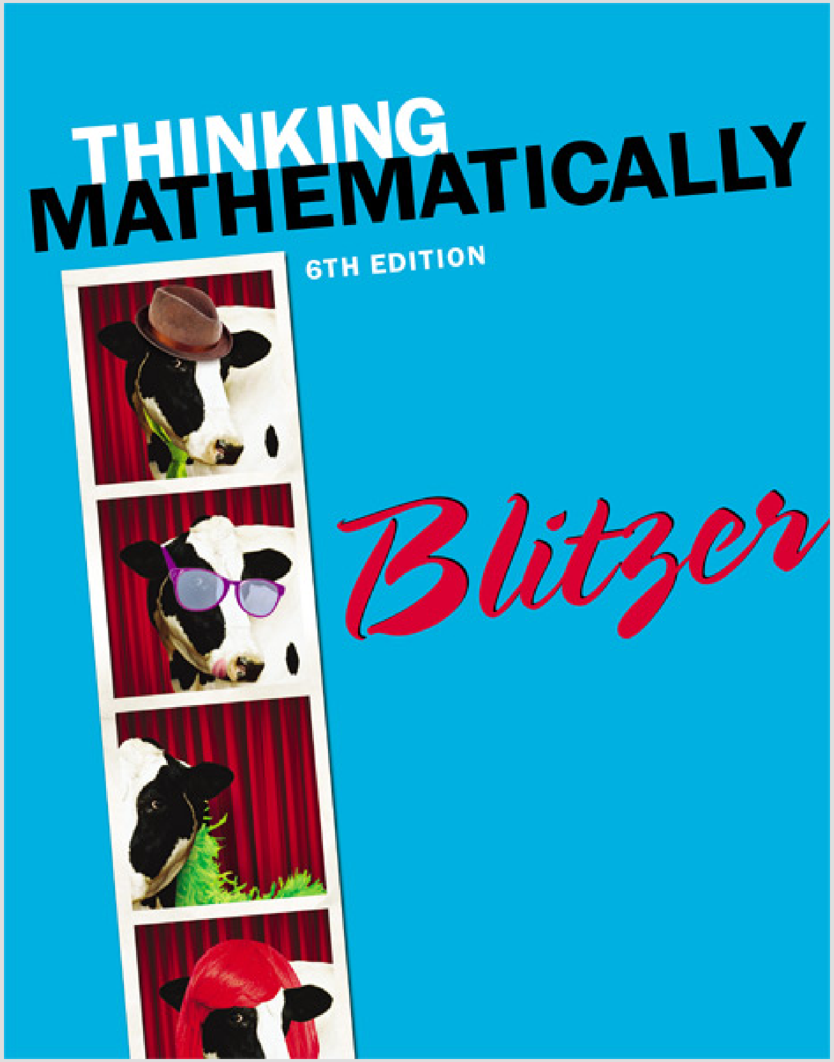
CHAPTER 3

Logic



3.4

Truth Tables for the Conditional and the Biconditional

Objectives

1. Understand the logic behind the definition of the conditional.
2. Construct truth tables for conditional statements.
3. Understand the definition of the biconditional.
4. Construct truth tables for biconditional statements.
5. Determine the true value of a compound statement for a specific case.

Truth Tables for Conditional Statements

$$p \rightarrow q$$

antecedent* → *consequent

If *p* then *q*.

A conditional is false only when the antecedent is true and the consequent is false.

Conditional		
p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example: Constructing a Truth Table

Construct a truth table for $\sim q \rightarrow \sim p$

p	q	$\sim q$	$\sim p$	$\sim q \rightarrow \sim p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

More on the Conditional Statement

You can reverse and negate the antecedent and consequent, and the statement’s truth value will not change.

If you’re cool, you won’t wear clothing with your school name on it.

If you wear clothing with your school name on it, you’re not cool.

> If the fashion tip above is true then so is this, and if it's false then this is false as well.

Example: Constructing a Truth Table

Construct a truth table for $[(p \vee q) \wedge \sim p] \rightarrow q$

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	$[(p \vee q) \wedge \sim p] \rightarrow q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

Biconditional Statements

$$p \leftrightarrow q$$

p if and only if *q*: $p \rightarrow q$ ***and*** $q \rightarrow p$

True only when the component statements have the same truth value.

Truth table for the Biconditional

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Definitions of Symbolic Logic

1. Negation $\sim$: not
   The negation of a statement has the opposite meaning, as well as the opposite truth value, from the statement.
2. Conjunction $\wedge$: and
   The only case in which a conjunction is true is when both component statements are true.
3. Disjunction $\vee$: or
   The only case in which a disjunction is false is when both component statements are false.
4. Conditional $\rightarrow$: if–then
   The only case in which a conditional is false is when the first component statement, the antecedent, is true and the second component statement, the consequent, is false.
5. Biconditional $\leftrightarrow$: if and only if
   The only cases in which a biconditional is true are when the component statements have the same truth value.

Example: Determining the Truth Value of a Compound Statement

You receive a letter that states that you have been assigned a Super Million Dollar Prize Entry Number -- 665567010. If your number matches the winning pre-selected number and you return the number before the deadline, you will win $1,000,000.00.

Suppose that your number does not match the winning pre-selected number, you return the number before the deadline and only win a free issue of a magazine. Under these conditions, can you sue the credit card company for making a false claim?

Example continued

Solution:

Assign letters to the simple statements in the claim.

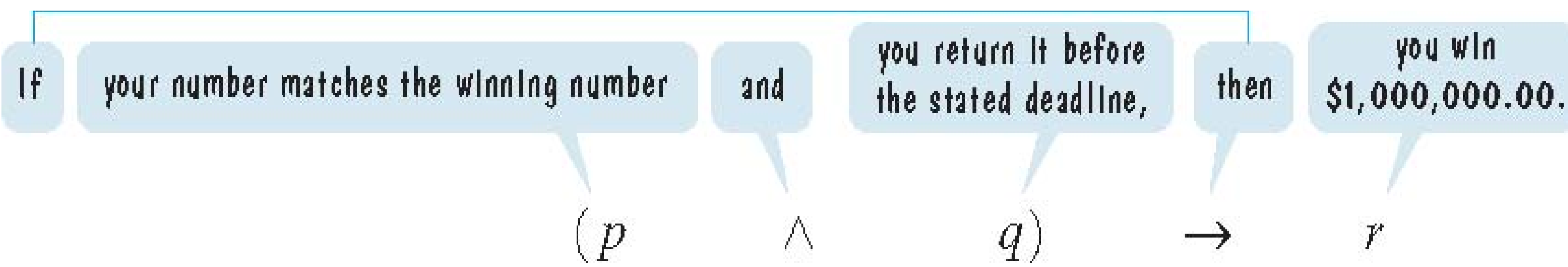
p: Your Super Million Dollar Prize Entry Number matches the winning preselected number. — false

q: You return the number before the stated deadline. — true

r: You win $1,000,000.00. — false; To make matters worse, you were duped into buying a magazine subscription.

Example continued

Now write the underlined claim in the letter in symbolic form:



Example continued

Substitute the truth values for *p*, *q*, and *r* to determine the truth value for the letter's claim.

$$(p \wedge q) \rightarrow r$$
$$(\mathrm{F} \wedge \mathrm{T}) \rightarrow \mathrm{F}$$
$$\mathrm{F} \rightarrow \mathrm{F}$$
$$\mathrm{T}$$

Our truth-value analysis indicates that you cannot sue the credit card company for making a false claim.