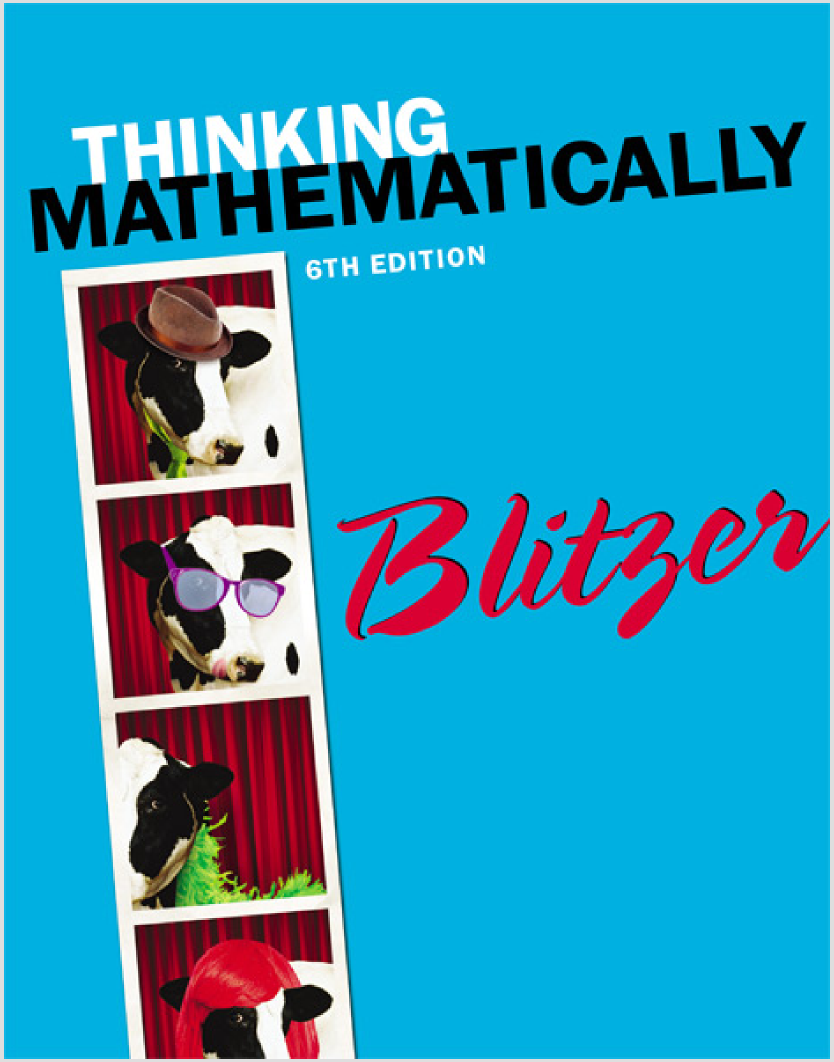
CHAPTER 3

Logic



3.2

Compound Statements and Connectives

Objectives

1. Express compound statements in symbolic form.

2. Express symbolic statements with parentheses in English.

3. Use the dominance of connectives.

Simple and Compound Statements

Simple statements convey one idea with no connecting words.

Compound statements combine two or more simple statements using connectives.

Connectives are words used to join simple statements. Examples are: **and**, **or**, **if…then**, and **if and only if**.

And Statements

If *p* and *q* are two simple statements, **then the compound statement "*p* and *q*" is symbolized by $p \wedge q$.**

The compound statement formed by connecting statements with the word *and* is called a conjunction. The symbol for *and* is $\wedge$.

Example: Translating from English to Symbolic Form

Let p and q represent the following simple statements:

p: It is after 5 P.M.

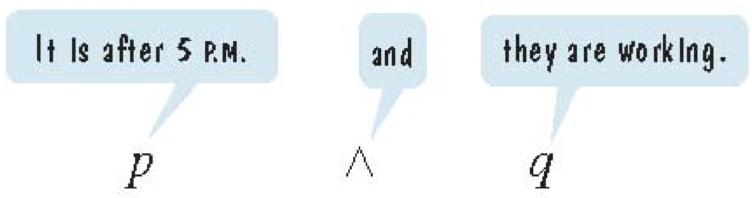
q: They are working.

Write each compound statement below in symbolic form:

a. It is after 5 P.M. and they are working.

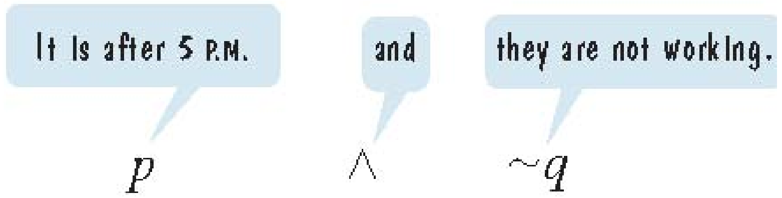
b. It is after 5 P.M. and they are not working.

a.



The symbolic form is $p \wedge q$.

b.



The symbolic form is $p \wedge \sim q$.

Common English Expressions for $p \wedge q$

Symbolic Statement	English Statement	Example *p*: It is after 5 P.M. *q*: They are working.
$p \wedge q$	*p* and *q*.	It is after 5 P.M. and they are working.
$p \wedge q$	*p* but *q*.	It is after 5 P.M., but they are working.
$p \wedge q$	*p* yet *q*.	It is after 5 P.M., yet they are working.
$p \wedge q$	*p* nevertheless *q*.	It is after 5 P.M.; nevertheless, they are working.

Or Statements

The connective *or* can mean two different things.

Consider the statement:

I visited London or Paris.

This statement can mean (**exclusive or**)

I visited London or Paris but not both.

It can also mean (**inclusive or**)

I visited London or Paris or both.

Or Statements

Disjunction is a compound statement formed using the **inclusive or** represented by the symbol $\vee$.

Thus, "*p* or *q* or both" is symbolized by $p \vee q$.

Example: Translating from English to Symbolic Form

Let p and q represent the following simple statements:

p: The bill receives majority approval.

q: The bill becomes a law.

Write each compound statement below in symbolic form:

a. The bill receives majority approval or the bill becomes a law. **Solution:** $p \vee q$

b. The bill receives majority approval or the bill does not become a law. **Solution:** $p \vee {\sim}q$

If -Then Statements

The compound statement "**If *p*, then *q*** is symbolized by $p \rightarrow q$.

This is called a **conditional statement.**

The statement before the $\rightarrow$ is called the ***antecedent.***

The statement after the $\rightarrow$ is called the ***consequent.***

If -Then Statements

This diagram shows a relationship that can be expressed 3 ways:

All poets are writers.

There are no poets that are not writers.

If a person is a poet, then that person is a writer.

Example: Translating from English to Symbolic Form

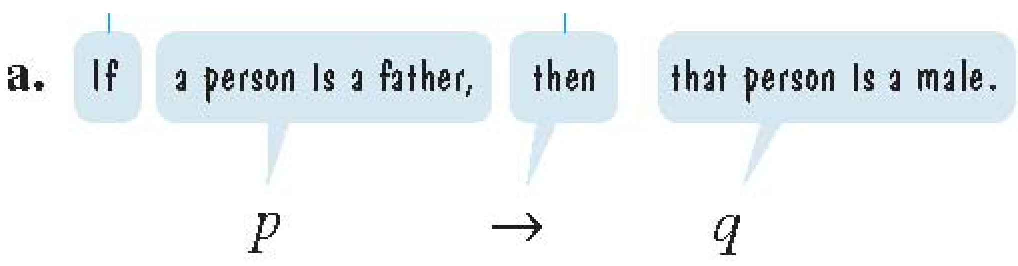
Let p and q represent the following simple statements:

p: A person is a father.

q: A person is a male.

Write each compound statement below in symbolic form:

a. If a person is a father, then that person is a male.



The symbolic form is $p \rightarrow q$.

Example: Translating from English to Symbolic Form

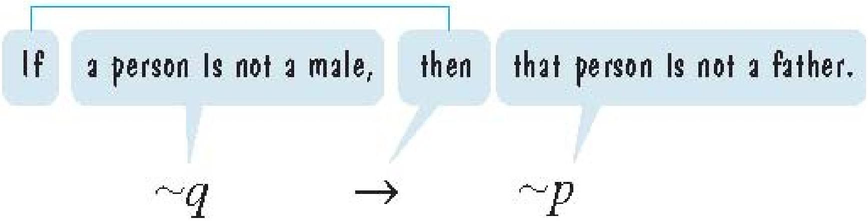
Let p and q represent the following simple statements:

p: A person is a father.

q: A person is a male.

Write each compound statement below in symbolic form:

c. If a person is not a male, then that person is not a father.



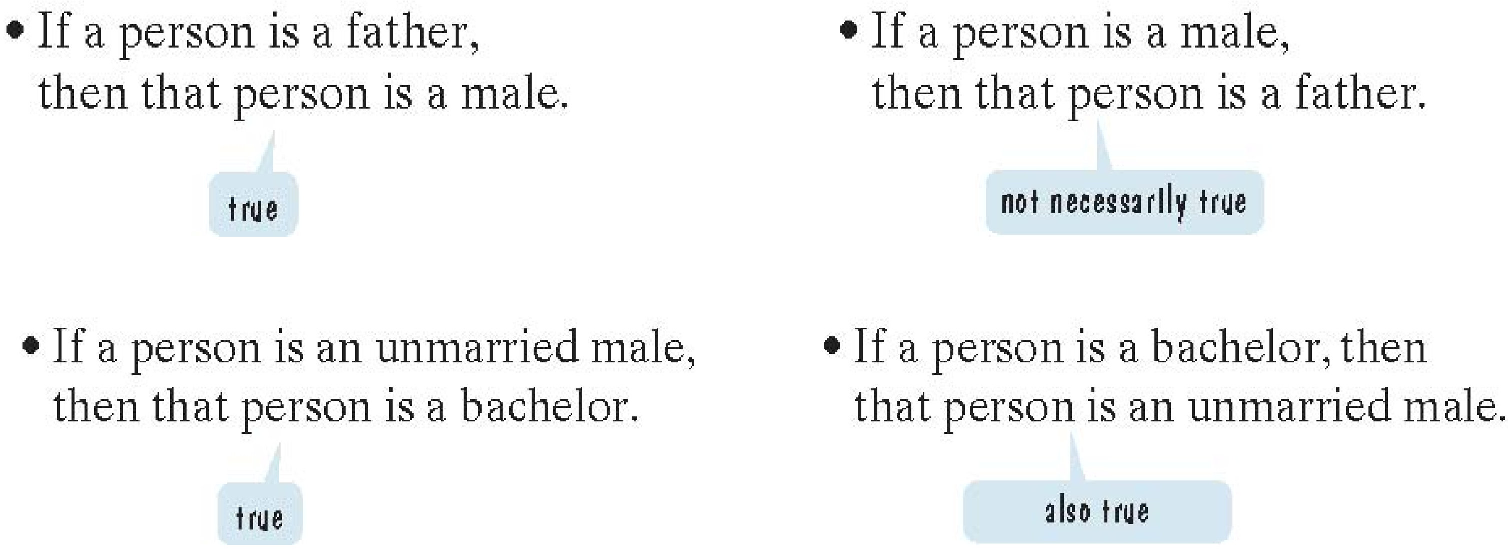
The symbolic form is $\sim q \rightarrow \sim p$.

Common English expressions for $p \to q$

Symbolic Statement	**English Statement**	**Example** ***p*: A person is a father.** ***q*: A person is a male.**
$p \to q$	If *p* then *q*.	If a person is a father, then that person is a male.
$p \to q$	*q* if *p*.	A person is a male, if that person is a father.
$p \to q$	*p* is sufficient for *q*.	Being a father is sufficient for being a male.
$p \to q$	*q* is necessary for *p*.	Being a male is necessary for being a father.
$p \to q$	*p* only if *q*.	A person is a father only if that person is a male.
$p \to q$	*Only if q, p*.	Only if a person is a male is that person a father.

If and *Only If* Statements

Biconditional statements are conditional statements that are true if the statement is still true when the antecedent and consequent are reversed.



The compound statement "***p* if and only if *q***" (abbreviated as *iff*) is symbolized by $\boldsymbol{p \leftrightarrow q}$.

Common English Expressions for $p \leftrightarrow q$.

Symbolic Statement	English Statement	Example ***p*: A person is an unmarried male.** ***q*: A person is a bachelor.**
$p \leftrightarrow q$	*p* if and only if *q*.	A person is an unmarried male if and only if that person is a bachelor.
$p \leftrightarrow q$	*q if and only if p.*	A person is a bachelor if and only if that person is an unmarried male.
$p \leftrightarrow q$	*If p* then *q, and if q then p.*	If a person is an unmarried male then that person is a bachelor, and if a person is a bachelor, then that person is an unmarried male.
$p \leftrightarrow q$	*p* is necessary and sufficient for *q*.	Being an unmarried male is necessary and sufficient for being a bachelor.
$p \leftrightarrow q$	*q is necessary and sufficient for p.*	Being a bachelor is necessary and sufficient for being an unmarried male.

Statements of Symbolic Logic

Name	Symbolic Form	Common English Translations
Negation	$\sim p$	Not *p*. It is not true that *p*.
Conjunction	$p \wedge q$	*p* and *q*, *p* but *q*.
Disjunction	$p \vee q$	*p* or *q*.
Conditional	$p \rightarrow q$	If *p*, then *q*, *p* is sufficient for *q*, *q* is necessary for *p*.
Biconditional	$p \leftrightarrow q$	*p* if and only if *q*, *p* is necessary and sufficient for *q*.

Example: Expressing Symbolic Statements with and without Parenthesis in English

Let p and q represent the following simple statements:

p: She is wealthy.

q: She is happy.

Write each of the following symbolic statements in words:

a. $\sim(p \wedge q)$

It is not true that she is wealthy and happy.

b. $\sim p \wedge q$

She is not wealthy and she is happy.

c. $\sim(p \vee q)$

She is neither wealthy nor happy. (Literally, it is not true that she is either wealthy or happy.)

Expressing Symbolic Statements with Parentheses in English

Symbolic Statement	Statements to Group Together	English Translation
$(q \wedge \sim p) \rightarrow \sim r$	$q \wedge \sim p$	If q and not p, then not r.
$q \wedge (\sim p \rightarrow \sim r)$	$\sim p \rightarrow \sim r$	q, and if not p then not r.

Notice that when we translate the symbolic statement into English, **the simple statements in parentheses appear on the same side of the comma.**

Example: Expressing Symbolic Statements with Parentheses in English

Let *p*, *q*, and *r* represent the following simple statements:

p: A student misses lecture.

q: A student studies.

r: A student fails.

Write each of these symbolic statements in words:

a. $(q \wedge \sim p) \rightarrow \sim r$

If a student studies and does not miss lecture, then the student does not fail.

Example: continued

Let p, q, and r represent the following simple statements:

p: A student misses lecture.

q: A student studies.

r: A student fails.

Write each of these symbolic statements in words:

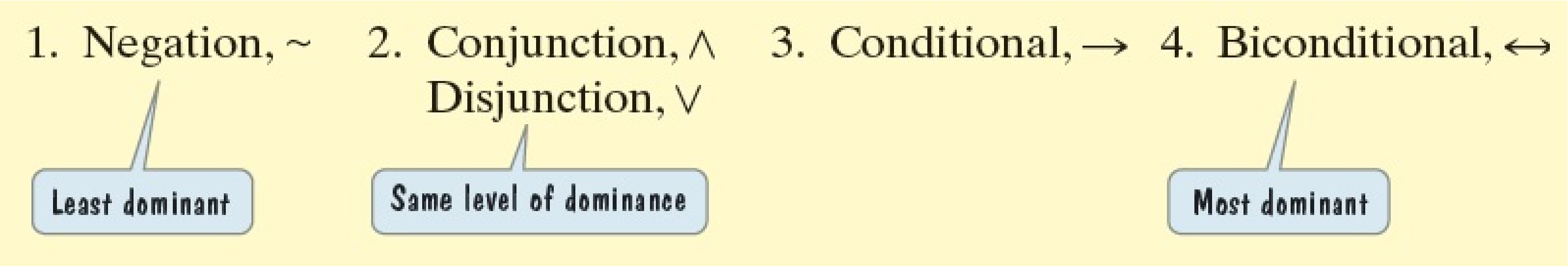
b. $q \wedge (\sim p \rightarrow \sim r)$

A student studies, and if the student does not miss lecture, then the student does not fail.

Dominance of Connectives

If a symbolic statement appears without parentheses, statements before and after the most *dominant connective* should be grouped.

The dominance of connectives used in symbolic logic is defined in the following order.



Using the Dominance of Connectives

Statement	Most Dominant Connective Highlighted in Red	Statements Meaning Clarified with Grouping Symbols	Type of Statement
$p \to q \wedge \sim r$	$p \to q \wedge \sim r$	$p \to (q \wedge \sim r)$	Conditional
$p \wedge q \to \sim r$	$p \wedge q \to \sim r$	$(p \wedge q) \to \sim r$	Conditional
$p \leftrightarrow q \to r$	$p \leftrightarrow q \to r$	$p \leftrightarrow (q \to r)$	Biconditional
$p \to q \leftrightarrow r$	$p \to q \leftrightarrow r$	$(p \to q) \leftrightarrow r$	Biconditional
$p \wedge \sim q \to r \vee s$	$p \wedge \sim q \to r \vee s$	$(p \wedge \sim q) \to (r \vee s)$	Conditional
$p \wedge q \vee r$	$p \wedge q \vee r$	The meaning is ambiguous.	?

Grouping symbols must be given with this statement to determine whether it means $(p \wedge q) \vee r$, a disjunction, or $p \wedge (q \vee r)$, a conjunction.

Example 8: Using the Dominance of Connectives

Let p, q, and r represent the following simple statements.

p: I fail the course.

q: I study hard.

r: I pass the final.

Write each compound statement in symbolic form:

a. I do not fail the course if and only if I study hard and I pass the final. $\sim p \leftrightarrow (q \wedge r)$

b. I do not fail the course if and only if I study hard, and I pass the final. $(\sim p \leftrightarrow q) \wedge r$

Example: Using the Dominance of Connectives

Write each compound statement below in symbolic form:

I do not fail the course if and only if I study hard and I pass the final.

Solution

We begin by assigning letters to the simple statements. Let each letter represent an English statement that is not negated. We can then represent any negated simple statement with the negation symbol.

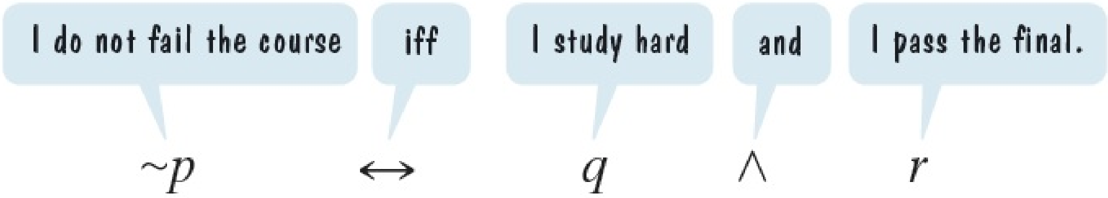
Example: Using the Dominance of Connectives

I do not fail the course if and only if I study hard and I pass the final.

p: I fail the course.

q: I study hard.

r: I pass the final.



Because the most dominant connective that appears is $\leftrightarrow$, the symbolic form with parentheses is $\sim p \leftrightarrow q \wedge r$