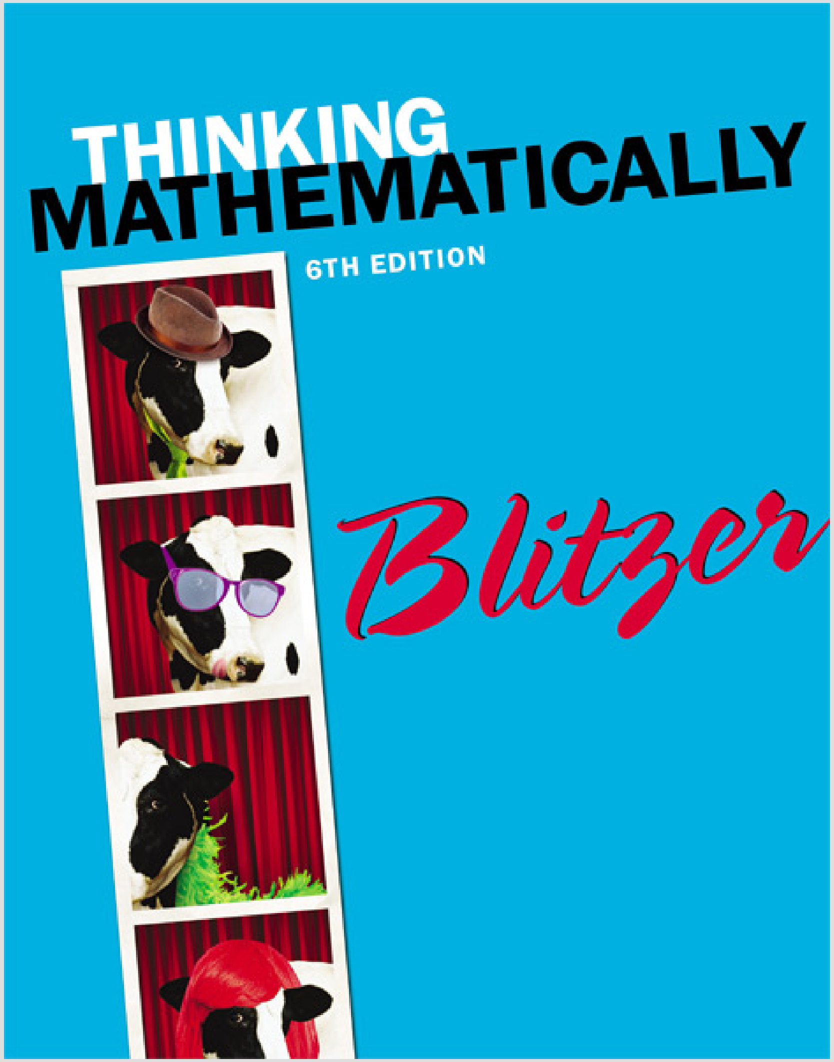
CHAPTER 12

Statistics



12.4

The Normal Distribution

Objectives

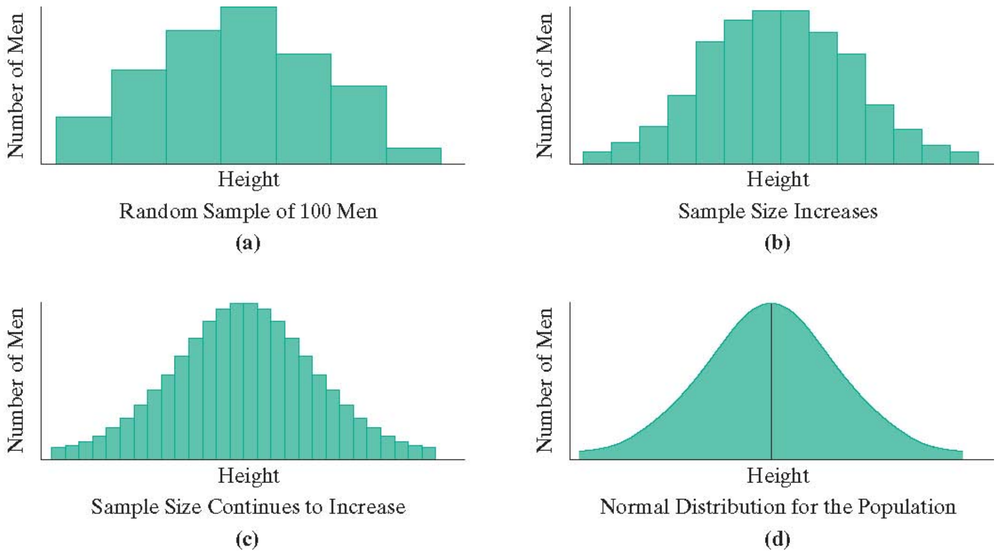
1. Recognize characteristics of normal distributions.
2. Understand the 68-95-99.7 Rule.
3. Find scores at a specified number of standard deviations from the mean.
4. Use the 68-95-99.7 Rule
5. Convert a data item to a z-score.
6. Understand percentiles and quartiles.
7. Use and interpret margins of error.
8. Recognize distributions that are not normal.

Normal Distribution

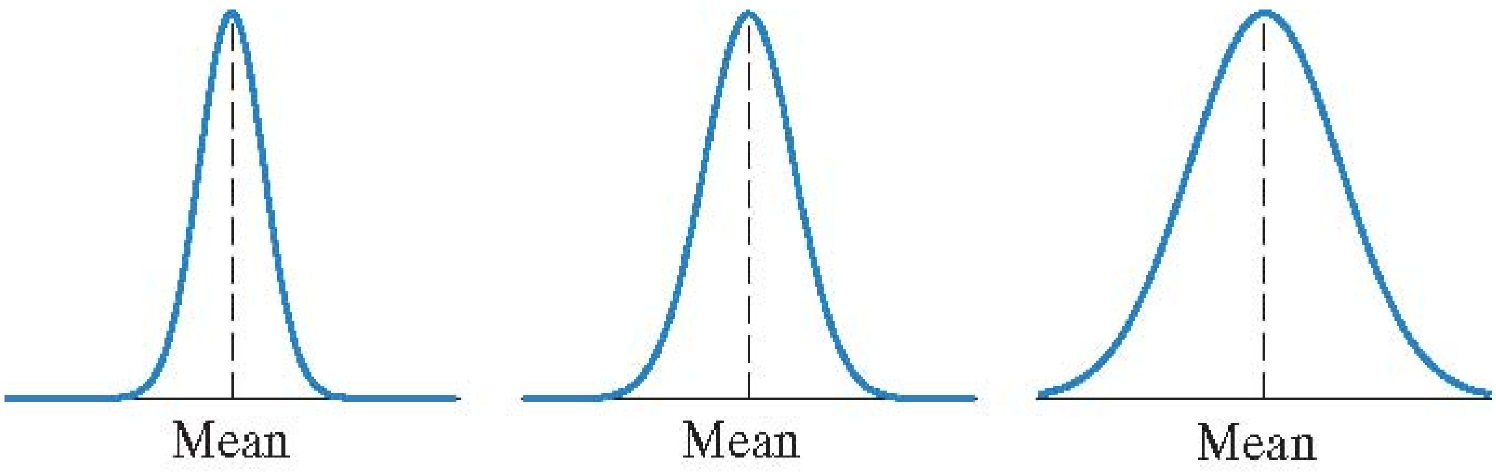
Also called the bell curve or Gaussian distribution.

Normal distribution is bell shaped and symmetric about a vertical line through its center.

Mean, median and mode are all equal and located at the center of the distribution.



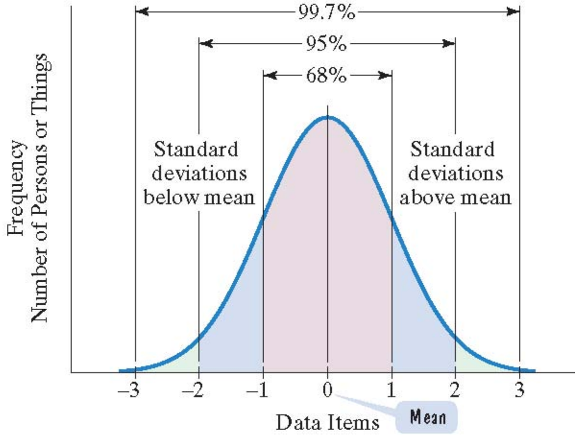
Normal Distribution continued



The shape of the normal distribution depends on the mean and the standard deviation. These three graphs have the same mean but different standard deviations. As the standard deviation increases, the distribution becomes more spread out.

Standard Deviation and the 68-95-99.7 Rule

1. Approximately 68% of the data items fall within 1 standard deviation of the mean (in both directions).
2. Approximately 95% of the data items fall within 2 standard deviations of the mean.
3. Approximately 99.7% of the data items fall within 3 standard deviations of the mean.



Example: Finding Scores at a Specified Standard Deviation From the Mean

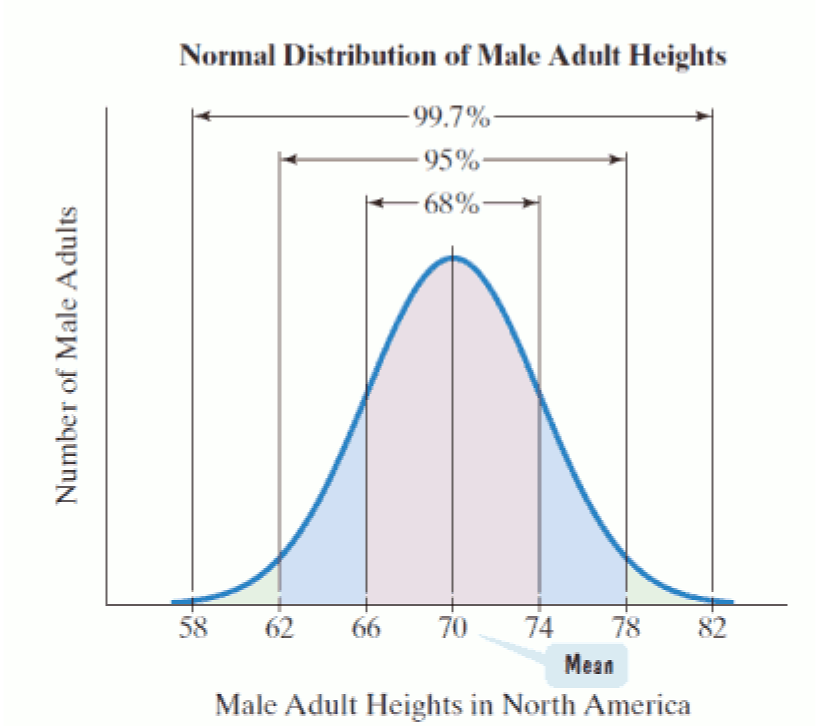
Male adult heights in North America are approximately normally distributed with a mean of 70 inches and a standard deviation of 4 inches. Find the height that is 2 standard deviations above the mean.

Solution:

$$\text{Height} = \text{mean} + 2 \cdot \text{standard deviation}$$
$$= 70 + 2 \cdot 4 = 70 + 8 = 78$$

Example: Using the 68-95-99.7 Rule

Use the distribution of male adult heights in the figure to find the percentage of men in North America with heights between 66 inches and 74 inches.



Example continued

Solution:

The 68-95-99.7 Rule states that approximately 68% of the data items fall within 1 standard deviation, 4, of the mean, 70.

Mean – 1·standard deviation

$$= 70 - 1 \cdot 4 = 66.$$

Mean + 1·standard deviation

$$= 70 + 1 \cdot 4 = 74.$$

68% of males have heights between 66 and 74 inches.

Computing z-Scores

COMPUTING Z-SCORES

A z-score describes how many standard deviations a data item in a normal distribution lies above or below the mean. The z-score can be obtained using

$$z\text{-score} = \frac{\text{data item} - \text{mean}}{\text{standard deviation}}.$$

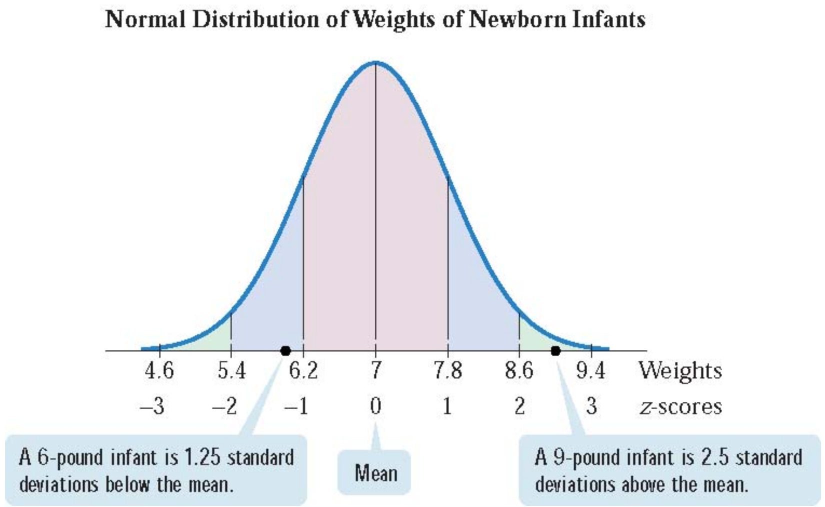
Data items above the mean have positive z-scores. Data items below the mean have negative z-scores. The z-score for the mean is 0.

Example: Computing Z-Scores

The mean weight of newborn infants is 7 pounds and the standard deviation is 0.8 pound.

The weights of newborn infants are normally distributed.

Find the *z*-score for a weight of 9 pounds.



Example continued

Solution:

The mean is 7 and the standard deviation is 0.8. The z-score written z_9, is:

$$z_9 = \frac{\text{data item - mean}}{\text{standard deviation}}$$

$$= \frac{9-7}{0.8} = \frac{2}{0.8}$$

$$= 2.5$$

Example: Understanding z-Scores

Intelligence quotients (IQs) on the Stanford–Binet intelligence test are normally distributed with a mean of 100 and a standard deviation of 16. What is the IQ corresponding to a z-score of -1.5?

Solution:

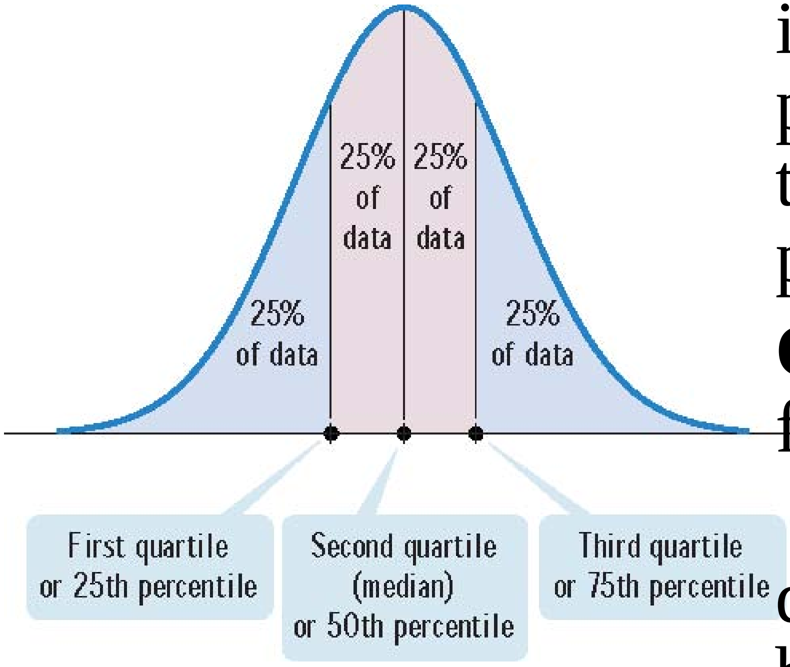
The negative sign in -1.5 tells us that the IQ is 1½ standard deviations below the mean.

Score = mean – 1.5 · standard deviation

= 100 – 1.5 (16) = 100 – 24 = 76.

The IQ corresponding to a z-score of -1.5 is 76.

Percentiles and Quartiles



Percentiles: If *n*% of the items in a distribution are less than a particular data item, we say that the data item is in the *n*th percentile of the distribution.

Quartiles: Divide data sets into four equal parts. The 25th percentile is the first quartile. 25% of the data fall below the first quartile. The 50th percentile is the second quartile. The 75th percentile is the third quartile.

A Percentile Interpretation for *z*-Scores

Using the *z*-score and a table, you can find the percentage of data items that are less than any data item in a normal distribution.

In a normal distribution, the mean, median, and mode all have a corresponding *z*-score of 0 and are the 50th percentile. Thus, 50% of the data items are greater than or equal to the mean, median and mode.

Polls and Margins of Error

Statisticians use properties of the normal distribution to estimate the probability that a result obtained from a single sample reflects what is truly happening.

If n is the sample size, there is a 95% probability that it lies within $\frac{1}{\sqrt{n}}$ of the true population statistic.

$\pm\frac{1}{\sqrt{n}}\times 100\%$ is called the **margin of error**.

Example: Using and Interpreting Margin of Error

In a random sample of 1172 children ages 6 through 14, 17% of the children said getting bossed around is a bad thing about being a kid.

a. Verify the margin of error.

Solution: The sample size is $n = 1172$. The margin of error is

$$\pm \frac{1}{\sqrt{n}} \times 100\% = \pm \frac{1}{\sqrt{1172}} \times 100\% \approx \pm 2.9\%$$

Example 7 continued

b. Write a statement about the percentage of children who feel that getting bossed around is a bad thing about being a kid.

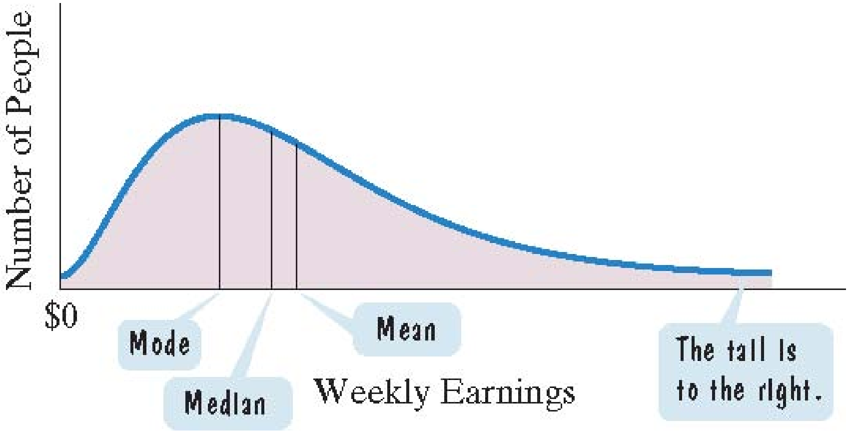
Solution: There is a 95% probability that the true population percentage lies between

$$17\% - 2.9\% = \mathbf{14.1\%}$$

and

$$17\% + 2.9\% = \mathbf{19.9\%}.$$

Other Kinds of Distributions



This graph represents the population distribution of weekly earnings in the United States. There is no upper limit on weekly earnings.

The relatively few people with very high weekly incomes pull the mean income to a value greater than the median.

The most frequent income, the mode, occurs towards the low end of the data items.

This is called a **skewed** distribution because a large number of data items are piled up at one end or the other with a "tail" at the other end.

This graph is **skewed to the right**.