CHAPTER 3

Logic



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3.1

Statements, Negations, and Quantified Statements

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Objectives

- 1. Identify English sentences that are statements.
- 2. Express statements using symbols.
- 3. Form the negation of a statement
- 4. Express negations using symbols.
- 5. Translate a negation represented by symbols into English.
- 6. Express quantified statements in two ways.
- 7. Write negations of quantified statements.

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Statements

DEFINITION OF A STATEMENT

A statement is a sentence that is either true or false, but not both simultaneously.

Examples of statements:

London is the capital of England William Shakespeare wrote the television series *Modern Family*.

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Not Statements

Commands, questions, and opinions, are not statements because they are neither true or false.

Examples:

Titanic is the greatest movie of all time. (*opinion*)
Read pages 23 – 57. (*order or command*)
If I start losing my memory, how will I know? (*question*)

Using Symbols to Represent Statements

In symbolic logic, we use lowercase letters such as p, q, r, and s to represent statements.

Here are two examples:

- *p*: London is the capital of England
- *q*: William Shakespeare wrote the television series *Modern Family*.

The letter *p* represents the first statement.

The letter q represents the second statement.

Negating Statements

The **negation** of a true statement is a false statement and the negation of a false statement is a true statement.

Example: Forming Negations

Form the negation of each statement.

a. Shakespeare wrote the television series *Modern Family*.

Shakespeare did not write the television series *Modern Family*.

It is not true that Shakespeare wrote the television series *Modern Family*

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Example (cont)

Form the negation of each statement.b. Today is not Monday.

It is not true that today is not Monday.

Today is Monday.

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Example: Expressing Negations Symbolically

Let p and q represent the following statements:

- *p*: Shakespeare wrote the television series *Modern Family*.
- *q*: Today is not Monday.

Express each of the following statements symbolically:

a. Shakespeare did not write the television series *Modern Family.* ~p

b. Today is Monday.

Quantified Statements

Quantifiers: The words all, some, and no (or none).

Statements containing a quantifier:All poets are writers.Some people are bigots.No math books have pictures.Some students do not work hard.

Equivalent Ways of Expressing Quantified Statements

Statement	An Equivalent Way to Express the Statement	Example (Two Equivalent Quantified Statements)
All A are B.	There are no A that are not B .	All poets are writers. There are no poets that are not writers.
Some A are B.	There exists at least one A that is a B .	Some people are bigots. At least one person is a bigot.
No <i>A</i> are <i>B</i> .	All A are not B.	No common colds are fatal. All common colds are not fatal.
Some <i>A</i> are not <i>B</i> .	Not all <i>A</i> are <i>B</i> .	Some students do not work hard. Not all students work hard.

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Negation of Quantified Statements

The statements diagonally opposite each other are



Here are some examples of quantified statements and



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negations.

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Example: Negating a Quantified Statement

The mechanic told me, "All piston rings were replaced." I later learned that the mechanic never tells the truth. What can I conclude?

Solution:

Let's begin with the mechanic's statement: **All piston rings were replaced.** Because the mechanic never tells the truth, I can conclude that the truth is the negation of what I

was told.

Example continued

The mechanic told me, "All piston rings were replaced." I later learned that the mechanic never tells the truth. What can I conclude?

Solution:

The negation of "All *A* are *B*" is "Some *A* are not *B*".

Thus, I can conclude that

Some piston rings were not replaced.

I can also correctly conclude that:

At least one piston ring was not replaced.