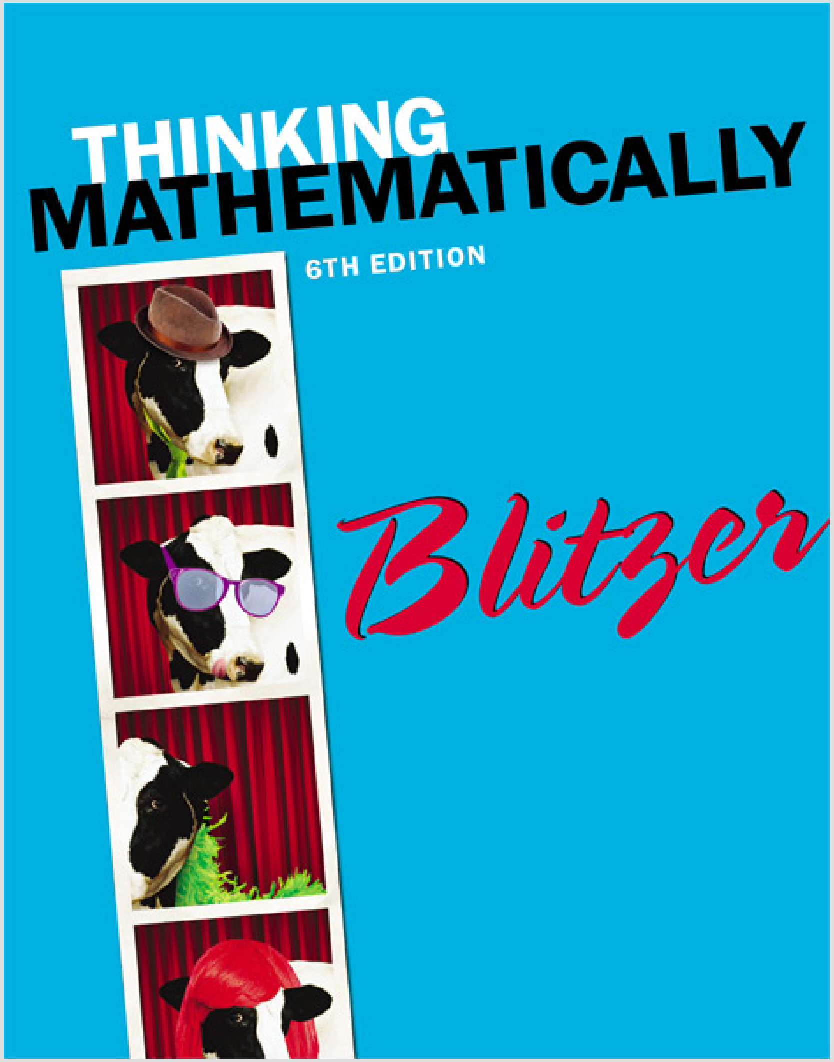
CHAPTER 11

Counting Methods and Probability Theory



11.1

The Fundamental Counting Principle

Objective

1. Use the Fundamental Counting Principle to determine the number of possible outcomes in a given situation.

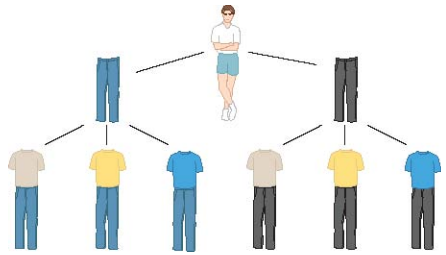
Fundamental Counting Principle

Definition

If you can choose one item from a group of *M* items and a second item from a group of *N* items, then the total number of two-item choices is $M \cdot N$.

Tree Diagram

A representation of all possible choices. This tree diagram shows that there are $2 \cdot 3 = 6$ different outfits from 2 pairs of jeans and three T-shirts.



Example: Applying the Fundamental Counting Principle

The Greasy Spoon Restaurant offers 6 appetizers and 14 main courses. In how many ways can a person order a two-course meal?

Solution:

Choosing from one of 6 appetizers and one of 14 main courses, the total number of two-course meals is:

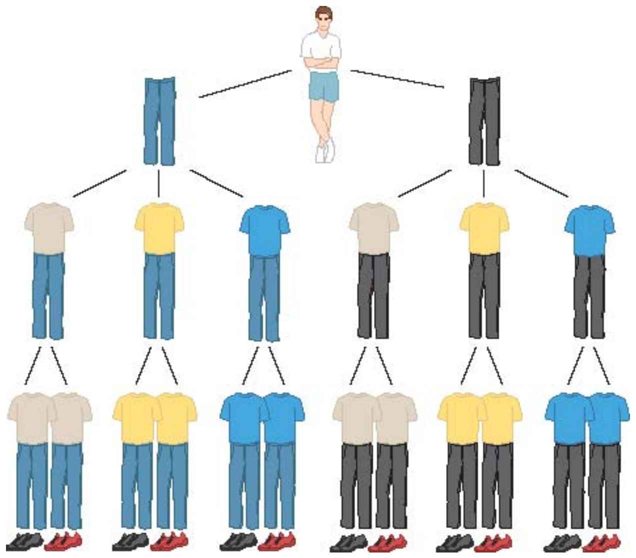
$$6 \cdot 14 = 84$$

The Fundamental Counting Principle with More than Two Groups of Items

Definition

The number of ways in which a series of successive things can occur is found by multiplying the number of ways in which each thing can occur.

The number of possible outfits from 2 pairs of jeans, 3 T-shirts and 2 pairs of sneakers are: $2 \cdot 3 \cdot 2 = 12$



Example: Options in Planning a Course Schedule

Next semester, you are planning to take three courses – math, English and humanities. There are 8 sections of math, 5 of English, and 4 of humanities that you find suitable. Assuming no scheduling conflicts, how many different three-course schedules are possible?

Solution:

This situation involves making choices with three groups of items.

Math	English	Humanities
{8 choices}	{5 choices}	{4 choices}

There are $8 \cdot 5 \cdot 4 = 160$ different three-course schedules.

Example: A Multiple-Choice Test

You are taking a multiple-choice test that has ten questions. Each of the questions has four answer choices, with one correct answer per question. If you select one of these four choices for each question and leave nothing blank, in how many ways can you answer the questions?

Solution:

This situation involves making choices with ten questions:

Question 1	**Question 2**	**Question 3** · · ·	**Question 9**	**Question 10**
{4 choices}	{4 choices}	{4 choices}	{4 choices}	{4 choices}

The number of different ways you can answer the questions is:

$$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^{10} = 1{,}048{,}576$$

Example: Telephone Numbers in the United States

Telephone numbers in the United States begin with three- digit area codes followed by seven-digit local telephone numbers. Area codes and local telephone numbers cannot begin with 0 or 1. How many different telephone numbers are possible?

Solution:

This situation involves making choices with ten groups of items. Here are the choices for each of the ten groups of items:

Area Code	Local Telephone Number
8 10 10	8 10 10 10 10 10 10

The total number of different telephone numbers is:

$$8 \cdot 10 \cdot 10 \cdot 8 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 6{,}400{,}000{,}000$$