CHAPTER 3

Logic
3.5

Equivalent Statements and Variation of Conditional Statements
Objectives

1. Use a truth table to show that statements are equivalent.

2. Write the contrapositive for a conditional statement.

3. Write the converse and inverse of a conditional statement.
Equivalent Statements

Equivalent compound statements are made up of the same simple statements and have the same corresponding truth values for all true-false combinations of these simple statements.

If a compound statement is true, then its equivalent statement must also be true.

If a compound statement is false, its equivalent statement must also be false.
Example: Showing that Statements are Equivalent

Show that $p \lor \sim q$ and $\sim p \rightarrow \sim q$ are equivalent.

**Solution:** Construct a truth table and see if the corresponding truth values are the same:

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$\sim q$</th>
<th>$p \lor \sim q$</th>
<th>$\sim p$</th>
<th>$\sim p \rightarrow \sim q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
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<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>
Variations of the Conditional Statement

\[ p \rightarrow q \]

**A CONDITIONAL STATEMENT AND ITS EQUIVALENT CONTRAPOSITIVE**

\[ p \rightarrow q \equiv \sim q \rightarrow \sim p \]

The truth value of a conditional statement does not change if the antecedent and consequent are reversed and both are negated. The statement \( \sim q \rightarrow \sim p \) is called the **contrapositive** of the conditional \( p \rightarrow q \).
Example: Writing Equivalent Contrapositives

Write the equivalent contrapositive for:
If you live in Los Angeles, then you live in California.

\[ p: \text{You live in Los Angeles.} \]
\[ q: \text{You live in California.} \]

If you live in Los Angeles, then you live in California.

\[ p \rightarrow q \]

\[ \sim q \rightarrow \sim p \]

If you do not live in California, then you do not live in Los Angeles.
Variations of the Conditional Statement

**VARIATIONS OF THE CONDITIONAL STATEMENT**

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbolic Form</th>
<th>English Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional</td>
<td>$p \rightarrow q$</td>
<td>If $p$, then $q$.</td>
</tr>
<tr>
<td>Converse</td>
<td>$q \rightarrow p$</td>
<td>If $q$, then $p$.</td>
</tr>
<tr>
<td>Inverse</td>
<td>$\sim p \rightarrow \sim q$</td>
<td>If not $p$, then not $q$.</td>
</tr>
<tr>
<td>Contrapositive</td>
<td>$\sim q \rightarrow \sim p$</td>
<td>If not $q$, then not $p$.</td>
</tr>
</tbody>
</table>

Conditional and Contrapositive are equivalent. Converse and Inverse are equivalent.
Example: Writing Variations of a Conditional Statement

Write the converse, inverse, and contrapositive of the following *conditional* statement:

If you are 17, then you are not eligible to vote. (true)

**Solution:** Use the following representations:

- $p$: You are 17.
- $q$: You are eligible to vote.
Example continued

Now work with $p \rightarrow \sim q$ to form the converse, inverse and contrapositive.

Then translate the symbolic form of each statement back into English.

(see next slide)
## Example continued

<table>
<thead>
<tr>
<th>Given Conditional Statement</th>
<th>Symbolic Statement</th>
<th>English Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given Conditional Statement</td>
<td>( p \rightarrow \sim q )</td>
<td>If you are 17, then you are not eligible to vote.</td>
</tr>
<tr>
<td>Converse: Reverse the components of ( p \rightarrow \sim q ).</td>
<td>( \sim q \rightarrow p )</td>
<td>If you are not eligible to vote, then you are 17.</td>
</tr>
<tr>
<td>Inverse: Negate the components of ( p \rightarrow \sim q ).</td>
<td>( \sim p \rightarrow \sim (\sim q) ) simplifies to ( \sim p \rightarrow q )</td>
<td>If you are not 17, then you are eligible to vote.</td>
</tr>
<tr>
<td>Contrapositive: Reverse and negate the components of ( p \rightarrow \sim q ).</td>
<td>( \sim (\sim q) \rightarrow \sim p ) simplifies to ( q \rightarrow \sim p )</td>
<td>If you are eligible to vote, then you are not 17.</td>
</tr>
</tbody>
</table>