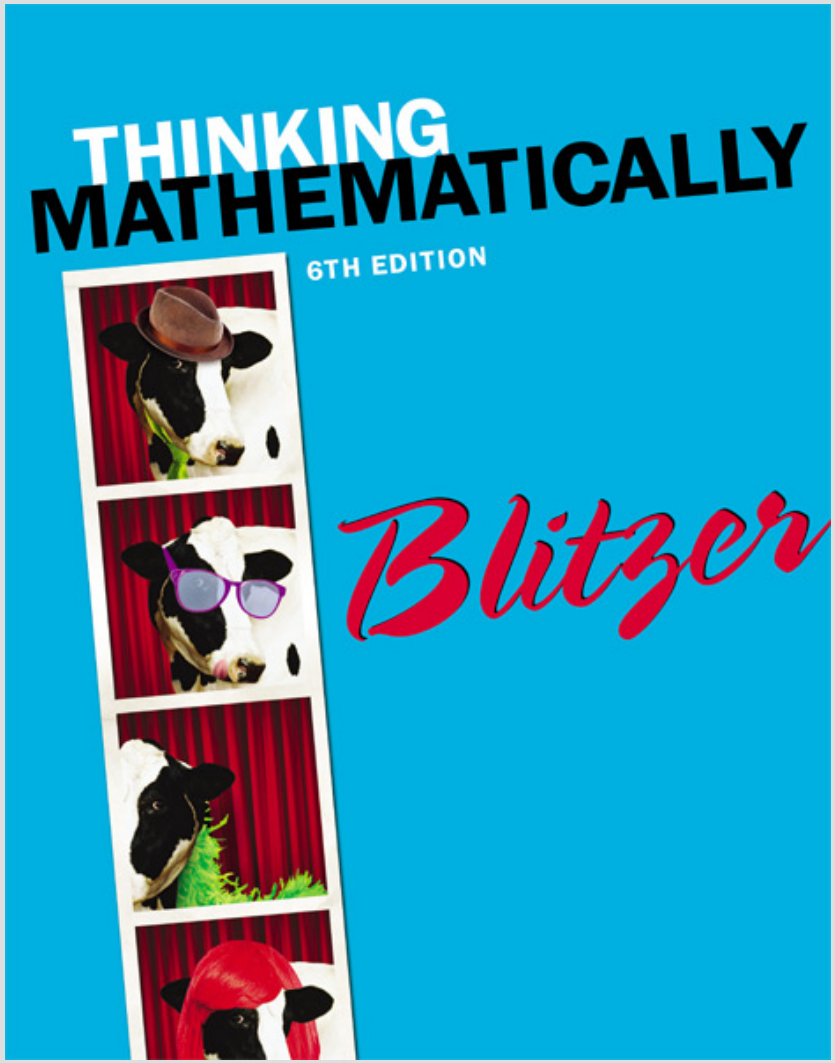
CHAPTER 2

Set Theory



2.4

Set Operations and Venn Diagrams with Three Sets

Objectives

1. Perform set operations with three sets.
2. Use Venn diagrams with three sets.
3. Use Venn diagrams to prove equality of sets.

Example: Set Operations with Three Sets

Given

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 3, 4, 5\}$ $B = \{1, 2, 3, 6, 8\}$

$C = \{2, 3, 4, 6, 7\}$

Find $A \cap (B \cup C')$

Step 1: Find the complement of C. $C' = \{1, 5, 8, 9\}$

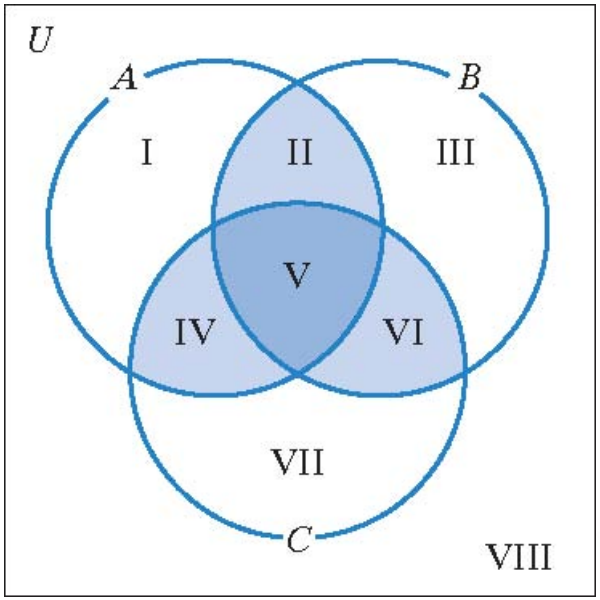
Step 2: Find the union of B and C′. $B \cup C' = \{1, 2, 3, 5, 6, 8, 9\}$

Step 3: Find the intersection.

$$A \cap B \cup C' = \{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 5, 6, 8, 9\}$$

$$A \cap B \cup C' = \{1, 2, 3, 5\}$$

Venn Diagrams with Three Sets



The Region Shown in Dark Blue

Region V — This region represents elements that are common to sets A, B, and C: $A \cap B \cap C$.

The Regions Shown in Light Blue

Region II — This region represents elements in both sets A and B that are not in set C: $(A \cap B) \cap C'$.

Region IV — This region represents elements in both sets A and C that are not in set B: $(A \cap C) \cap B'$.

Region VI — This region represents elements in both sets B and C that are not in set A: $(B \cap C) \cap A'$.

The Regions Shown in White

Region I — This region represents elements in set A that are in neither sets B nor C: $A \cap (B' \cap C')$.

Region III — This region represents elements in set B that are in neither sets A nor C: $B \cap (A' \cap C')$.

Region VII — This region represents elements in set C that are in neither sets A nor B: $C \cap (A' \cap B')$.

Region VIII — This region represents elements in the universal set U that are not in sets A, B, or C: $A' \cap B' \cap C'$.

Example: Determining Sets from a Venn Diagram with Three Intersecting Sets

Use the Venn diagram to find:

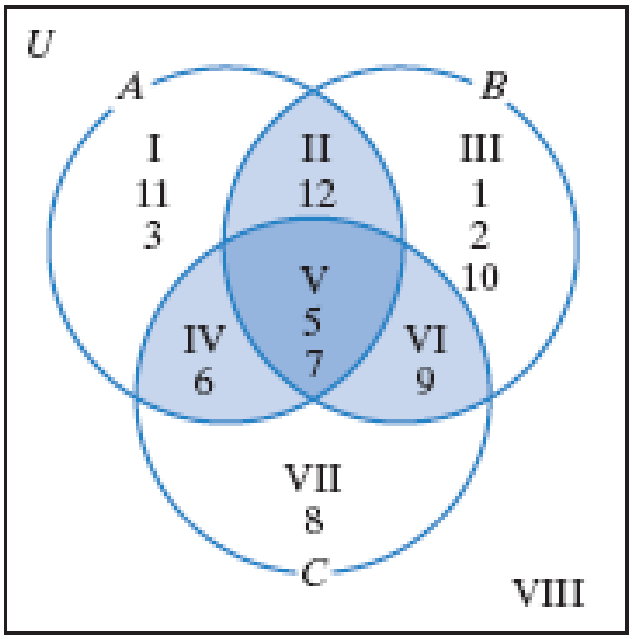
a. A

b. $A \cup B$

c. $B \cap C$

d. C'

e. $A \cap B \cap C$

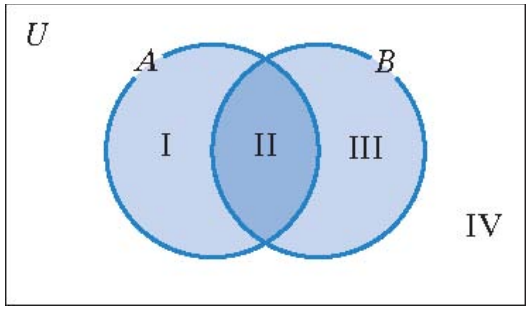


SOLUTION

Set to Dermine	Description of Set	Regions in Venn Diagram	Set in Roster Form
a. A	set of elements in A	I, II, IV, V	{11, 3, 12, 6, 5, 7}
b. $A \cup B$	set of elements in A or B or both	I, II, III, IV, V, VI	{11, 3, 12, 1, 2, 10, 6, 5, 7, 9}
c. $B \cap C$	set of elements in both B and C	V, VI	{5, 7, 9}
d. C'	set of elements in U that are not in C	I, II, III, VIII	{11, 3, 12, 1, 2, 10, 4}
e. $A \cap B \cap C$	set of elements in A and B and C	V	{5, 7}

Example: Proving the Equality of Sets

Prove that $(A \cap B)' = A' \cup B'$



Solution

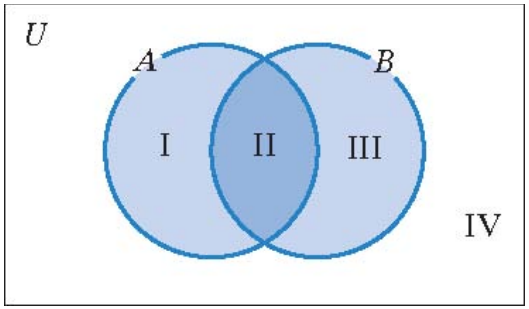
We can apply deductive reasoning using a Venn diagram to prove this statement is true for *all* sets A and B.

If both sets represent the same regions in this general diagram then this proves that they are equal.

Example continued

Prove that $(A \cap B)' = A' \cup B'$

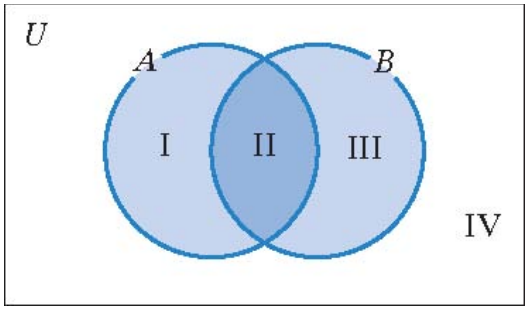
Solution: Begin with regions represented by $(A \cap B)'$.



Set	Regions
A	I, II
B	II, III
$(A \cap B)$	II
$(A \cap B)'$	I, III, IV

Example continued

Next, find the regions Represented by $A' \cup B'$.



Set	Regions
A'	III, IV
B'	I,IV
$A' \cup B'$	I, III,IV

Since both $(A \cap B)'$ and $A' \cup B'$ are represented by the same regions, the result proves that they are equal.

De Morgan's Laws

DE MORGAN'S LAWS

$(A \cap B)' = A' \cup B'$:	The complement of the intersection of two sets is the union of the complements of those sets.
$(A \cup B)' = A' \cap B'$:	The complement of the union of two sets is the intersection of the complements of those sets.