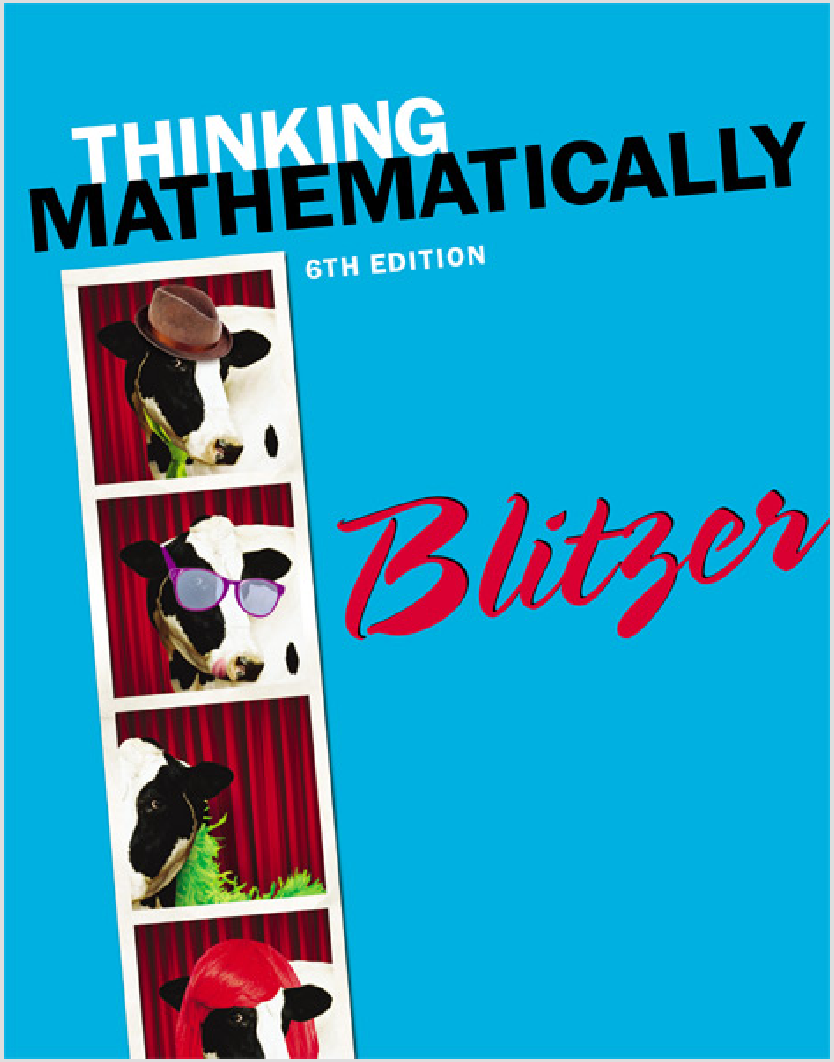
CHAPTER 2

Set Theory



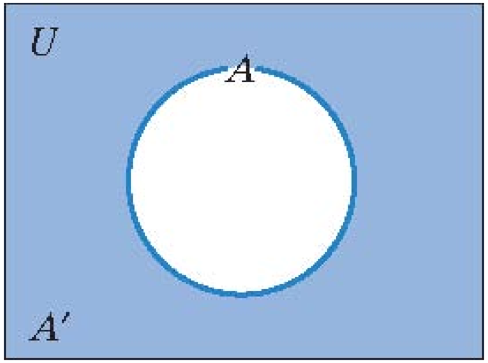
2.3

Venn Diagrams and Set Operation

Objectives

1. Understand the meaning of a universal set.
2. Understand the basic ideas of a Venn diagram.
3. Use Venn diagrams to visualize relationships between two sets.
4. Find the complement of a set
5. Find the intersection of two sets.
6. Find the union of two sets.
7. Perform operations with sets.
8. Determine sets involving set operations from a Venn diagram.
9. Understand the meaning of *and* and *or*.
10. Use the formula for $n(A \cup B)$.

Universal Sets and Venn Diagrams



The **universal set** is a general set that contains all elements under discussion.

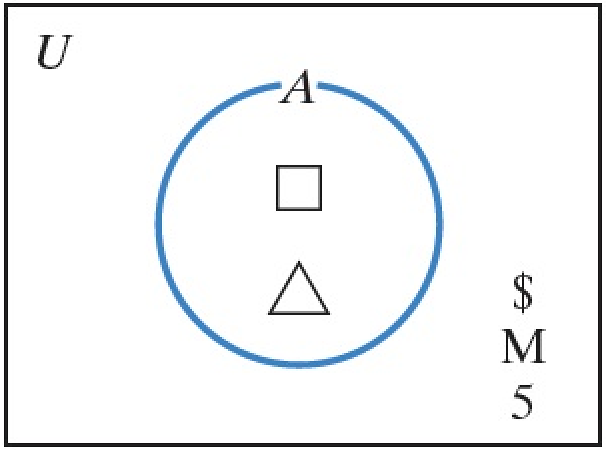
John Venn (1843 – 1923) created Venn diagrams to show the visual relationship among sets.

Universal set is represented by a rectangle

Subsets within the universal set are depicted by circles, or sometimes ovals or other shapes.

Example: Determining Sets From a Venn Diagram

Use the Venn diagram to determine each of the following sets:



a. U

$U = \{\square, \Delta, \$, \text{M}, 5\}$

b. A

$A = \{\square, \Delta\}$

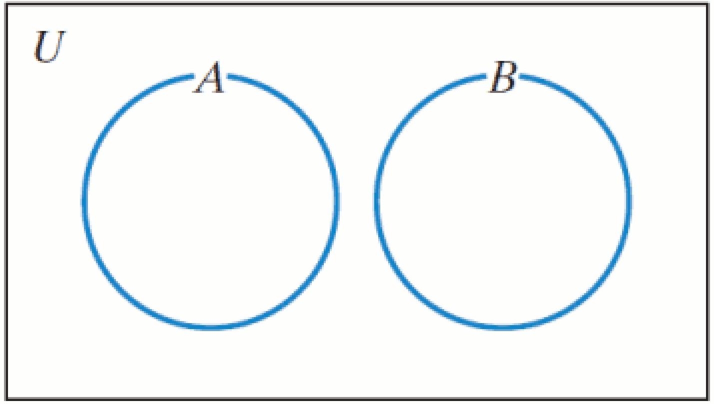
c. The set of elements in U that are not in A.

$\{\$, \text{M}, 5\}$

Representing Two Sets in a Venn Diagram

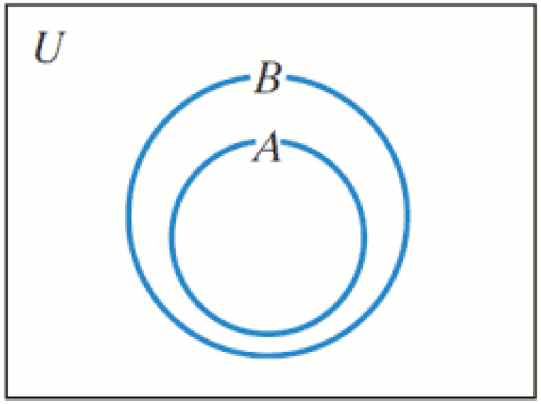
Disjoint Sets:

Two sets that have no elements in common.



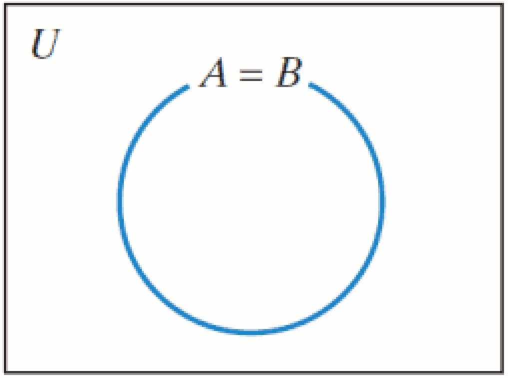
Proper Subsets:

All elements of set *A* are elements of set *B*.



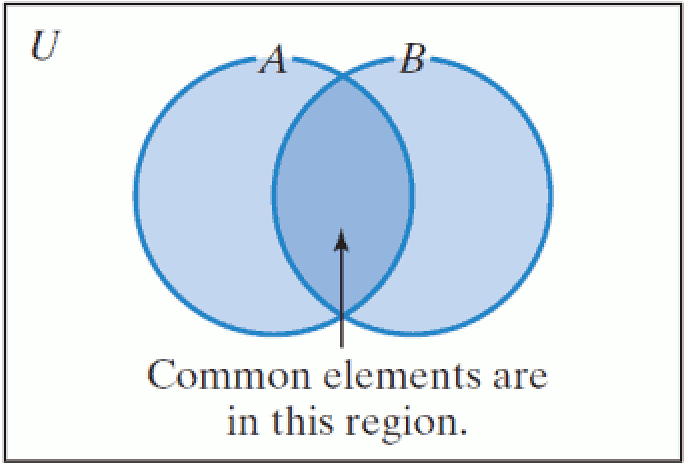
Representing Two Sets in a Venn Diagram

Equal Sets: If $A = B$ then $A \subseteq B$ and $B \subseteq A$.



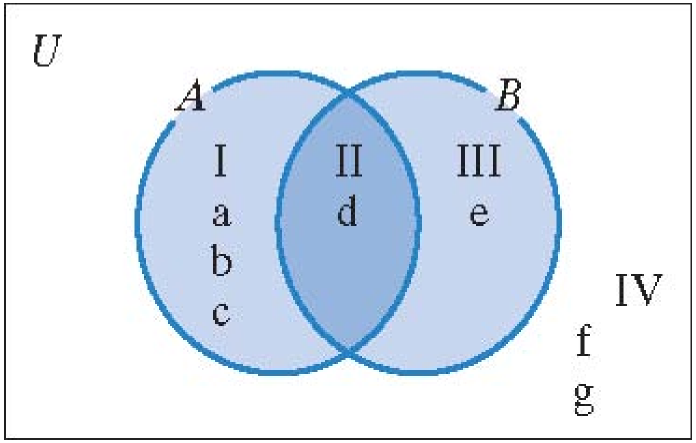
Sets with Some Common Elements

If set A and set B have at least one element in common, then the circles representing the sets must overlap.



Example: Determining Sets from a Venn Diagram

Use the Venn Diagram to determine:



a. *U*

U = {a, b, c, d, e, f, g}

b. *B*

B = {d, e}

c. the set of elements in *A* but not *B*

{a, b, c}

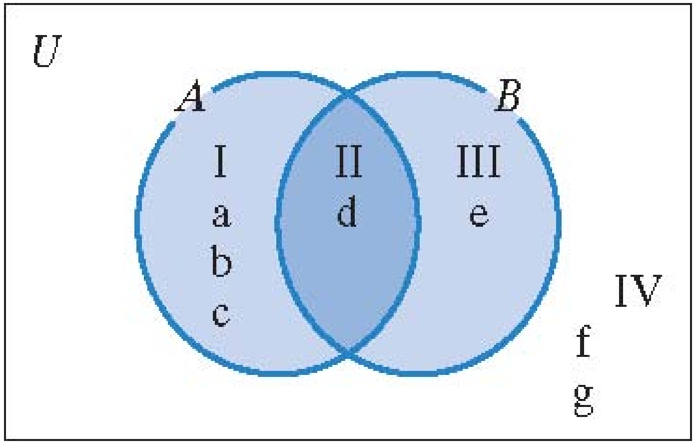
Example: Determining Sets from a Venn Diagram (cont)

d. the set of elements in *U* that are not in *B*

{a, b, c, f, g}

e. the set of elements in both *A* and *B*.

{d}



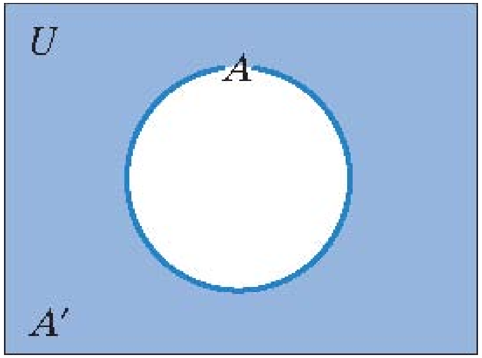
The Complement of a Set

DEFINITION OF THE COMPLEMENT OF A SET

The **complement** of set A, symbolized by A', is the set of all elements in the universal set that are ***not*** in A. This idea can be expressed in set-builder notation as follows:

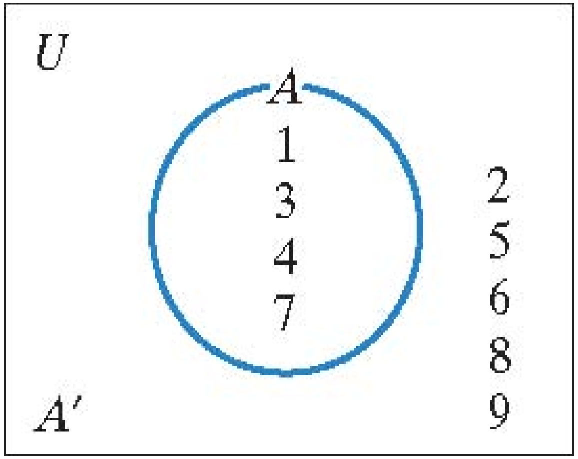
$$A' = \{x \mid x \in U \text{ and } x \notin A\}.$$

The shaded region represents the complement of set A or A'. This region lies outside the circle.



Example: Finding a Set's Complement

Let U = { 1, 2, 3, 4, 5, 6, 7, 8, 9} and
A = {1, 3, 4, 7 }. Find A'.



Solution:

Set A' contains all the elements of set U that are not in set A.

Because set A contains the elements 1, 3, 4, and 7, these elements cannot be members of set A': $A' = \{2, 5, 6, 8, 9\}$.

The Intersection of Sets

DEFINITION OF THE INTERSECTION OF SETS

The **intersection** of sets A and B, written $A \cap B$, is the set of elements common to both set A and set B. This definition can be expressed in set-builder notation as follows:

$$A \cap B = \{x \mid x \in A \quad \text{and} \quad x \in B\}.$$

Example: Finding the Intersection of Two Sets

Find each of the following intersections:

a. $\{7, 8, 9, 10, 11\} \cap \{6, 8, 10, 12\}$

$\{8, 10\}$

b. $\{1, 3, 5, 7, 9\} \cap \{2, 4, 6, 8\}$

$\emptyset$

c. $\{1, 3, 5, 7, 9\} \cap \emptyset$

$\emptyset$

The Union of Sets

DEFINITION OF THE UNION OF SETS

The **union** of sets A and B, written $A \cup B$, is the set of elements that are members of set A or of set B or of both sets. This definition can be expressed in set-builder notation as follows:

$$A \cup B = \{x \mid x \in A \quad \text{or} \quad x \in B\}.$$

Example: Finding the Union of Two Sets

Find each of the following unions:

a. $\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\}$

$\{6, 7, 8, 9, 10, 11, 12\}$

b. $\{1, 3, 5, 7, 9\} \cup \{2, 4, 6, 8\}$

$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

c. $\{1, 3, 5, 7, 9\} \cup \emptyset$

$\{1, 3, 5, 7, 9\}$

The Empty Set in Intersection and Union

THE EMPTY SET IN INTERSECTION AND UNION

For any set A,

1. $A \cap \varnothing = \varnothing$
2. $A \cup \varnothing = A$.

Performing Set Operations

Some problems involve more than one set operation. The set notation specifies the order in which we perform these operations. **Always begin by performing any operations inside parentheses.**

Example: Performing Set Operations

Given:

$U = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{ 1, 3, 7, 9 \}$

$B = \{ 3, 7, 8, 10 \}$

find

a. $(A \cup B)'$

Solution:

$A \cup B = \{1, 3, 7, 8, 9, 10\}$

$(A \cup B)' = \{2, 4, 5, 6\}$

Example: Performing Set Operations (cont)

Given:

$U = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{ 1, 3, 7, 9 \}$

$B = \{ 3, 7, 8, 10 \}$

find

b. $A' \cap B'$

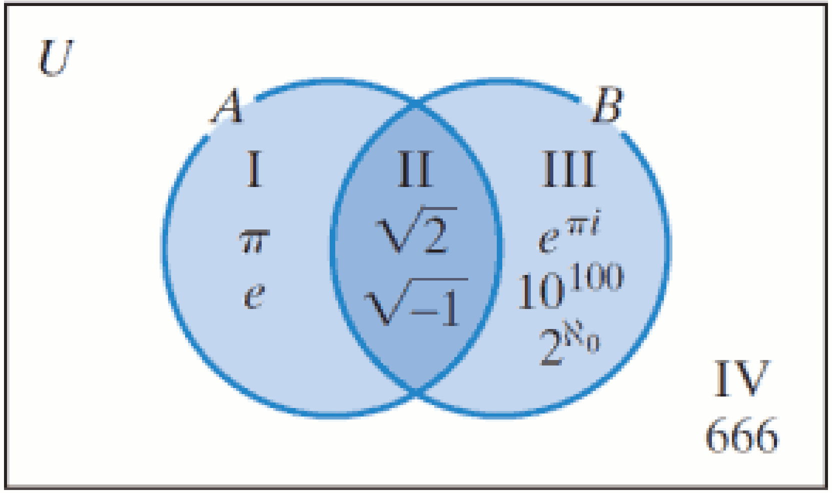
Solution

$A' = \{2, 4, 5, 6, 8, 10\}$

$B' = \{1, 2, 4, 5, 6, 9\}$

$A' \cap B' = \{2, 4, 5, 6\}$

Example: Determining Sets from a Venn Diagram



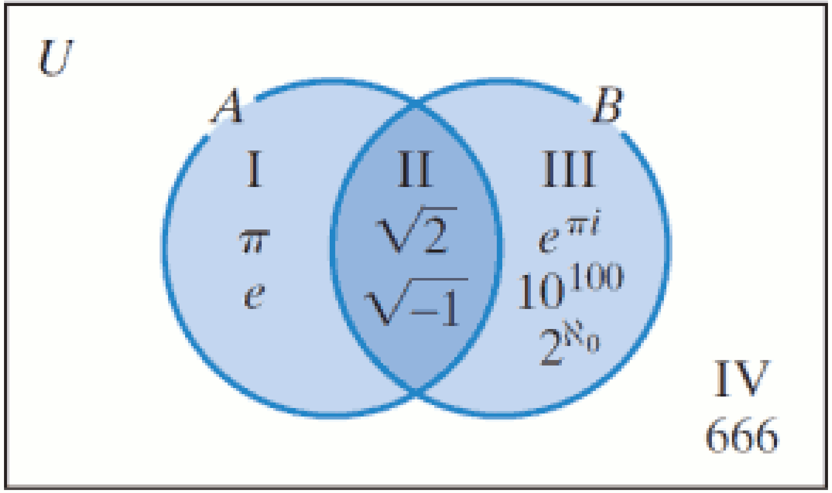
Use the diagram to determine each of the following s

a. $A \cup B$ **b.** $(A \cup B)'$ **c.** $A \cap B$

d. $(A \cap B)'$ **e.** $A' \cap B$ **f.** $A \cup B'$

Example: Determining Sets from a Venn Diagram (cont)

Solution



Set to Determine	Description of Set	Regions in Venn Diagram	Set in Roster Form
a. $A \cup B$	set of elements in A or B or both	I, II, III	$\{\pi, e, \sqrt{2}, \sqrt{-1}, e^{\pi i}, 10^{100}, 2^{\aleph_0}\}$
b. $(A \cup B)'$	set of elements in U that are not in $A \cup B$	IV	$\{666\}$
c. $A \cap B$	set of elements in both A and B	II	$\{\sqrt{2}, \sqrt{-1}\}$
d. $(A \cap B)'$	set of elements in U that are not in $A \cap B$	I, III, IV	$\{\pi, e, e^{\pi i}, 10^{100}, 2^{\aleph_0}, 666\}$
e. $A' \cap B$	set of elements that are not in A and are in B	III	$\{e^{\pi i}, 10^{100}, 2^{\aleph_0}\}$
f. $A \cup B'$	set of elements that are in A or not in B or both	I, II, IV	$\{\pi, e, \sqrt{2}, \sqrt{-1}, 666\}$

Sets and Precise Use of Everyday English

Set operations and Venn diagrams provide precise ways of organizing, classifying, and describing the vast array of sets and subsets we encounter every day.

Or refers to the union of sets

And refers to the intersection of sets

The Cardinal Number of the Union of Two Finite Sets

FORMULA FOR THE CARDINAL NUMBER OF THE UNION OF TWO FINITE SETS

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

The number of elements in A or B | is | the number of elements in A plus the number of elements in B | minus the number of elements in A and B.

Example: The Cardinal Number of the Union of Two Finite Sets

Some of the results of the campus blood drive survey indicated that 490 students were willing to donate blood, 340 students were willing to help serve a free breakfast to blood donors, and 120 students were willing to do both.

How many students were willing to donate blood **or** serve breakfast?

Example: continued

number of blood donors or breakfast servers

number of blood donors

number of breakfast servers

number of blood donors and breakfast servers

$$
\begin{aligned}
n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\
&= 490 + 340 - 120 \\
&= 830 - 120 \\
&= 710
\end{aligned}
$$