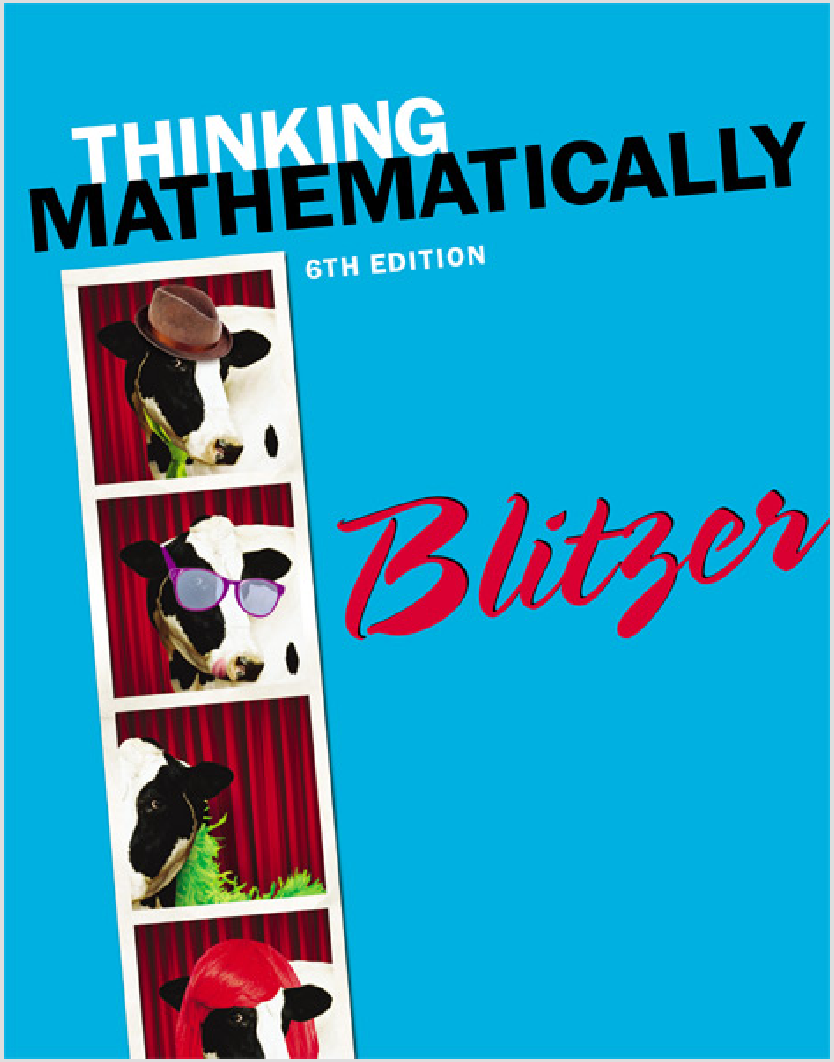
CHAPTER 11

Counting Methods and Probability Theory



11.4

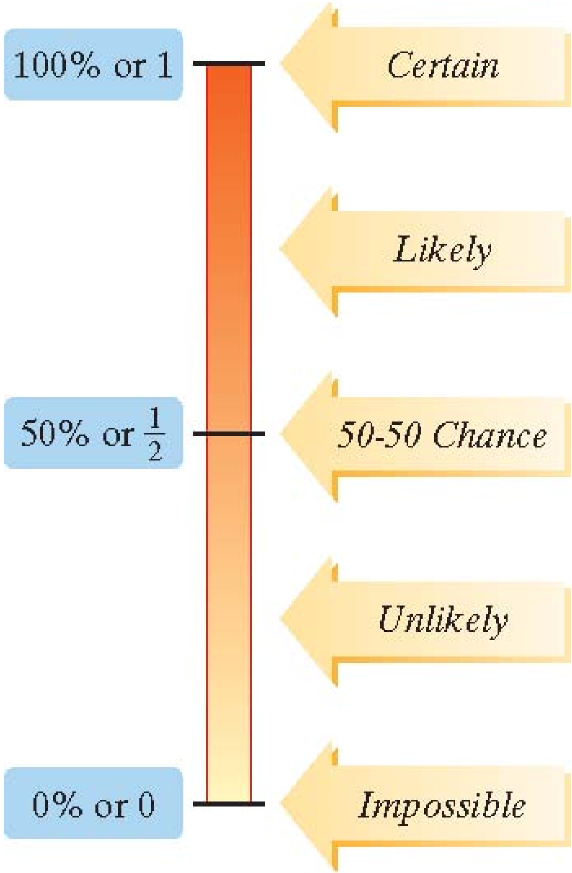
Fundamentals of Probability

Objectives

1. Compute theoretical probability.

2. Compute empirical probability.

Probability



Possible Values for Probabilities

Probabilities are assigned values from 0 to 1.

The closer the probability of a given event is to 1, the more likely it is that the event will occur.

The closer the probability of a given event is to 0, the less likely that the event will occur.

Theoretical Probability

Experiment is any occurrence for which the outcome is uncertain.

Sample space is the set of all possible outcomes of an experiment , denoted by *S*.

Event, denoted by *E* is any subset of a sample space.

Sum of the theoretical probabilities of all possible outcomes is 1.

Computing Theoretical Probability

If an event E has $n(E)$ equally likely outcomes and its sample space S has $n(S)$ equally-likely outcomes, the theoretical probability of event E, denoted by $P(E)$, is:

$$P(E) = \frac{\text{number of outcomes in event } E}{\text{total number of possible outcomes}} = \frac{n(E)}{n(S)}$$

Example: Computing Theoretical Probability

A die is rolled once. Find the probability of rolling

a. 3

b. an even number

Solution: The sample space is $S = \{1,2,3,4,5,6\}$

a. There is only one way to roll a 3 so $n(E) = 1$.

$$P(3) = \frac{\text{number of outcomes that result in 3}}{\text{total number of possible outcomes}} = \frac{n(E)}{n(S)} = \frac{1}{6}$$

b. Rolling an even number describes the event $E = \{2,4,6\}$. This event can occur in 3 ways: $n(E) = 3$.

P(even number) =

$$\frac{\text{number of outcomes that result in even number}}{\text{total number of possible outcomes}} = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

Example: Probability and a Deck of 52 Cards

You are dealt one card from a standard 52-card deck. Find the probability of being dealt a

a. a king b. a heart

Solution:

$$\text{a. } P(king) = \frac{\text{king outcomes}}{\text{total possible outcomes}} = \frac{4}{52} = \frac{1}{13}$$

$$\text{b. } P(heart) = \frac{\text{heart outcomes}}{\text{total possible outcomes}} = \frac{13}{52} = \frac{1}{4}$$

Empirical Probability

Applies to situations in which we observe how frequently an event occurs.

Computing Empirical Probability

The empirical probability of event E is:

$$P(E) = \frac{\text{observed number of times } E \text{ occurs}}{\text{total number of observed occurrences}} = \frac{n(E)}{n(S)}$$

Example: Computing Empirical Probability

	Married	Never Married	Divorced	Widowed	Total
Male	65	40	10	3	118
Female	65	34	14	11	124
Total	130	74	24	14	242

If one person is randomly selected from the population described above, find the probability that the person is female.

Solution: The probability of selecting a female is the observed number of females, 124 (million), divided by the total number of U.S. adults, 242 (million).

$$P(\textit{female}) = \frac{\text{females}}{\text{total number of adults}} = \frac{124}{242} \approx 0.51$$