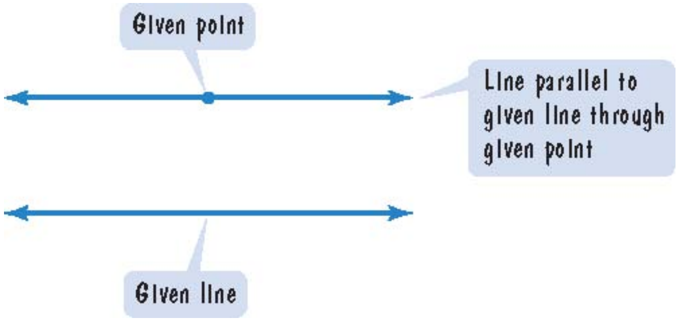
Closed geometric figures

> If you start at any point and trace along the sides, you end up at the starting point.

Euclid's Theorem

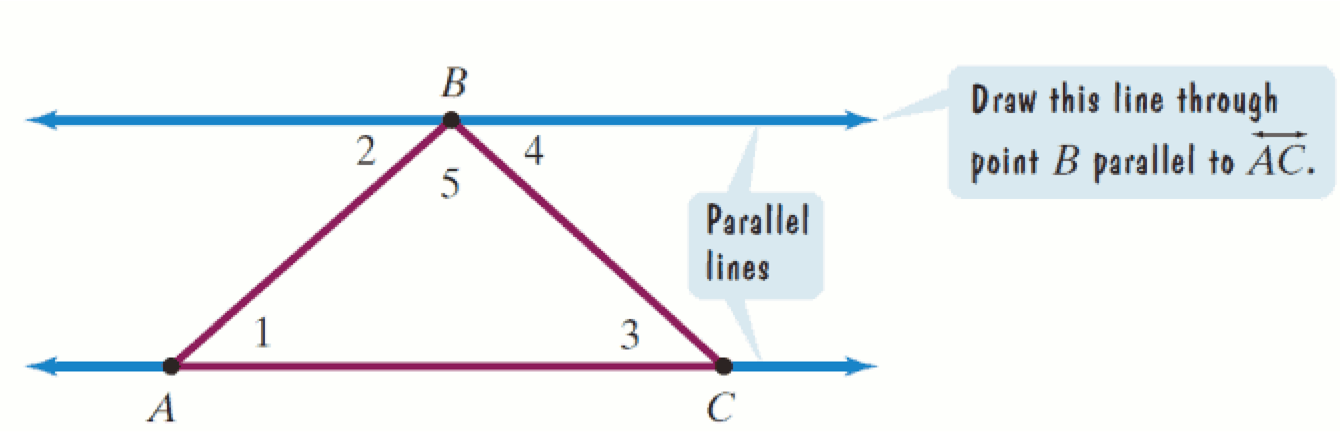
Theorem: A conclusion that is proved to be true through deductive reasoning.

Euclid's assumption: Given a line and a point not on the line, one and only one line can be drawn through the given point parallel to the given line.



Euclid’s Theorem (cont.)

Euclid’s Theorem: The sum of the measures of the three angles of any triangle is 180º.



Euclid's Theorem (cont.)

Proof:

$m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$ (alternate interior angles)

Angles 2, 5, and 4 form a straight angle (180º)

$$m\angle 2 + m\angle 5 + m\angle 4 = 180°$$

Because $m\angle 1 = m\angle 2$, replace $m\angle 2$ with $m\angle 1$.

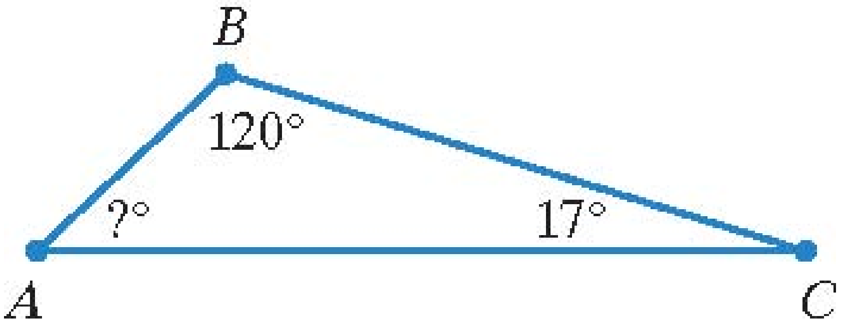
Because $m\angle 3 = m\angle 4$, replace $m\angle 4$ with $m\angle 3$.

$$m\angle 1 + m\angle 5 + m\angle 3 = 180°$$

Therefore, $m\angle 1 + m\angle 5 + m\angle 3 = 180º$.

Example: Using Angle Relationships in Triangles

Find the measure of angle *A* for the triangle *ABC*.



Solution:

$m\angle A + m\angle B + m\angle C = 180^{\circ}$

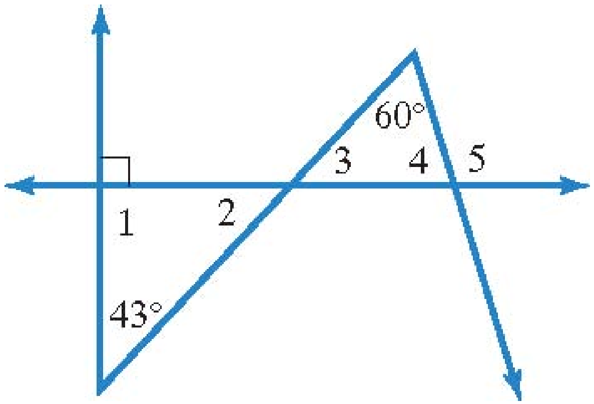
$m\angle A + 120^{\circ} + 17^{\circ} = 180^{\circ}$

$m\angle A + 137^{\circ} = 180^{\circ}$

$m\angle A = 180^{\circ} - 137^{\circ} = 43^{\circ}$

Example: Using Angle Relationships in Triangles

Find the measures of angles 1 through 5.



Solution:

$m\angle 1 + m\angle 2 + 43^{\circ} = 180^{\circ}$

$90^{\circ} + m\angle 2 + 43^{\circ} = 180^{\circ}$

$m\angle 2 + 133^{\circ} = 180^{\circ}$

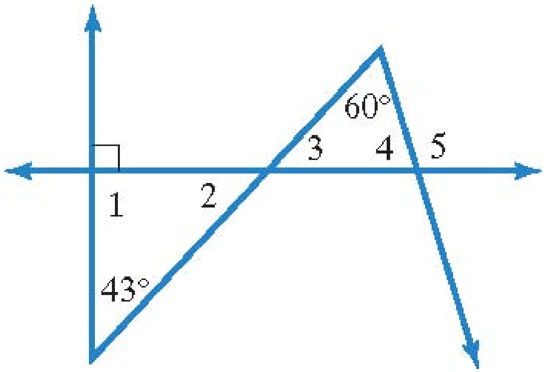
$m\angle 2 = 47^{\circ}$

Example continued

$$m\angle 3 + m\angle 4 + 60^\circ = 180^\circ$$

$$47^\circ + m\angle 4 + 60^\circ = 180^\circ$$

$$m\angle 4 = 180^\circ - 107^\circ = 73^\circ$$



$$m\angle 4 + m\angle 5 = 180^\circ$$

$$73^\circ + m\angle 5 = 180^\circ$$

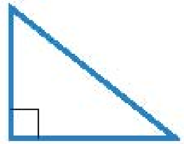
$$m\angle 5 = 107^\circ$$

Triangles and Their Characteristics

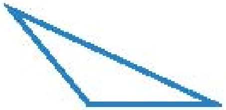
Classification by Angles

Acute Triangle
All angles are acute.

Right Triangle
One angle measures 90°.

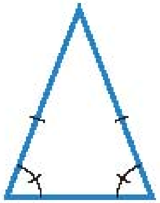


Obtuse Triangle
One angle is obtuse.

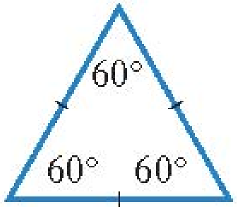


Classification by Sides

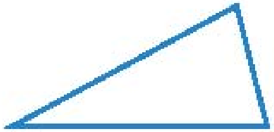
Isoceles Triangle
Two sides have equal length. (Angles opposite these sides have the same measure.)



Equilateral Triangle
All sides have equal length. (Each angle measures 60°.)



Scalene Triangle
No two sides are equal in length.



Similar Triangles

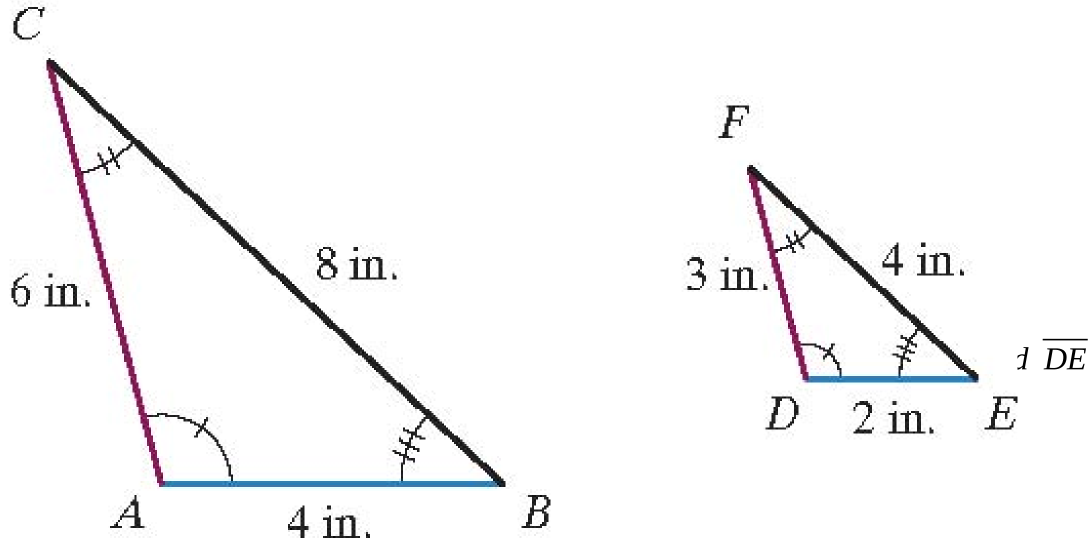
Similar figures have the same shape, but not necessarily the same size.

In **similar triangles**, the angles are equal but the sides may or may not be the same length.

Corresponding angles are angles that have the same measure in the two triangles.

Corresponding sides are the sides opposite the corresponding angles.

Similar Triangles (continued)

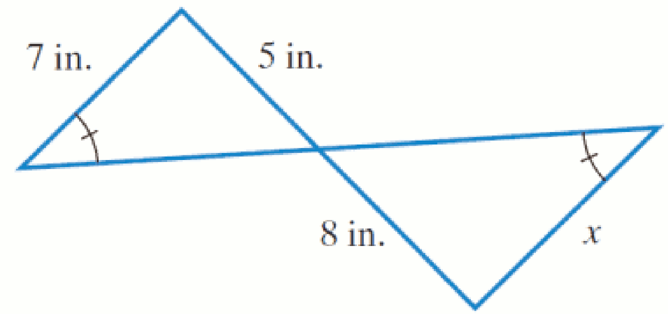


Triangles ABC and DEF are similar:

Corresponding Angles	Corresponding Sides
Angles A and D	Sides $\overline{AC}$ *and* $\overline{DF}$
Angles C and F	Sides $\overline{AB}$ *and* $\overline{DE}$
Angles B and E	Sides $\overline{BC}$ *and* $\overline{EF}$

Example: Using Similar Triangles

Find the missing length *x*.



Solution: Because the triangles are similar, their corresponding sides are proportional:

Opposite the angle with one tick in the small △

Opposite the angle with one tick in the large △

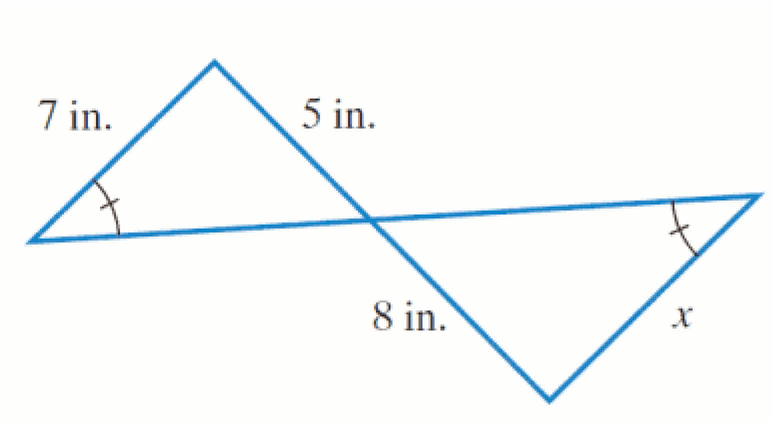
$$\frac{5}{8} = \frac{7}{x}$$

Opposite the angle with two ticks in the small △

Opposite the angle with two ticks in the large △

Example: Using Similar Triangles

Find the missing length *x*.



$$\frac{5}{8}=\frac{7}{x}$$

$$5x=8\cdot 7$$

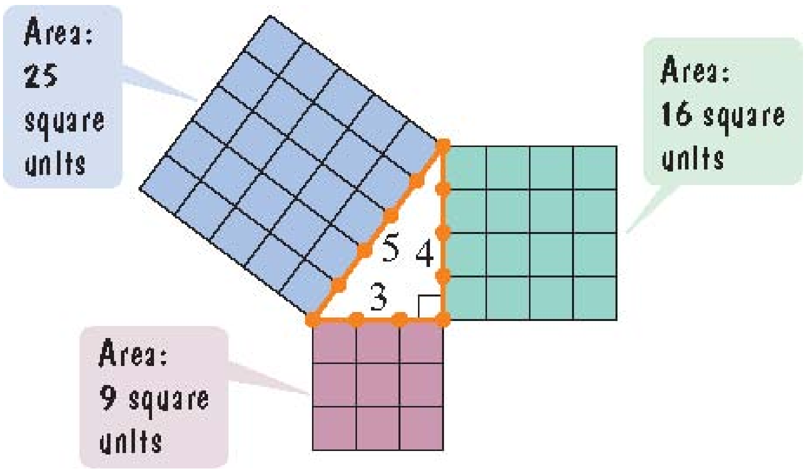
$$\frac{5x}{5}=\frac{56}{5}$$

$$x=11.2$$

The missing length of *x* is 11.2 inches.

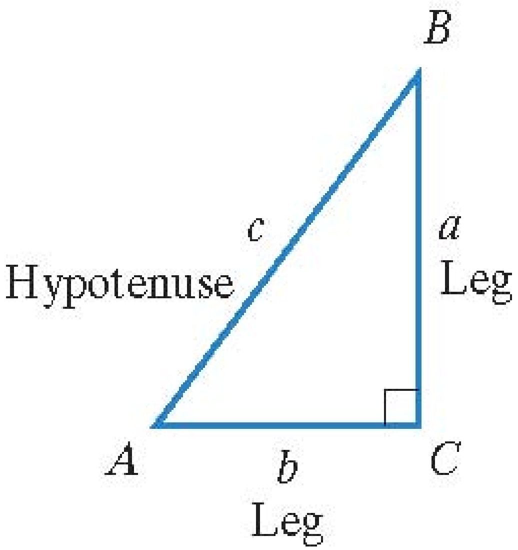
Pythagorean Theorem

The sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.



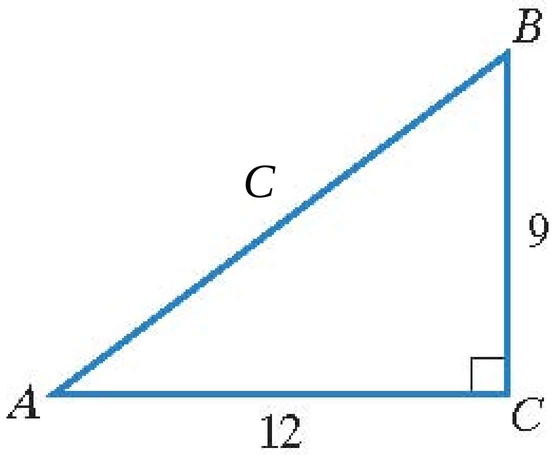
If the legs have lengths *a* and *b* and the hypotenuse has length *c*, then

$$a^2 + b^2 = c^2$$



Example: Using the Pythagorean Theorem

Find the length of the hypotenuse c in this right triangle:



Solution:

Let $a = 9$ and $b = 12$

$$c^2 = a^2 + b^2$$

$$c^2 = 9^2 + 12^2$$

$$c^2 = 81 + 144$$

$$c^2 = 225$$

$$c = \sqrt{225} = 15$$