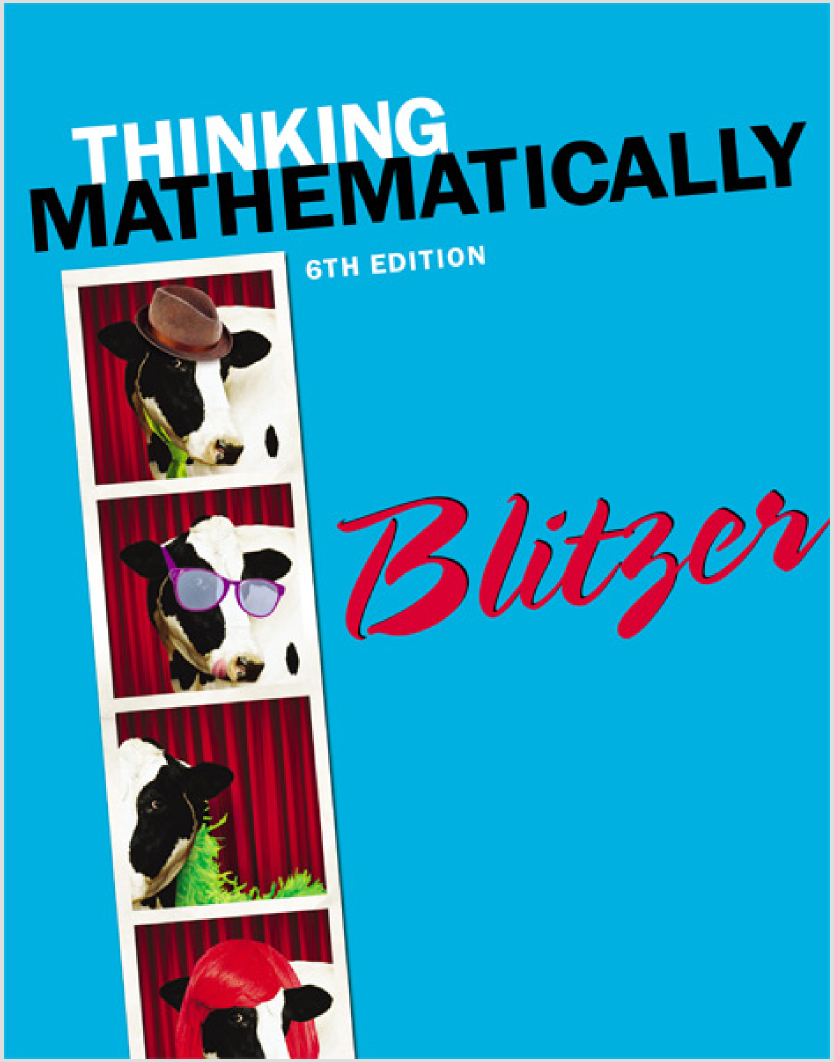
CHAPTER 11

Counting Methods and Probability Theory



11.7

Events Involving *And;* Conditional Probability

Objectives

1. Find the probability of one event and a second event occurring.
2. Compute conditional probabilities.

And Probabilities with Independent Events

Independent Events: Two events are independent events if the occurrence of either of them has no effect on the probability of the other.

***And* Probabilities with Independent Events**

If A and B are independent events, then

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Example: Independent Events on a Roulette Wheel

A U.S. roulette wheel has 38 numbered slots (1 through 36, 0, and 00). 18 are black, 18 are red, and 2 are green. The ball can land on any slot with equal probability. What is the probability of red occurring on 2 consecutive plays?

Solution:

The probability of red occurring on a play is 18/38 or 9/19.

$$P(\text{red and red}) = P(\text{red}) \cdot P(\text{red}) = \frac{9}{19} \cdot \frac{9}{19} = \frac{81}{361} \approx 0.224$$

Example: Independent Events In a Family

If two or more events are independent, we can find the probability of them all occurring by multiplying their probabilities. The probability of a baby girl is ½, so the probability of nine girls in a row is ½ used as a factor nine times.

Solution:

$P(\text{ nine girls in a row}) = \frac{1}{2}\cdot \frac{1}{2}\cdot \frac{1}{2}\cdot \frac{1}{2}\cdot \frac{1}{2}\cdot \frac{1}{2}\cdot \frac{1}{2}\cdot \frac{1}{2}\cdot \frac{1}{2}$

$$= \left(\frac{1}{2}\right)^9 = \frac{1}{512}$$

Example: Hurricanes and Probabilities

If the probability that South Florida will be hit by a hurricane in any single year is 5/19,

a. What is the probability that South Florida will be hit by a hurricane in three consecutive years?

Solution:

$$P(\text{hurricane and hurricane and hurricane})$$
$$= P(\text{hurricane}) \cdot P(\text{hurricane}) \cdot P(\text{hurricane}) = \frac{5}{19} \cdot \frac{5}{19} \cdot \frac{5}{19} = \frac{125}{6859} \approx 0.018$$

Example: Hurricanes and Probabilities

If the probability that South Florida will be hit by a hurricane in any single year is 5/19,

b. What is the probability that South Florida will not be hit by a hurricane in the next ten years?

Solution:

$$P(\text{no hurricane}) = 1 - \frac{5}{19} = \frac{14}{19} \approx 0.737$$

The probability of not being hit by a hurricane in a single year is 14/19. The probability of not being hit by a hurricane ten years in a row is 14/19 used as a factor ten times.

$$= \left(\frac{14}{19}\right)^{10} \approx (0.737)^{10} \approx 0.047$$

And Probabilities with Dependent Events

Dependent Events: Two events are dependent events if the occurrence of one of them has an effect on the probability of the other.

And Probabilities with Dependent Events

If A and B are dependent events, then

P(A and B) = $P(A) \cdot P(B$ given that A has occurred).

Example: *And* Probabilities with Dependent Events

You have won a free trip to Madrid and can take two people with you, all expenses paid. Bad news: Ten of your cousins have appeared out of nowhere and are begging you to take them. You write each cousin's name on a card, place the cards in a hat, and select one name. Then you select a second name without replacing the first card. If three of your ten cousins speak Spanish, find the probability of selecting two Spanish-speaking cousins.

Solution

$$P(\text{speaks Spanish}) \cdot P(\text{speaks Spanish given that a Spanish cousin was selected first})$$

$$= \frac{3}{10} \cdot \frac{2}{9} = \frac{6}{90} = \frac{1}{15} \approx 0.067$$

Example: *And* Probability With Three Dependent Events

Three people are randomly selected, one person at a time, from 5 freshmen, 2 sophomores, and 4 juniors. Find the probability that the first two people selected are freshmen and the third is a junior.

Solution:

$P(\text{first two are freshmen and the third is a junior})$

$$= P(\text{freshman}) \cdot P\left(\begin{array}{c}\text{freshman given that a}\\ \text{freshman was selected first}\end{array}\right) \cdot P\left(\begin{array}{c}\text{junior given that a freshman was}\\ \text{selected first and a freshman was}\\ \text{selected second}\end{array}\right)$$

$$= \frac{5}{11} \cdot \frac{4}{10} \cdot \frac{4}{9}$$

- $\frac{5}{11}$: There are 11 people, five of whom are freshmen.
- $\frac{4}{10}$: After picking a freshman, there are 10 people left, four of whom are freshmen.
- $\frac{4}{9}$: After the first two selections, 9 people are left, four of whom are juniors.

$$= \frac{8}{99}$$

Conditional Probability

CONDITIONAL PROBABILITY

The probability of event B, assuming that the event A has already occurred, is called the **conditional probability** of B, given A. This probability is denoted by $P(B \mid A)$.

Example: Finding Conditional Probability

A letter is randomly selected from the letters of the English alphabet. Find the probability of selecting a vowel, given that the outcome is a letter that precedes h.

Solution:

We are looking for P(vowel | letter precedes h).

This is the probability of a vowel if the sample space is restricted to the set of letters that precede h.

S = {a, b, c, d, e, f, g}.

Example: Finding Conditional Probability continued

A letter is randomly selected from the letters of the English alphabet. Find the probability of selecting a vowel, given that the outcome is a letter that precedes h.

Solution:

There are 7 possible outcomes in the sample space. We can select a vowel from this set in one of two ways: *a* or *e*.

The probability of selecting a vowel that precedes h, is 2/7.

Example: Conditional Probabilities with Real-World Data

Mammography Screening on 100,000 U.S. Women, Ages 40 to 50			
	Breast Cancer	No Breast Cancer	Total
Positive Mammogram	720	6,944	7,664
Negative Mammogram	80	92,256	92,336
Total	800	99,200	100,000

Assuming that these numbers are representative of all U.S. women age 40 to 50, find the probability that a woman in this age range has a positive mammogram, given that she does not have breast cancer.

Example: Conditional Probabilities with Real-World Data continued

Mammography Screening on 100,000 U.S. Women, Ages 40 to 50			
	Breast Cancer	No Breast Cancer	Total
Positive Mammogram	720	6,944	7,664
Negative Mammogram	80	92,256	92,336
Total	800	99,200	100,000

Solution:

$P(\text{positive mammogram}|\text{no cancer})$

There are 6944 + 92,256 or 99,200 women without breast cancer.

$$P(\text{positive mammogram}|\text{no breast cancer}) = \frac{6944}{99,200} = 0.07$$