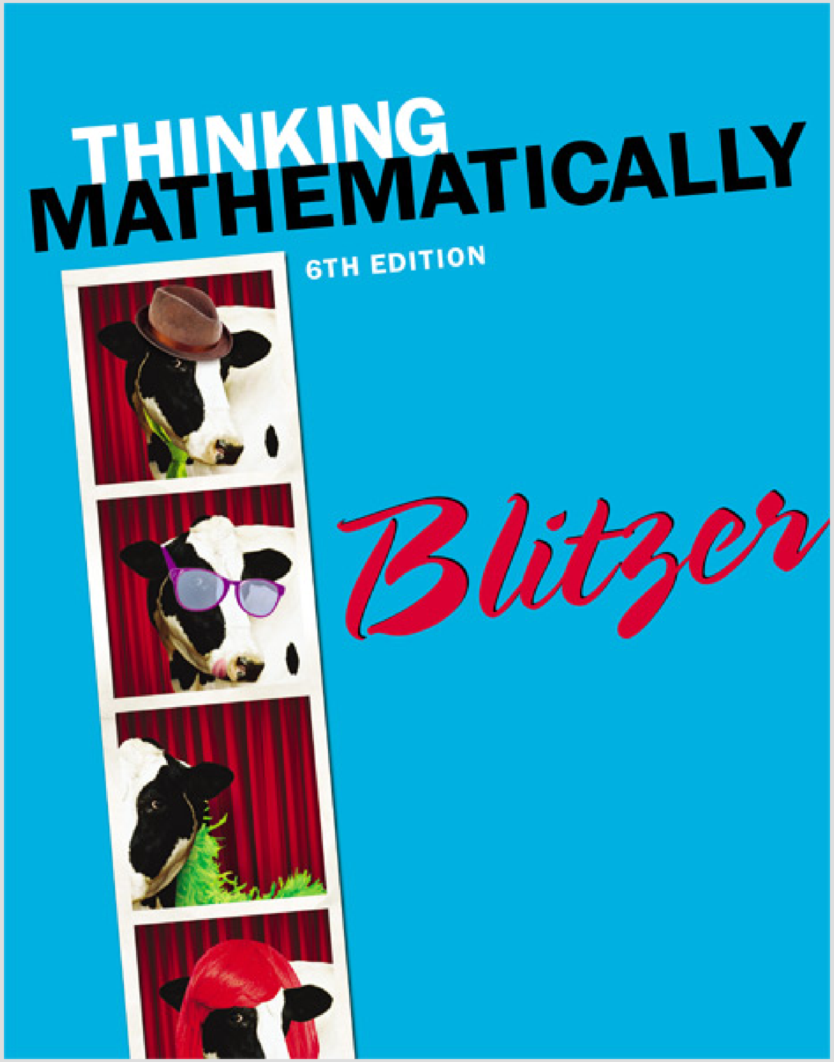
CHAPTER 11

Counting Methods and Probability Theory



11.2

Permutations

Objectives

1. Use the Fundamental Counting Principle to count permutations.

2. Evaluate factorial expressions.

3. Use the permutations formula.

4. Find the number of permutations of duplicate items.

Permutations

Permutation is an ordered arrangement of items that occurs when:

No item is used more than once.

The order of arrangement makes a difference.

Example: Counting Permutations

You need to arrange seven of your favorite books along a small shelf. How many different ways can you arrange the books, assuming that the order of the books makes a difference to you?

Solution:

You can choose any one of the seven books for the first position on the shelf. This leave six choices for the second position. After the first two positions are filled, there are five books to choose and so on.

$$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$$

Factorial Notation

The product $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ is called 7 factorial and is written 7!

Definition:

If n is a positive integer, the notation $n!$ (read "*n* factorial") is the product of all positive integers from n down through 1.

$$n! = n(n-1)(n-2)\cdots(3)(2)(1)$$

0! (zero factorial), by definition, is 1.

$$0! = 1$$

Example: Using Factorial Notation

Evaluate without using your calculator:

$$a.\ \frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5!} = 8 \cdot 7 \cdot 6 = 336$$

$$b.\ \frac{26!}{21!} = \frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21!}{21!} = 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 = 7{,}893{,}600$$

A Formula for Permutations

The number of possible permutations of *r* items are taken from *n* items is:

$$_nP_r = \frac{n!}{(n-r)!}$$

Example: Using the Formula for Permutations

You and 19 of your friends have decided to form a business. The group needs to choose three officers– a CEO, an operating manager, and a treasurer. In how many ways can those offices be filled?

Solution:

Your group is choosing r = 3 officers from a group of n = 20 people. The order matters because each officer has different responsibilities:

$$_{n}P_{r} = {}_{20}P_{3} = \frac{20!}{(20-3)!} = \frac{20!}{17!} = \frac{20 \cdot 19 \cdot 18 \cdot \cancel{17!}}{\cancel{17!}} = 20 \cdot 19 \cdot 18 = 6840$$

A Formula for Permutations of Duplicate Items

Permutations of Duplicate Items

The number of permutations of n items, where p items are identical, q items are identical, r items are identical, and so on, is given by:

$$\frac{n!}{p!q!r!\ldots}$$

Example: Using the Formula for Permutations of Duplicate Items

In how many distinct ways can the letters of the word MISSISSIPPI be arranged?

Solution:

The word contains 11 letters (n = 11) where four Is are identical (p = 4), four Ss are identical (q = 4) and 2 Ps are identical (r = 2). The number of distinct permutations are:

$$\frac{n!}{p!q!r!} = \frac{11!}{4!4!2!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}\, 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 34,650$$