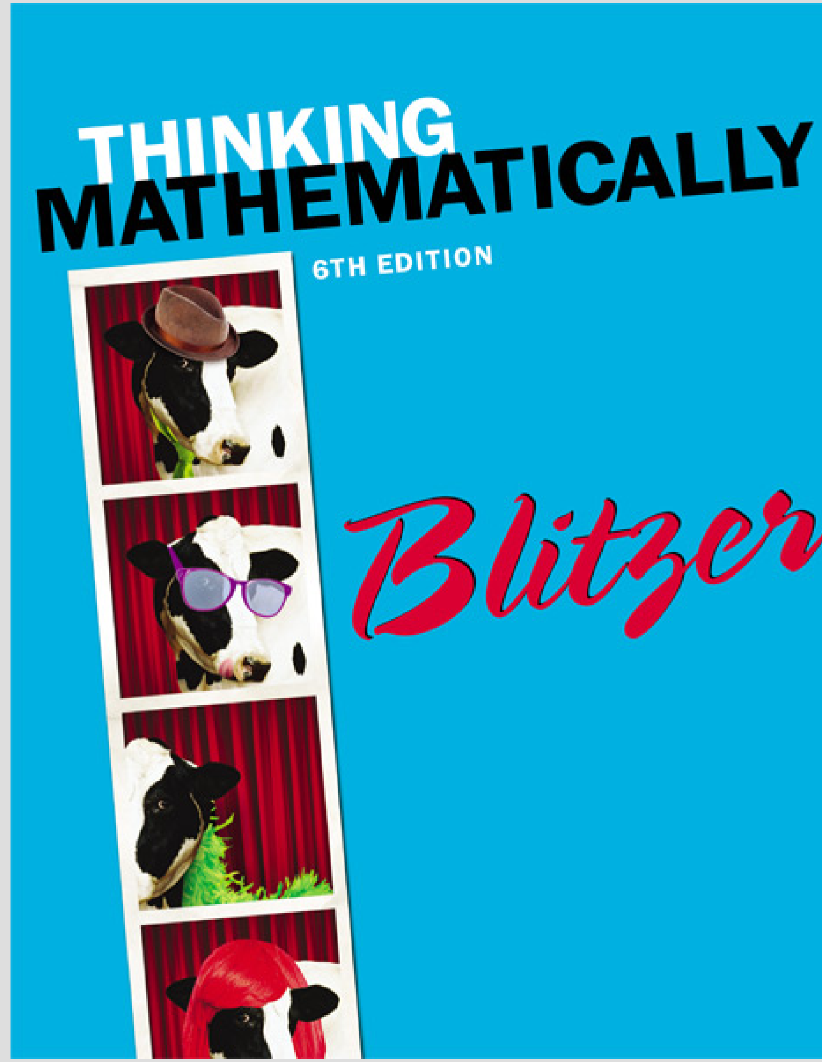
CHAPTER 11

Counting Methods and Probability Theory



11.8

Expected Value

Objectives

1. Compute expected value.
2. Use expected value to solve applied problems.
3. Use expected value to determine the average payoff or loss in a game of chance.

Expected Value

Expected value is a mathematical way to use probabilities to determine what to expect in various situations over the long run.

Expected value is used to

- determine premiums on insurance policies.
- weigh the risks versus the benefits in alternatives in business ventures.
- indicate to a player of any game of chance what will happen if the game is played repeatedly.

Standard way to compute expected value is to multiply each possible outcome by its probability and then add these products.

Example: Computing Expected Value

Find the expected value for the number of girls for a family with three children.

Solution:

A family with three children can have 0,1,2, or 3 girls. There are eight ways these outcomes can occur.

No girls : Boy Boy Boy — One way
One girl : Girl Boy Boy, Boy Girl Boy, Boy Boy Girl — Three ways
Two girls : Girl Girl Boy, Girl Boy Girl, Boy Girl Girl — Three ways
Three girls : Girl Girl Girl — One way

The expected value, *E*, is computed by multiplying each outcome by its probability and then adding these products.

Example: Computing Expected Value continued

$$E = 0\cdot\frac{1}{8}+1\cdot\frac{3}{8}+2\cdot\frac{3}{8}+3\cdot\frac{1}{8}=\frac{0+3+6+3}{8}=\frac{12}{8}=\frac{3}{2}=1.5$$

The expected value is 1.5. This means that if we record the number of girls in many different three-child families, the average number of girls for all these families will be 1.5 or half the children.

Example: Determining an Insurance Premium

An automobile insurance company has determined the probabilities for various claim amounts (to the nearest \$2000) for drivers ages 16 through 21 as shown in the table. Calculate the expected value and describe what this means in practical terms.

Amount of Claim	Probability
\$0	0.70
\$2000	0.15
\$4000	0.08
\$6000	0.05
\$8000	0.01
\$10,000	0.01

$E = \$0(0.70) + \$2000(0.15) + \$4000(0.08) + \$6000(0.05) + 8000(0.01)+\$10{,}000(0.01)$

$= \$0 + \$300 + \$320 + \$300 + \$80 + \$100 = \$1100.$

This means that in the long run, the average cost of a claim is \$1100 which is the very least the insurance company should charge to break even.

Expected Value and Games of Chance

To find the expected value of a game, multiply the gain or loss for each possible outcome by its probability. Then add the products.

Example: Expected Value and Roulette

Find the expected value of betting $1 on the number 20 in roulette.

If the ball lands on that number, you are awarded $35 and get to keep the $1 that you paid to play.

If the ball lands on any of the other 37 slots, you are awarded nothing and the $1 that you bet is collected.

Example: Expected Value and Roulette continued

Playing one Number with a 35 to 1 Payoff in Roulette		
Outcome	Gain or Loss	Probability
Ball lands on 20	\$35	$\frac{1}{38}$
Ball doesn't land on 20	–\$1	$\frac{37}{38}$

$$E = 35\left(\frac{1}{38}\right) + (-\$1)\left(\frac{37}{38}\right) = \frac{\$35 - \$37}{38} = \frac{-\$2}{38} \approx -\$0.05$$

The expected value is approximately –\$0.05. This means that in the long run, a player can expect to lose about 5¢ for each game played.