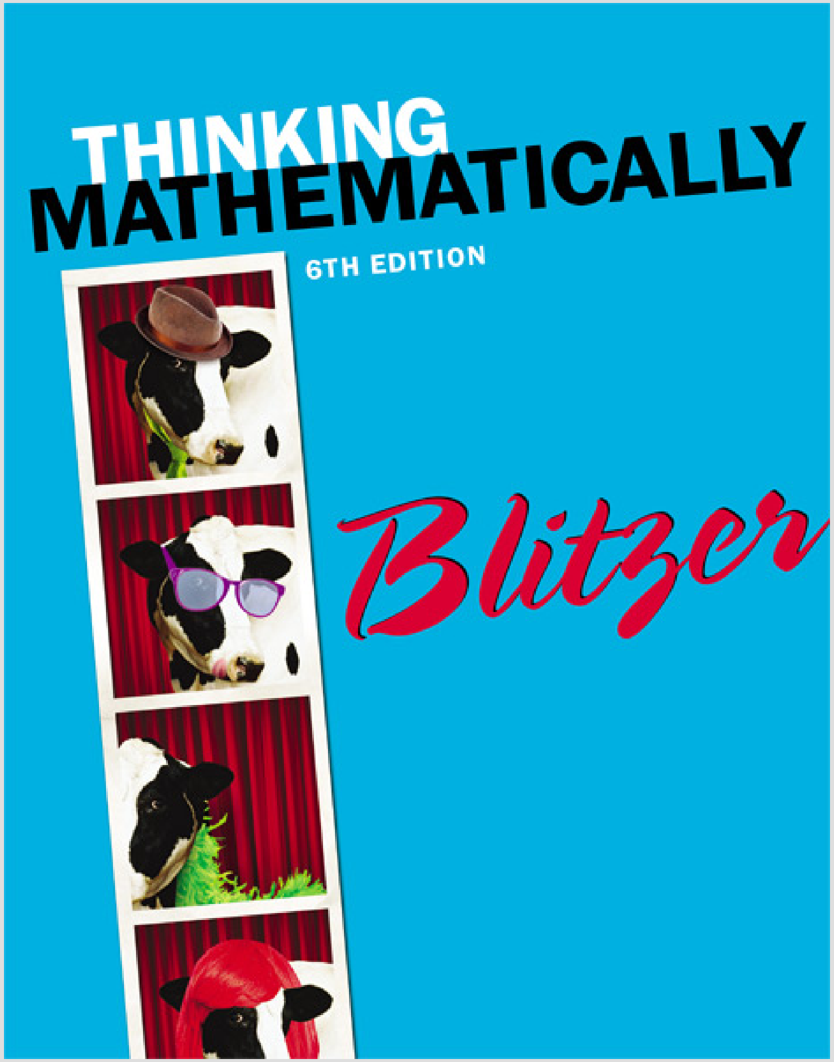
CHAPTER 11

Counting Methods and Probability Theory



11.3

Combinations

Objectives

1. Distinguish between permutation and combination problems.

2. Solve problems involving combinations using the combinations formula.

Combinations

A **combination** of items occurs when

The items are selected from the same group.

No item is used more than once.

The order of items makes no difference.

Note:

Permutation problems involve situations in which **order matters.**

Combination problems involve situations in which the **order** of items **makes no difference**.

Example: Distinguishing Between Permutations and Combinations

Determine which involve permutations and which involve combinations.

Six students are running for student government president, vice-president and treasure. The student with the greatest number of votes becomes the president, the second highest vote-getter becomes vice-president, and the student who gets the third largest number of votes will be treasurer. How many different outcomes are possible?

Solution:

Order matters since the number of the three highest votes determine the office. Permutations

Example: Distinguishing Between Permutations and Combinations

Six people are on the board of supervisors for your neighborhood park. A three-person committee is needed to study the possibility of expanding the park. How many different committees could be formed from the six people?

Solution:

The order in which the three people are selected does not matter since they are not filling different roles. Combinations

Comparing Combinations and Permutations

Given the letters: A,B,C,D: We can compare how many permutations and how many combinations are possible if we chose 3 letters at a time:

ABC,	ABD,	ACD,	BCD,
ACB,	ADB,	ADC,	BDC,
BAC,	BAD,	CAD,	CBD,
BCA,	BDA,	CDA,	CDB,
CAB,	DAB,	DAC,	DBC,
CBA,	DBA,	DCA,	DCB.
This column contains only one combination, ABC.	This column contains only one combination, ABD.	This column contains only one combination, ACD.	This column contains only one combination, BCD.

A Formula for Combinations

The number of possible combinations if *r* items are taken from *n* items is:

$$ {}_{n}C_{r} = \frac{n!}{(n-r)!r!} $$

Example: Using the Formula for Combinations

How many three-person committees could be formed from 8 people?

Solution: We are selecting 3 people (r = 3) from 8 people (n = 8)

$${}_nC_r = {}_8C_3 = \frac{8!}{(8-3)!3!} = \frac{8!}{5!3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 3 \cdot 2 \cdot 1} = 56$$

Example: Using the Formula for Combinations and the Fundamental Counting Principle

In December, 2011, the U.S Senate consisted of 51 Democrats and 47 Republicans and 2 Independents. How many distinct five-person committees can be formed if each committee must have 3 Democrats and 2 Republicans?

Solution:

The order in which members are selected does not matter so this is a problem of combinations.

Picking 3 Democrats out of 51.

$$ {}_{n}C_{r} = {}_{51}C_{3} = \frac{51!}{(51-3)!3!} = \frac{51!}{48!3!} = \frac{51\cdot 50\cdot 49\cdot \cancel{48}!}{\cancel{48}!\cdot 3\cdot 2\cdot 1} = \frac{51\cdot 50\cdot 49}{3\cdot 2\cdot 1} = 20{,}825 $$

Example: Using the Formula for Combinations and the Fundamental Counting Principle continued

Select 2 Republicans out of 47.

$$ {}_{n}C_{r} = {}_{47}C_{2} = \frac{47!}{(47-2)!2!} = \frac{47!}{45!2!} = \frac{47 \cdot 46 \cdot \cancel{45}!}{\cancel{45}! \cdot 2 \cdot 1} = \frac{47 \cdot 46}{2 \cdot 1} = 1081 $$

Use the Fundamental Counting Principle to find the number of committees that can be formed.

$$ {}_{51}C_{3} \cdot {}_{47}C_{2} = 20,825 \cdot 1081 = 22,511,825 $$