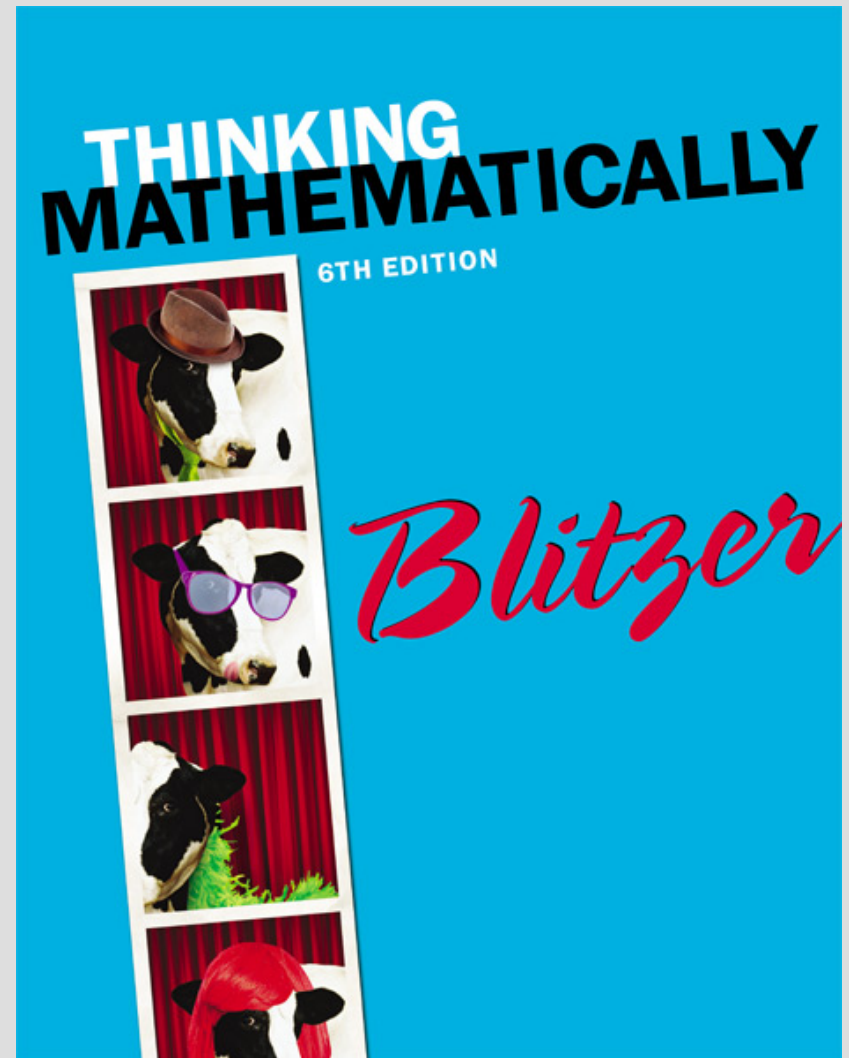


# CHAPTER 12

## Statistics



12.2

## **Measures of Central Tendency**

# Objectives

1. Determine the mean for a data set.
2. Determine the median for a data set.
3. Determine the mode for a data set.
4. Determine the midrange for a data set.

# The Mean

**Mean:** The sum of the data items divided by the number of items.

$$\text{Mean} = \frac{\Sigma x}{n},$$

where  $\Sigma x$  represents the sum of all the data items and  $n$  represents the number of items.

# Example: Calculating the Mean

The table shows the ten highest-earning TV actors and the ten highest-earning TV actresses for the 2010–2011 television season. Find the mean earnings, in millions of dollars, for the ten highest-earning actors.

Actor	Earnings (millions of dollars)	Actress	Earnings (millions of dollars)
Charlie Sheen	\$40	Eva Longoria	\$13
Ray Romano	\$20	Tina Fey	\$13
Steve Carell	\$15	Marcia Cross	\$10
Mark Harmon	\$13	Mariska Hargitay	\$10
Jon Cryer	\$11	Marg Helgenberger	\$10
Laurence Fishburne	\$11	Teri Hatcher	\$9
Patrick Dempsey	\$10	Felicity Huffman	\$9
Simon Baker	\$9	Courtney Cox	\$7
Hugh Laurie	\$9	Ellen Pompeo	\$7
Chris Meloni	\$9	Julianna Margulies	\$7

# Example continued

Find the mean by adding the earnings for the actors and dividing this sum by 10, the number of data items.

$$\begin{aligned}\text{Mean} &= \frac{\sum x}{n} \\ &= \frac{40 + 20 + 15 + 13 + 11 + 11 + 10 + 9 + 9 + 9}{10} = \frac{147}{10} = 14.7\end{aligned}$$

# Calculating the Mean for a Frequency Distribution

When many data values occur more than once and a frequency distribution is used to organize the data, we can use the following formula to calculate the mean:

$$\text{Mean} = \bar{x} = \frac{\sum xf}{n},$$

where

$x$  represents each data value.

$f$  represents the frequency of that data value.

$\sum xf$  represents the sum of all the products obtained by multiplying each data value by its frequency.

$n$  represents the *total frequency* of the distribution.

# Example: Calculating the Mean for a Frequency Distribution

The table to the right shows the students' responses to the question "How stressed have you felt in the last 2 ½ weeks, on a scale of 0 to 10, with 0 being not stressed at all and 10 being as stressed as possible?" Use the frequency distribution to find the mean of the stress-level ratings.

Stress Rating $x$	Frequency $f$
0	2
1	1
2	3
3	12
4	16
5	18
6	13
7	31
8	26
9	15
10	14

## Example continued

### Solution:

First find  $xf$ , obtained by multiplying each data value,  $x$ , by its frequency,  $f$ .

Then use the formula to find the mean.

$$\text{Mean} = \bar{x} = \frac{\sum xf}{n},$$

Stress Rating $x$	Frequency $f$	Data value $x$ frequency $xf$
0	2	$0 \cdot 2 = 0$
1	1	$1 \cdot 1 = 1$
2	3	$2 \cdot 3 = 6$
3	12	$3 \cdot 12 = 36$
4	16	$4 \cdot 16 = 64$
5	18	$5 \cdot 18 = 90$
6	13	$6 \cdot 13 = 78$
7	31	$7 \cdot 31 = 217$
8	26	$8 \cdot 26 = 208$
9	15	$9 \cdot 15 = 135$
10	14	$10 \cdot 14 = 140$

## Example 2 continued

*Solution:*

Totals:  $n = 151$

$$\sum xf = 975$$

$$\bar{x} = \frac{\sum xf}{n} = \frac{975}{151} \approx 6.46$$

Stress Rating $x$	Frequency $f$	Data value $x$ frequency $xf$
0	2	$0 \cdot 2 = 0$
1	1	$1 \cdot 1 = 1$
2	3	$2 \cdot 3 = 6$
3	12	$3 \cdot 12 = 36$
4	16	$4 \cdot 16 = 64$
5	18	$5 \cdot 18 = 90$
6	13	$6 \cdot 13 = 78$
7	31	$7 \cdot 31 = 217$
8	26	$8 \cdot 26 = 208$
9	15	$9 \cdot 15 = 135$
10	14	$10 \cdot 14 = 140$

# The Median

**Median** is the data item in the middle of each set of ranked, or ordered, data.

To find the **median** of a group of data items,

1. Arrange the data items in order, from smallest to largest.
2. If the number of data items is odd, the median is the data item in the middle of the list.
3. If the number of data items is even, the median is the mean of the two middle data items.

## Example: Finding the Median

Find the median for each of the following groups of data:

a. 84, 90, 98, 95, 88

## Solution:

Arrange the data items in order from smallest to largest.

The number of data items in the list, five, is odd.

Thus, the median is the middle number.

84, 88, 90, 95, 98      The median is 90.

## Example continued

b. 68, 74, 7, 13, 15, 25, 28, 59, 34, 47

### Solution:

Arrange the data items in order from smallest to largest.

The number of data items in the list, ten, is even.

Thus, the median is the mean of the two middle numbers.

7, 13, 15, 25, 28, | 34, 47, 59, 68, 74

$$\text{The median is } \frac{28 + 34}{2} = \frac{62}{2} = 31$$

## Example: Finding the Median using the Position Formula

Listed below are number of letters in the nine longest words in the English language. Find the median number of letters for the nine longest words. The data items are arranged from smallest to largest:

25, 25, 27, 27, 28, 29, 29, 34, 45

### **Solution:**

The median is in the  $\frac{n+1}{2}$  position =  $\frac{9+1}{2} = 5^{\text{th}}$  position.

The median is 28.

# Example: Finding the Median for a Frequency Distribution

Stress Rating $x$	Frequency $f$
0	2
1	1
2	3
3	12
4	16
5	18
6	13
7	31
8	26
9	15
10	14

Find the median stress-level rating.

## Solution:

There are 151 data items in this table so  $n = 151$ .

The median is the value in the  $\frac{151 + 1}{2} = \frac{152}{2} = 76^{\text{th}}$  position

Count down the frequency column in the distribution until we identify the  $76^{\text{th}}$  data item.

# Example continued

$x$	$f$
0	2
1	1
2	3
3	12
4	16
5	18
6	13
7	31
8	26
9	15
10	14

We count down the frequency column.

1, 2

3

4, 5, 6

7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34

35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52,

53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65

66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76

Stop counting. We've reached the 76th data item.

The 76th data item is 7. The median stress-level rating is 7.

# Example: Comparing the Median and the Mean

Five employees in a manufacturing plant earn salaries of \$19,700, \$20,400, \$21,500, \$22,600 and \$23,000 annually. The section manager has an annual salary of \$95,000.

a. Find the median annual salary for the six people

**Solution:** First arrange the salaries in order.

\$19,700, \$20,400, \$21,500, \$22,600 \$23,000 \$95,000

Since there is an even number of data items, six, the median is the average of the two middle items.

$$\text{Median} = \frac{\$21,500 + \$22,600}{2} = \frac{\$44,100}{2} = \$22,050$$

## Example continued

- b. Find the mean annual salary for the six people.

**Solution:** We find the mean annual salary by adding the six annual salaries and dividing by 6.

$$\begin{aligned}\text{Mean} &= \frac{\$19,700 + \$20,400 + \$21,500 + \$22,600 + \$23,000 + \$95,000}{6} \\ &= \frac{\$202,200}{6} \\ &= \$33,700\end{aligned}$$

# The Mode

**Mode** is the data value that occurs most often in a data set. If more than one data value has the highest frequency, then each of these data values is a mode. If no data items are repeated, then the data set has no mode.

# Example: Finding the mode

Find the mode for the following groups of data:

7, 2, 4, 7, 8, 10

**Solution:**

The mode is 7.

# The Midrange

**Midrange** is found by adding the lowest and highest data values and dividing the sum by 2.

$$\text{Midrange} = \frac{\text{lowest data value} + \text{highest data value}}{2}$$

# Example: Finding the Midrange

*Newsweek* magazine examined factors that affect women's lives, including justice, health, education, economics, and politics. Using these five factors, the magazine graded each of 165 countries on a scale from 0 to 100. The 12 best places to be a woman and the 12 worst places to be a woman are shown below. Find the midrange among the 12 best countries to be a woman.  
(see next slide)

## Example: Finding the Midrange

Best Places to Be a Woman		Worst Places to Be a Woman	
Country	Score	Country	Score
Iceland	100.0	Chad	0.0
Canada	99.6	Afghanistan	2.0
Sweden	99.2	Yemen	12.1
Denmark	95.3	Democratic Republic of the Congo	13.6
Finland	92.8	Mali	17.6
Switzerland	91.9	Solomon Islands	20.8
Norway	91.3	Niger	21.2
United States	89.8	Pakistan	21.4
Australia	88.2	Ethiopia	23.7
Netherlands	87.7	Sudan	26.1
New Zealand	87.2	Guinea	28.5
France	87.2	Sierra Leone	29.0

$$\begin{aligned}
 \text{Midrange} &= \frac{\text{lowest score} + \text{highest score}}{2} \\
 &= \frac{\$87.2 + 100.0}{2} = \frac{187.2}{2} = 93.6
 \end{aligned}$$