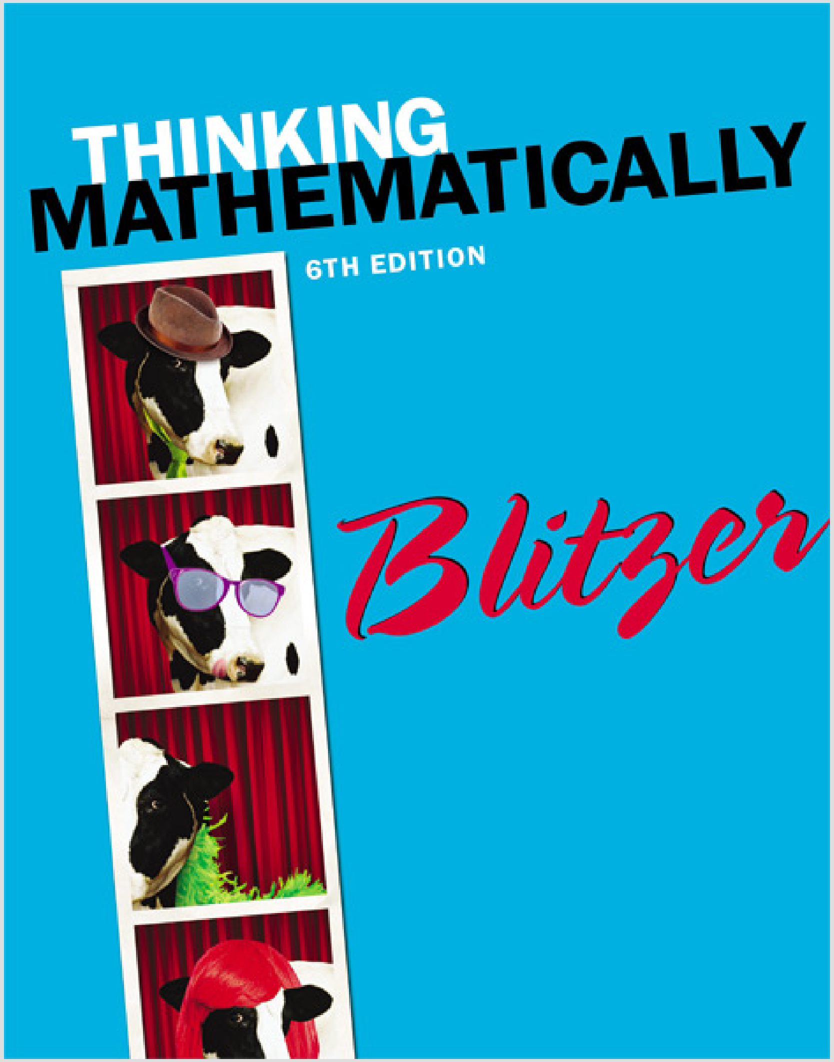
CHAPTER 10

Geometry



10.5

Volume and Surface Area

Objectives

1. Use volume formulas to compute the volumes of three-dimensional figures and solve applied problems.

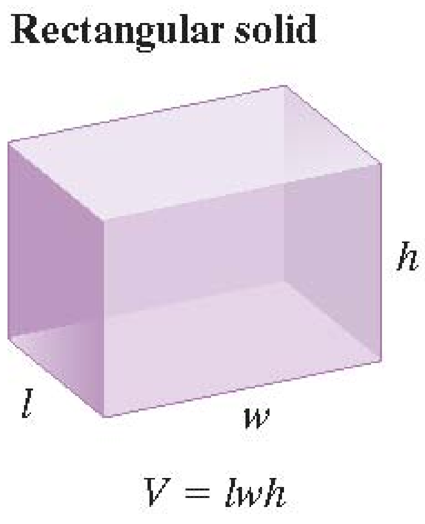
2. Compute the surface area of a three-dimensional figure.

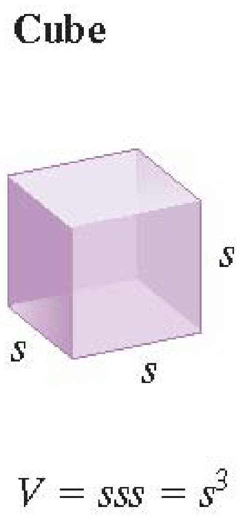
Formulas for Volume

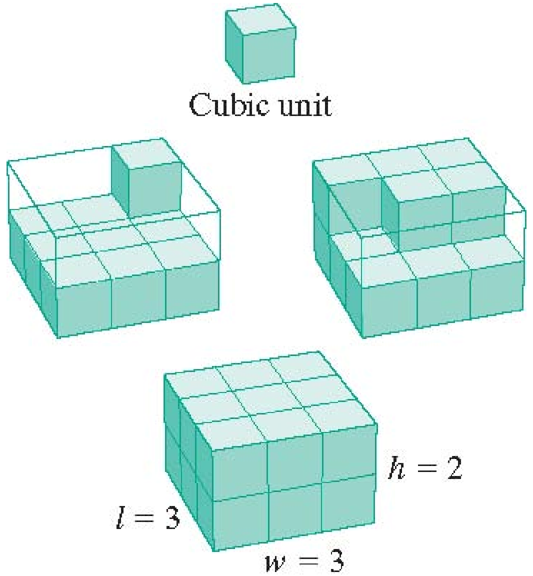
Volume of a rectangular solid, V, is the product of its length, l, its width, w, and its height, h:

$$V = lwh$$

Volumes of Boxlike Shapes







Example: Solving a Volume Problem

You are about to begin work on a swimming pool in your yard. The first step is to have a hole dug that is 90 feet long, 60 feet wide, and 6 feet deep. You will use a truck that can carry 10 cubic yards of dirt and charges $35 per load. How much will it cost you to have all the dirt hauled away?

Solution: Begin by converting feet to yards:

$$90\text{ ft} = \frac{90\ \cancel{\text{ft}}}{1} \cdot \frac{1\text{yd}}{3\ \cancel{\text{ft}}} = \frac{90}{3}\text{yd} = 30\text{ yd}$$

Similarly, 60 ft = 20 yd and 6 ft = 2 yd.

Example continued

Next, we find the volume of dirt that needs to be dug out and hauled off.

$$V = lwh = 30 \text{ yd} \cdot 20 \text{ yd} \cdot 2 \text{ yd} = 1200 \text{ yd}^3$$

Now, find the number of truckloads by dividing the number of cubic yards of dirt by 10 yards.

$$\text{Number of truckloads} = \frac{1200\text{yd}^3}{\frac{10\text{yd}^3}{\text{trip}}}$$

$$= \frac{1200\text{yd}^3}{1} \cdot \frac{\text{trip}}{10\text{yd}^3} = \frac{1200}{10}\text{trips} = 120 \text{ trips}$$

Example continued

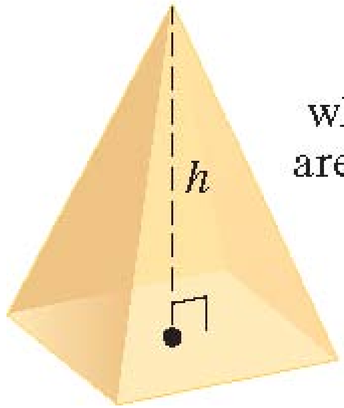
The truck charges \$35 per trip, the cost to have all the dirt hauled away is:

$$\frac{120 \text{ trips}}{1} \cdot \frac{\$35}{\text{trip}} = 120(\$35) = \$4200$$

The dirt-hauling phase of the pool project will cost you \$4200.

Example: Using the Formula for a Pyramid's Volume

The Transamerica Tower in San Francisco is a pyramid with a square base.
It is 256 meters tall and each side of the square base is 52 meters long.
Find its volume.



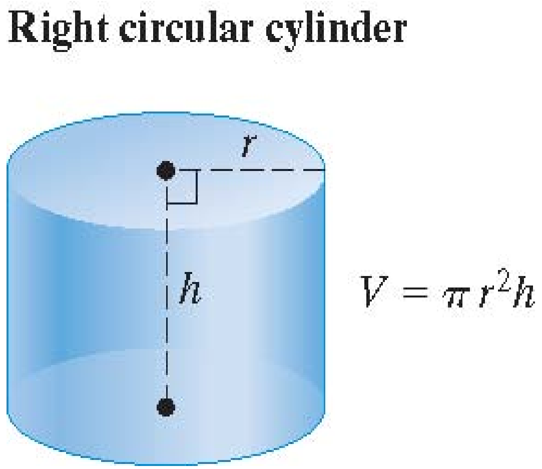
Solution: The area of the square base is:

$$B = 52 \text{ m} \cdot 52 \text{ m} = 2704 \text{ m}^2$$

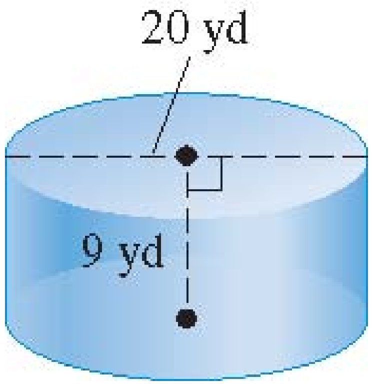
The volume of the pyramid is:

$$V = \tfrac{1}{3}Bh = \tfrac{1}{3} \cdot 2704 \text{ m}^2 \cdot 256 \text{ m} \approx 230{,}742 \text{ m}^3$$

Example:
Volume of a Right Circular Cylinder



Find the volume of this cylinder with diameter = 20 yards and height = 9 yards.



Solution:

The radius is ½ the diameter

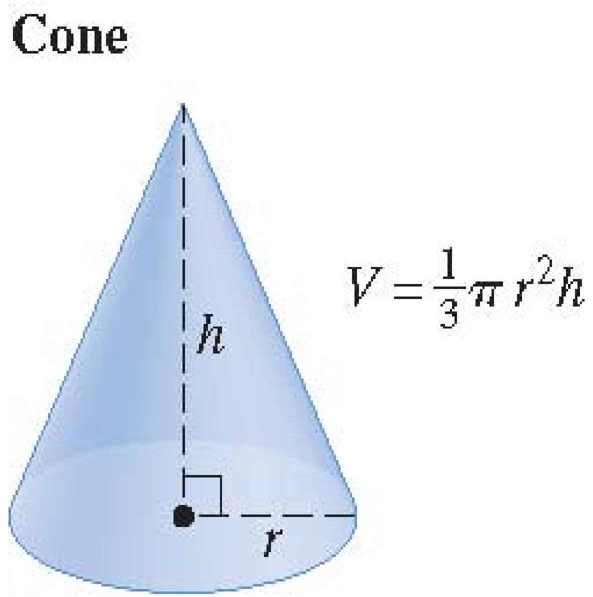
= 10 yards

$V = \pi r^2 h = \pi(10\text{ yd})^2 \cdot 9\text{ yd}$

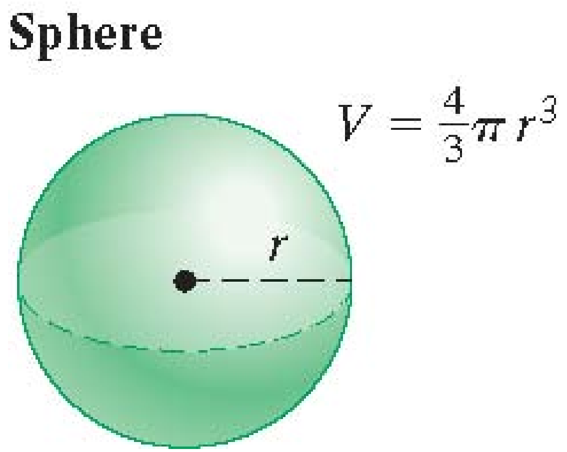
$= 900\pi\text{ yd}^3 \approx 2827\text{ yd}^3$

Volumes of a Cone and a Sphere

The Volume, *V* of a right circular cone that has height *h* and radius *r* is given by the formula:

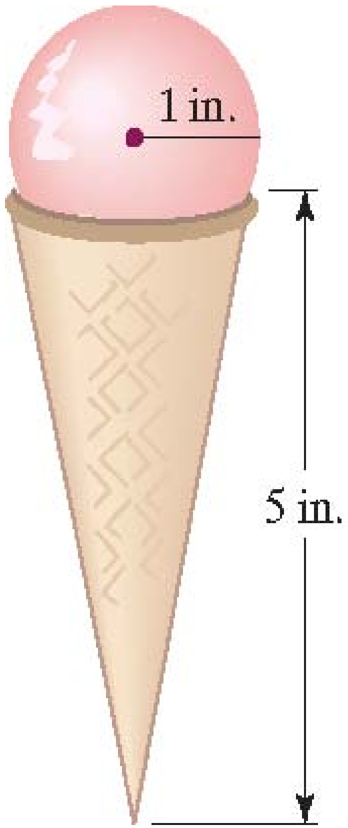


The Volume, *V* of a sphere of radius *r* is given by the formula:



Example: Applying Volume Formulas

An ice cream cone is 5 inches deep and has a radius of 1 inch. A spherical scoop of ice cream also has a radius of 1 inch. If the ice cream melts into the cone, will it overflow?

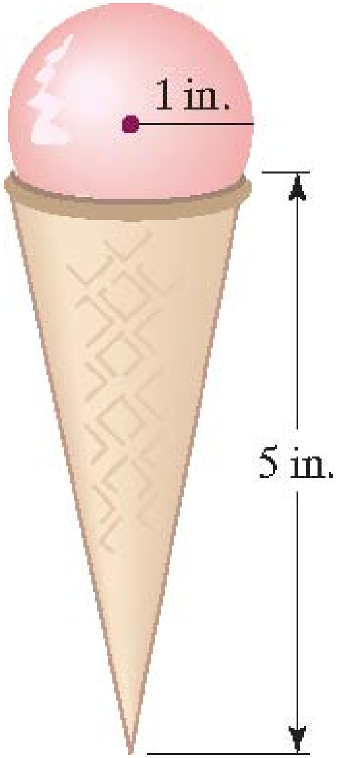


Solution: The ice cream will overflow if the volume of the ice cream, a sphere, is greater than the volume of the cone. Find the volume of each.

Example continued

$$V_{\text{cone}} = \frac{1}{3}\pi^2 h = \frac{1}{3}\pi\left(1\text{ in.}\right)^2 \cdot 5\text{ in.} = \frac{5\pi}{3}\text{in.}^3 \approx 5\text{in.}^3$$

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(1\text{ in.}\right)^3 = \frac{4\pi}{3}\text{in.}^3 \approx 4\text{in.}^3$$



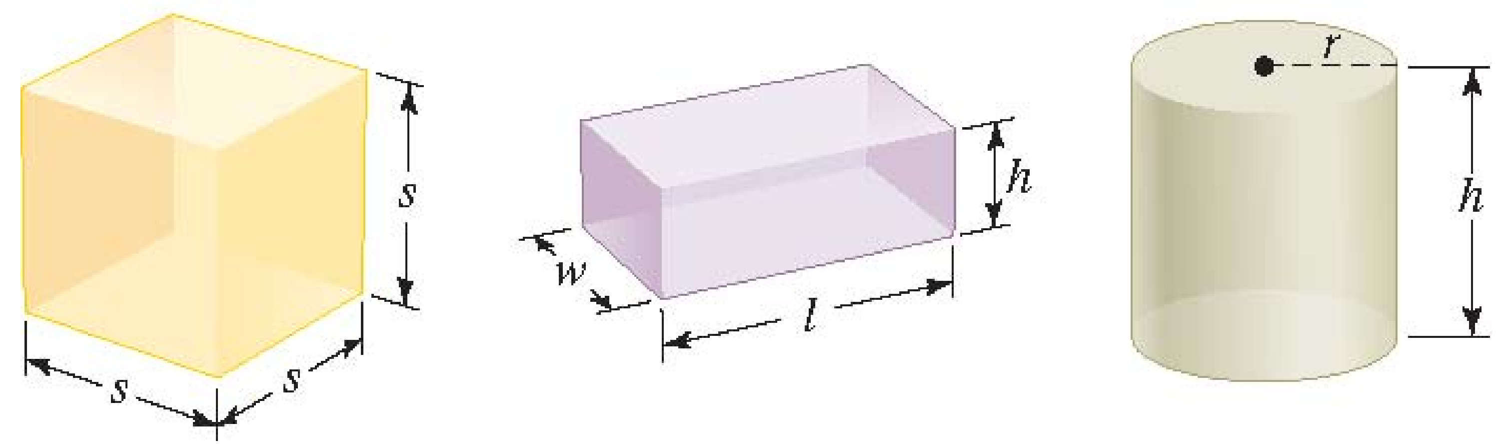
The volume of the spherical scoop of ice cream is less than the volume of the cone so there will be no overflow.

Surface Area

The area of the outer surface of a three-dimensional object.

Measured in square units

Cube	Rectangular Solid	Circular Cylinder
$SA = 6s^2$	$SA = 2lw + 2lh + 2wh$	$SA = 2\pi r^2 + 2\pi rh$

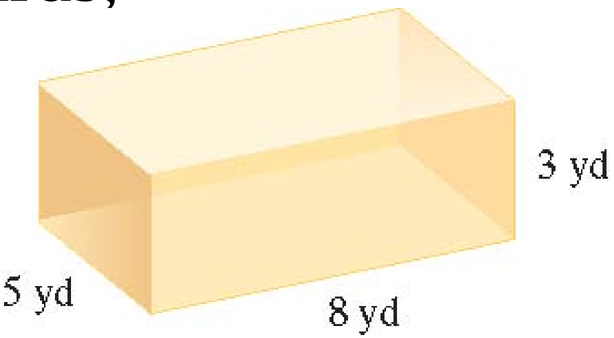


Example: Finding the Surface Area of a Solid

Find the surface area of this rectangular solid.

Solution:

The length is 8 yards, the width is 5 yards, and the height is 3 yards.



Thus, $l = 8$, $w = 5$, $h = 3$.

$$SA = 2lw + 2lh + 2wh$$

$$= 2 \cdot 8 \text{ yd} \cdot 5 \text{ yd} + 2 \cdot 8 \text{ yd} \cdot 3 \text{ yd} + 2 \cdot 5 \text{ yd} \cdot 3 \text{ yd}$$

$$= 80 \text{ yd}^2 + 48 \text{ yd}^2 + 30 \text{ yd}^2 = 158 \text{ yd}^2$$