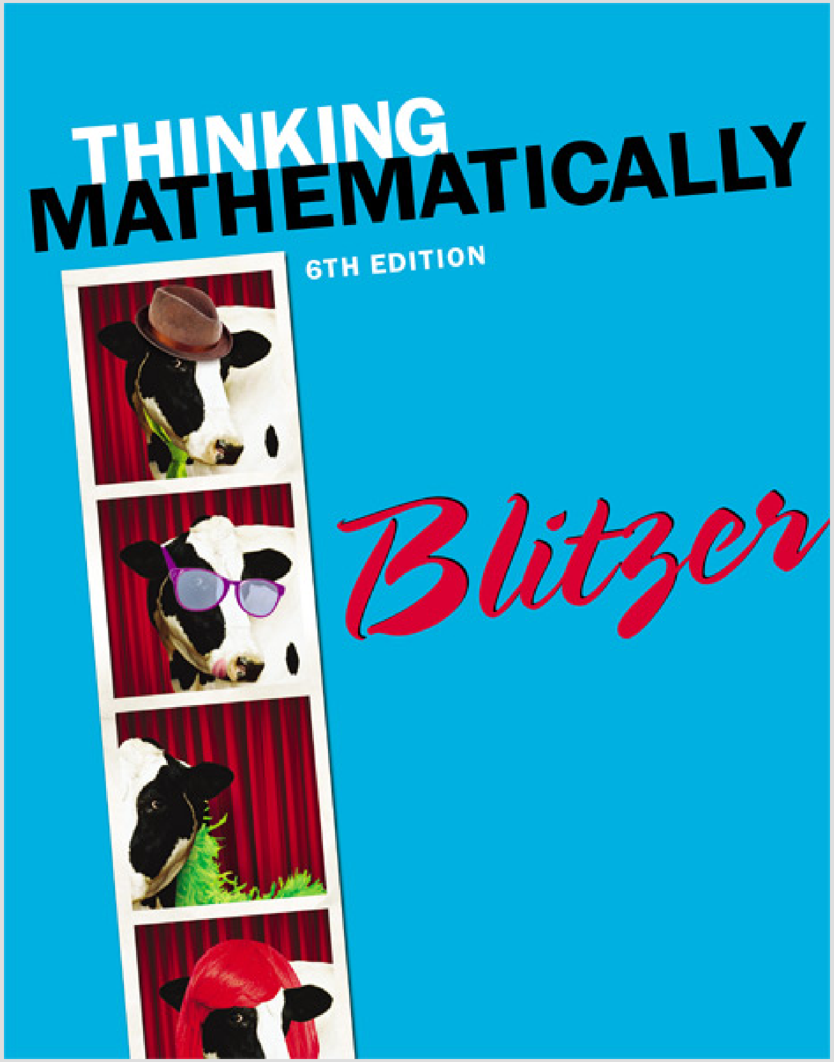
CHAPTER 11

Counting Methods and Probability Theory



11.5

Probability with the Fundamental Counting Principle, Permutations, and Combinations

Objectives

1. Compute probabilities with permutations.

2. Compute probabilities with combinations.

Example: Probability and Permutations

Six jokes about books by Groucho Marx, Bob Blitzer, Steven Wright, Henry Youngman, Jerry Seinfeld, and Phyllis Diller are each written on one of six cards. The cards are placed in a hat and drawn one at a time. What is the probability that a man's joke will be delivered first and a woman's joke last?

Solution:

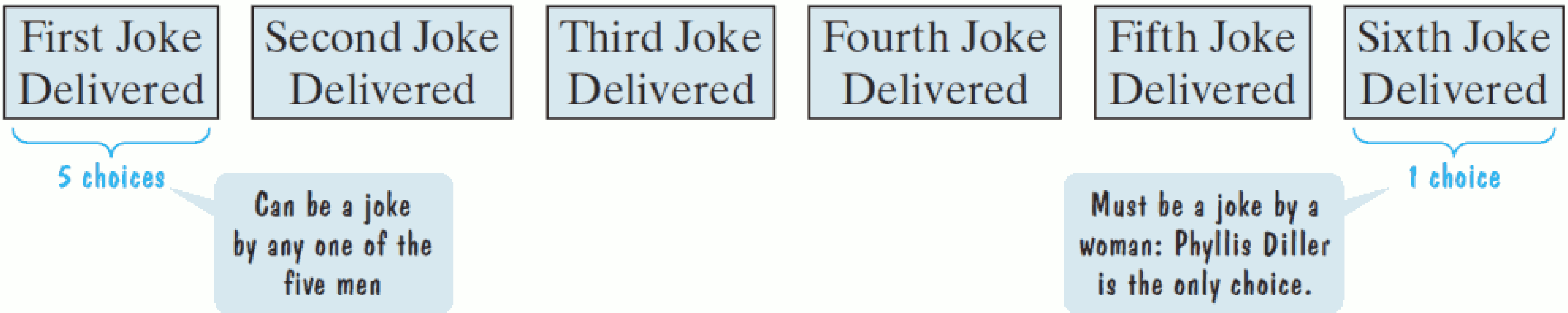
$$P(\text{man first, woman last}) = \frac{\text{number of permutations man's joke first, woman's joke last}}{\text{total number of possible permutations}}$$

First Joke Delivered	Second Joke Delivered	Third Joke Delivered	Fourth Joke Delivered	Fifth Joke Delivered	Sixth Joke Delivered
6 choices	5 choices	4 choices	3 choices	2 choices	1 choice

$$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720 \text{ permutations}$$

Example: Probability and Permutations continued

Use the Fundamental Counting Principle to find the number of permutations with a man's joke first and a woman's joke last:



$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 120 \text{ permutations}$$

$$P(\text{man first, woman last}) = \frac{120}{720} = \frac{1}{6}$$

Example: Probability and Combinations: Winning the Lottery

Powerball is a multi-state lottery played in most U.S. states. It is the first lottery game to randomly draw numbers from two drums. The game is set up so that each player chooses five different numbers from 1 to 59 and one Powerball number from 1 to 35. Twice per week 5 white balls are drawn randomly from a drum with 59 white balls, numbered 1 to 59, and then one red Powerball is drawn randomly from a drum with 35 red balls, numbered 1 to 35. A player wins the jackpot by matching all five numbers drawn from the white balls in any order and matching the number on the red Powerball. With one $2 Powerball ticket, what is the probability of winning the jackpot?

Example: Probability and Combinations: Winning the Lottery

Because the order of the five numbers shown on the white balls does not matter, this is a situation involving combinations. We begin with the formula for probability.

$$P(\text{winning}) = \frac{\text{number of ways of winning}}{\text{total number of possible combinations}}$$

$$_{59}C_5 = \frac{59!}{(59-5)!5!} = \frac{59!}{54!5!} = \frac{59 \cdot 58 \cdot 57 \cdot 56 \cdot 55 \cdot \cancel{54}}{\cancel{54}! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 5{,}006{,}386$$

Next, we must determine the number of ways of selecting the red Powerball. Because there are 35 red Powerballs in the second drum, there are 35 possible combinations of numbers.

Example: Probability and Combinations: Winning the Lottery

We can use the Fundamental Counting Principle to find the total number of possible number combinations in Powerball.

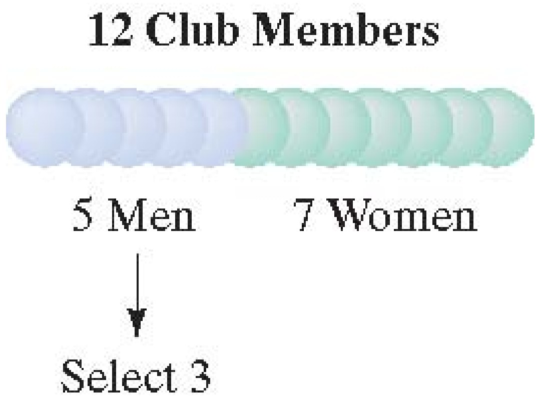
$$ {}_{59}C_5 \cdot 35 = 5,006,386 \cdot 35 = 175,223,510 $$

There are 175,223,510 number combinations in Powerball. If a person buys one $2 ticket, that person has selected only one combination of the numbers. With one Powerball ticket, there is only one way of winning the jackpot.

$$ \frac{1}{175,223,510} \approx 5.707 \times 10^{-9} $$

Example: Probability and Combinations

A club consists of five men and seven women. Three members are selected at random to attend a conference. Find the probability that the selected group consists of 3 men.



Solution:

Order of selection does not matter, so this is a problem involving combinations.

$$P(3\text{ men}) = \frac{\text{number of ways of selecting 3 men}}{\text{total number of possible combinations}}$$

Example: Probability and Combinations continued

A club consists of five men and seven women. Three members are selected at random to attend a conference.

Find the probability that the selected group consists of 3 men.

Solution:

$$P(3\text{ men}) = \frac{\text{number of ways of selecting 3 men}}{\text{total number of possible combinations}}$$

$${}_{12}C_3 = \frac{12!}{(12-3)!3!} = \frac{12!}{9!3!} = \frac{12\cdot 11\cdot 10\cdot 9!}{9!\cdot 3\cdot 2\cdot 1} = 220$$

There are 220 possible three-person selections.

Example: Probability and Combinations continued

A club consists of five men and seven women. Three members are selected at random to attend a conference. Find the probability that the selected group consists of 3 men.

Solution: Determine the numerator. We are interested in the number of ways of selecting three men from 5 men.

$$_5C_3 = \frac{5!}{(5-3)!3!} = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3!}{3! \cdot 2 \cdot 1} = 10$$

$$P(3\text{ men}) = \frac{\text{number of ways of selecting 3 men}}{\text{total number of possible combinations}} = \frac{10}{220} = \frac{1}{22}$$