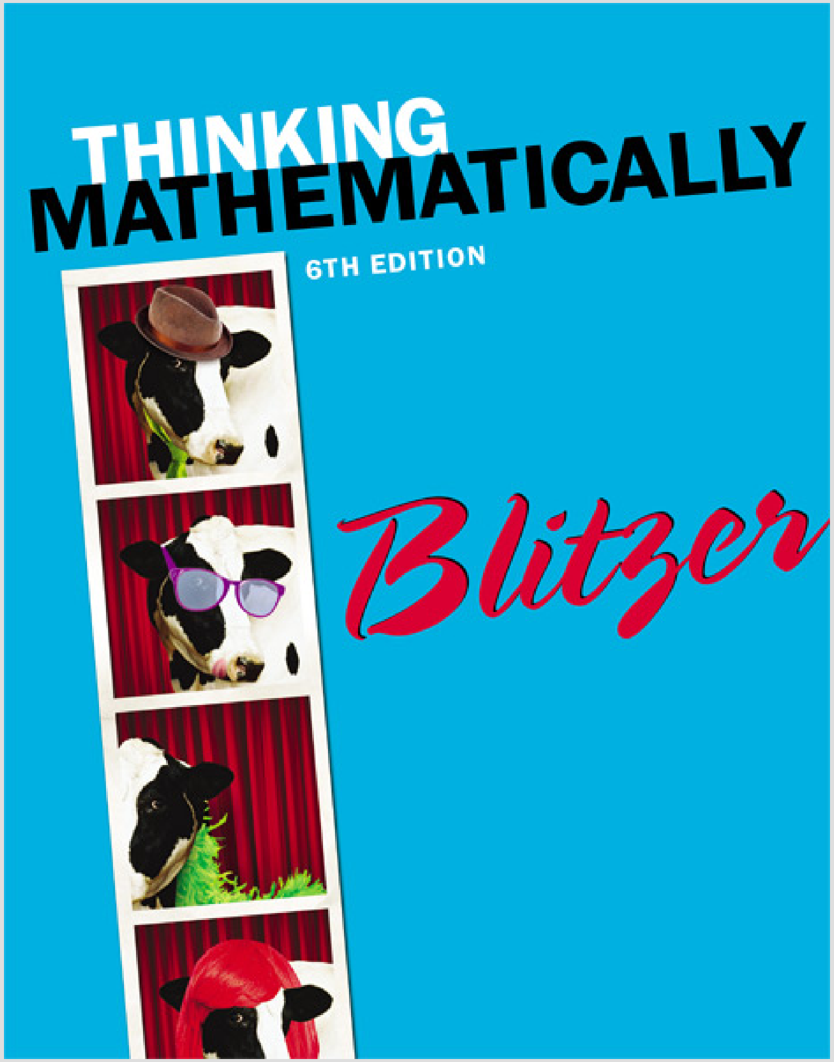
CHAPTER 2

Set Theory



2.2

Subsets

Objectives

1. Recognize subsets and use the notation $\subseteq$.
2. Recognize proper subsets and use the notation $\subset$.
3. Determine the number of subsets of a set.
4. Apply concepts of subsets and equivalent sets to infinite sets.

Subsets

DEFINITION OF A SUBSET OF A SET

Set A is a **subset** of set B, expressed as

$$A \subseteq B,$$

if every element in set A is also an element in set B.

The notation $A \not\subseteq B$ means that A **is not a subset** of B. Set A is not a subset of set B if there is at least one element of set A that is not an element of set B.

Every set is a subset of itself.

Example: Using the Symbols $\subseteq$ and $\not\subseteq$

Write $\subseteq$ or $\not\subseteq$ in the blank to form a true statement.

$A = \{1, 3, 5, 7\}$ $B = \{1, 3, 5, 7, 9, 11\}$

$A \; \underline{\;\subseteq\;} \; B$

Set A is a subset of B.

$A = \{x \mid x \text{ is a letter in the word } \textit{proof}\}$

$A = \{y \mid y \text{ is a letter in the word } \textit{roof}\}$

$A \; \underline{\;\not\subseteq\;} \; B$

Set A is not a subset of B.

Proper Subsets

DEFINITION OF A PROPER SUBSET OF A SET

Set A is a **proper subset** of set B, expressed as $A \subset B$, if set A is a subset of set B and sets A and B are not equal ($A \neq B$).

Example: Using the Symbols $\subseteq$ and $\not\subseteq$

Write $\subseteq$, $\subset$, or both in the blank to form a true statement:

$A = \{x \mid x$ is a person and x lives in San Francisco$\}$

$B = \{x \mid x$ is a person and x lives in California$\}$

A _____ B

Solution: A $\subseteq$, $\subset$ B

b. $A = \{ 2, 4, 6, 8\}$

$B = \{ 2, 8, 4, 6\}$

A _____ B

Solution: A $\subseteq$ B

Subsets and the Empty Set

THE EMPTY SET AS A SUBSET

1. For any set B, $\emptyset \subseteq B$.
2. For any set B other than the empty set, $\emptyset \subset B$.

The Number of Subsets of a Given Set

Set	Number of Elements	List of All Subsets	Number of Subsets
$\{\ \}$	0	$\{\ \}$	1
$\{a\}$	1	$\{a\}, \{\ \}$	2
$\{a, b\}$	2	$\{a, b\}, \{a\}, \{b\}, \{\ \}$	4
$\{a, b, c\}$	3	$\{a, b, c\}$, $\{a, b\}, \{a, c\}, \{b, c\}$, $\{a\}, \{b\}, \{c\}, \{\ \}$	8

As we increase the number of elements in the set by one, the number of subsets doubles.

The number of subsets of a set with n elements is 2^n.

The number of proper subsets of a set with n elements is $2^n - 1$.

Example: Finding the Number of Subsets and Proper Subsets

Find the number of subsets and the number of proper subsets.

a. {a, b, c, d, e }

There are 5 elements so there are $2^5 = 32$ subsets and $2^5 - 1 = 31$ proper subsets.

b. $\{x \mid x \in \mathbf{N} \text{ and } 9 \leq x \leq 15\}$

In roster form, we see that there are 7 elements: { 9, 10, 11, 12, 13, 14, 15 }

There are $2^7 = 128$ subsets and $2^7 - 1 = 127$ proper subsets.

The Number of Subsets of Infinite Sets

There are $\aleph_0$ natural numbers.

- It has $2^{\aleph_0}$ subsets.
- It has $2^{\aleph_0} - 1$ proper subsets
- $2^{\aleph_0} > \aleph_0$
- Denote $2^{\aleph_0}$ by $\aleph_1$
- $\aleph_1 > \aleph_0$

$\aleph_0$ is the "smallest" transfinite cardinal number in an infinite hierarchy of different infinities.