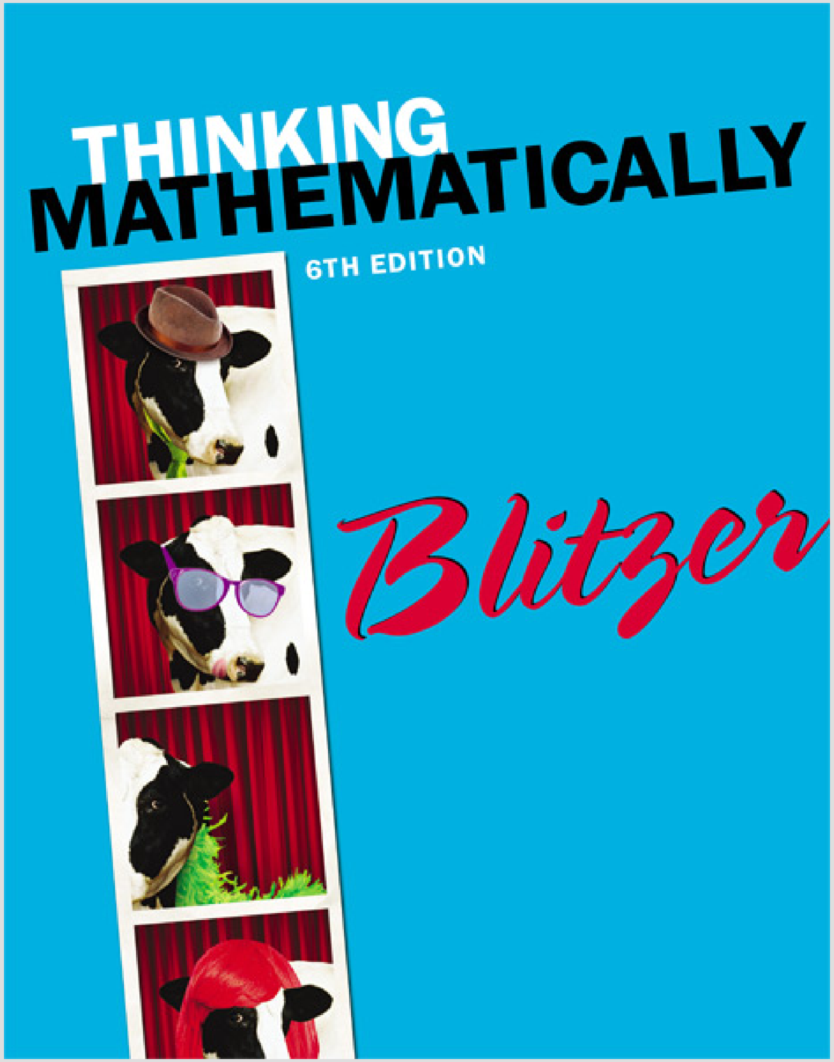
CHAPTER 11

Counting Methods and Probability Theory



11.6

Events Involving *Not* and *Or*; Odds

Objectives

1. Find the probability that an event will not occur.
2. Find the probability of one event or a second event occurring.
3. Understand and use odds.

Probability of an Event Not Occurring

Complement of E: If we know $P(E)$, the probability of an event E, we can determine the probability that the event will not occur, denoted by $P(\text{not } E)$.

The probability that an event E will not occur is equal to 1 minus the probability that it will occur.

$$P(\text{not } E) = 1 - P(E)$$

The probability that an event E will occur is equal to 1 minus the probability that it will not occur.

$$P(E) = 1 - P(\text{not } E)$$

Using set notation, if E' is the complement of E, then

$$P(E') = 1 - P(E) \text{ and } P(E) = 1 - P(E')$$

Example: The Probability of an Event Not Occurring

If you are dealt one card from a standard 52-card deck, find the probability that you are not dealt a queen.

Solution:

Because $P(\text{not } E) = 1 - P(E)$ then

$$\begin{aligned} P(\text{not a queen}) &= 1 - P(\text{queen}) \\ &= 1 - \frac{1}{13} \\ &= \frac{13}{13} - \frac{1}{13} \\ &= \frac{12}{13} \end{aligned}$$

Or Probabilities with Mutually Exclusive Events

Mutually Exclusive Events: Events A and B are mutually exclusive if it is impossible for them to occur simultaneously.

Or **Probabilities with Mutually Exclusive Events:**

If A and B are mutually exclusive events, then

$$P(A \text{ or } B) = P(A) + P(B)$$

Using set notation,

$$P(A \cup B) = P(A) + P(B)$$

Example: The Probability of Either of Two Mutually Exclusive Events Occurring

If one card is randomly selected from a deck of cards, what is the probability of selecting a king or a queen?

Solution:

$$\text{P (king or queen)} = P(\text{king}) + P(\text{queen})$$

$$= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

Or Probabilities with Events That Are Not Mutually Exclusive

If A and B are not mutually exclusive events then the probability that A or B will occur is determined by adding their individual probabilities and then subtracting the probability that A and B occur simultaneously.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Using set notation,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example: An *OR* Probability With Events That Are Not Mutually Exclusive

In a group of 25 baboons, 18 enjoy grooming their neighbors, 16 enjoy screeching wildly, while 10 enjoy doing both. If one baboon is selected at random, find the probability that it enjoys grooming its neighbors or screeching wildly.

Solution:

Since 10 of the baboons enjoy both grooming their neighbors and screeching wildly, these events are not mutually exclusive.

Example: An *OR* Probability With Events That Are Not Mutually Exclusive continued

$$P\begin{pmatrix}\text{grooming}\\ \text{or screeching}\end{pmatrix} = P(\text{grooming}) + P(\text{screeching}) - P\begin{pmatrix}\text{grooming}\\ \text{and screeching}\end{pmatrix}$$

$$= \frac{18}{25} + \frac{16}{25} - \frac{10}{25}$$

18 of the 25 baboons enjoy grooming.

16 of the 25 baboons enjoy screeching.

10 of the 25 baboons enjoy both.

$$= \frac{18 + 16 - 10}{25} = \frac{24}{25}$$

Probability to Odds

If we know the probability of an event, we can determine the odds *in favor*, or the odds *against*, the event.

1. The **odds in favor of *E*** are found by taking the probability that *E* will occur and dividing by the probability that *E* will not occur.

$$\text{Odd in favor of } E = \frac{P(E)}{P(notE)}$$

2. The **odds against E** are found by taking the probability that E will not occur and dividing by the probability that E will occur.

$$\text{Odd against } E = \frac{P(\text{not } E)}{P(E)}$$

The odds against *E* can also be found by reversing the ratio representing the odds in favor of *E*

Example: From Probability to Odds

You roll a single, six-sided die. Find the odds in favor of rolling a 2

Solution:

Let E represent the event of rolling a 2. Find the probability of E occurring and the probability of E not occurring.

$$S = \{1,2,3,4,5,6\} \qquad E = \{2\}$$

$$P(E) = \frac{1}{6} \qquad P(\text{not } E) = 1 - \frac{1}{6} = \frac{5}{6}$$

$$\frac{P(E)}{P(\text{not}E)} \frac{\frac{1}{6}}{\frac{5}{6}} = \frac{1}{6} \cdot \frac{6}{5} = \frac{1}{5}$$

Odds to Probability

Odds to Probability

If the odds in favor of event E are a to b, then the probability of the event is given by:

$$P(E) = \frac{a}{a+b}.$$

Example: Odds to Probability

The odds in favor of a particular horse winning a race are 2 to 5. What is the probability this horse will win the race?

Solution:

Because odds in favor, *a* to *b*, means a probability of $\frac{a}{a+b}$ then the odds in favor, 2 to 5 means a probability of

$$\frac{2}{2+5}=\frac{2}{7}$$

The probability that this horse will win the race is 2/7.