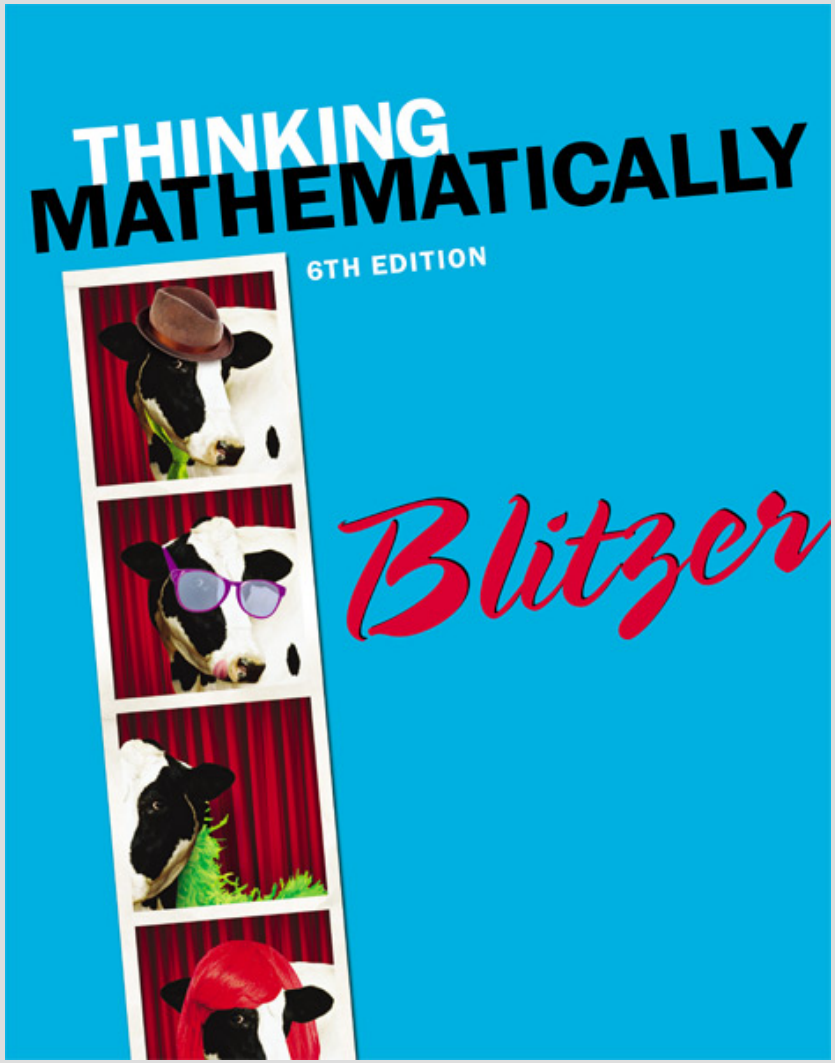
CHAPTER 12

Statistics



12.5

Problem Solving with the Normal Distribution

Objectives

1. Solve applied problems involving normal distribution.

Example: Finding the Percentage of Data Items Less Than a Given Data Item

According to the Department of Health and Education, cholesterol levels are normally distributed. For men between 18 and 24 years, the mean is 178.1 and the standard deviation is 40.7. What percentage of men in this age range have a cholesterol level less than 239.15?

Solution:

Compute the z-score for a 239.15 cholesterol level.

$$z_{239.15} = \frac{\text{data item - mean}}{\text{standard deviation}} = \frac{239.15 - 178.1}{40.7} = \frac{61.05}{40.7} = 1.5$$

Example continued

We must find the percentage of men with a cholesterol level less than $z = 1.5$. The table gives this percentage as a percentile. Finding 1.5 in the z-score column gives a percentile of 93.32. Thus , 93.32% of men between 18 and 24 have a cholesterol level less than 239.15.

z-score	Percentile
1.4	91.92
1.5	93.32
1.6	94.52

Finding the Percentage of Data Items Between Two Given Data Items.

FINDING THE PERCENTAGE OF DATA ITEMS BETWEEN TWO GIVEN ITEMS IN A NORMAL DISTRIBUTION

1. Convert each given data item to a z-score:

$$z = \frac{\text{data item} - \text{mean}}{\text{standard deviation}}.$$

2. Use **Table 12.14** to find the percentile corresponding to each z-score in step 1.
3. Subtract the lesser percentile from the greater percentile and attach a % sign.

Example: Finding the Percentage of Data Items Between Two Given Data Items

The amount of time that self-employed Americans work each week is normally distributed with a mean of 44.6 hours and a standard deviation of 14.4 hours. What percentage of self-employed individuals in the United States work between 37.4 and 80.6 hours per week?

Solution:

Step 1 Convert each given data item to a z-score.

$$z_{37.4} = \frac{\text{data item - mean}}{\text{standard deviation}} = \frac{37.4 - 44.6}{14.4} = \frac{-7.2}{14.4} = -0.5$$

$$z_{80.6} = \frac{\text{data item - mean}}{\text{standard deviation}} = \frac{80.6 - 44.6}{14.4} = \frac{36}{14.4} = 2.5$$

Example continued

Step 2 Use the Table to find the percentile corresponding to these z-scores. The percentile corresponding to -0.50 is 30.85. This means that 30.85% of self-employed Americans work fewer than 37.4 hours per week.

The Table also shows that the percentile that corresponds to a z-score of 2.5 is 99.38.

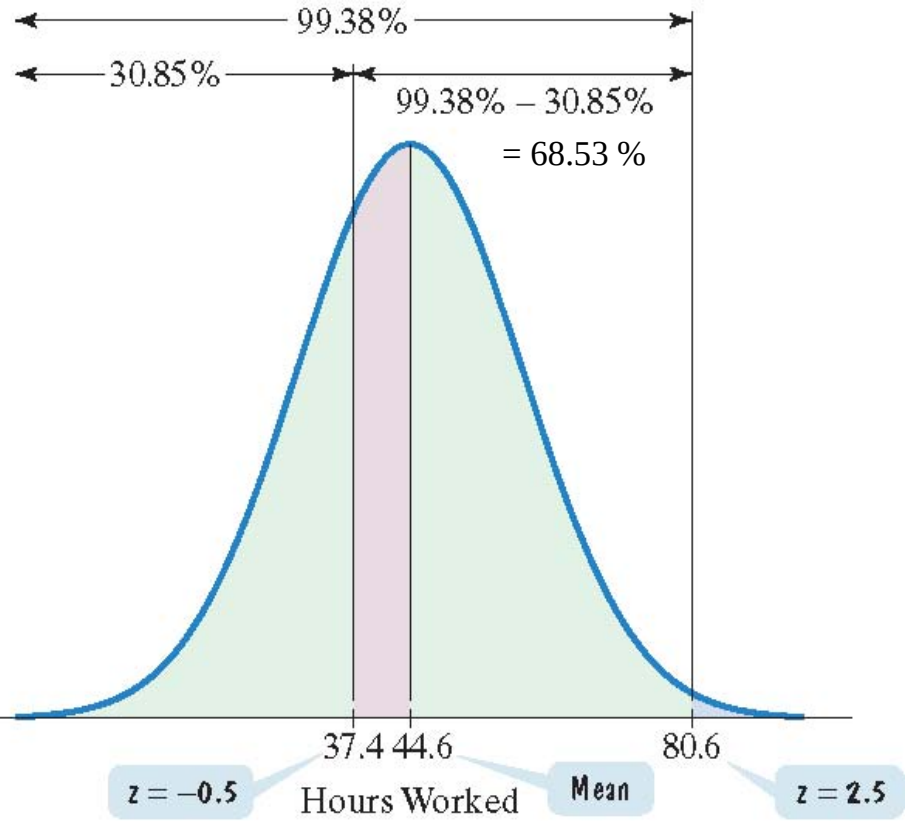
That means that 99.38% of self-employed Americans work fewer than 80.6 hours per week.

Example continued

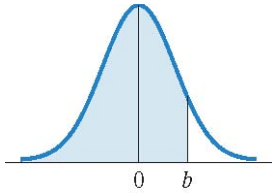
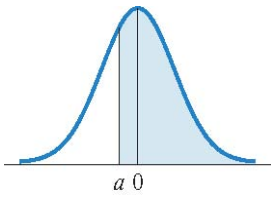
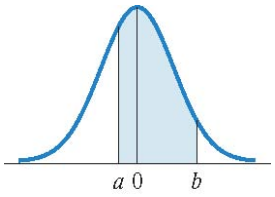
Step 3 Subtract the lesser percentile from the greater percentile and attach a % sign.

$$99.38 - 30.85 = 68.53.$$

Thus, 68.53% of self-employed Americans work between 37.4 and 80.6 hours per week.



Summary of Computing Percentage of Data Items for Normal Distributions

Description of Percentage	Graph	Computation of Percentage
Percentage of data items less than a given data item with $z = b$		Use the table percentile for $z = b$ and add a % sign
Percentage of data items greater than a given data item with $z = a$		Subtract the table percentile for $z = a$ from 100 and add a % sign.
Percentage of data items between two given data items with $z = a$ and $z = b$		Subtract the table percentile for $z = a$ from the table percentile for $z = b$ and add a % sign.