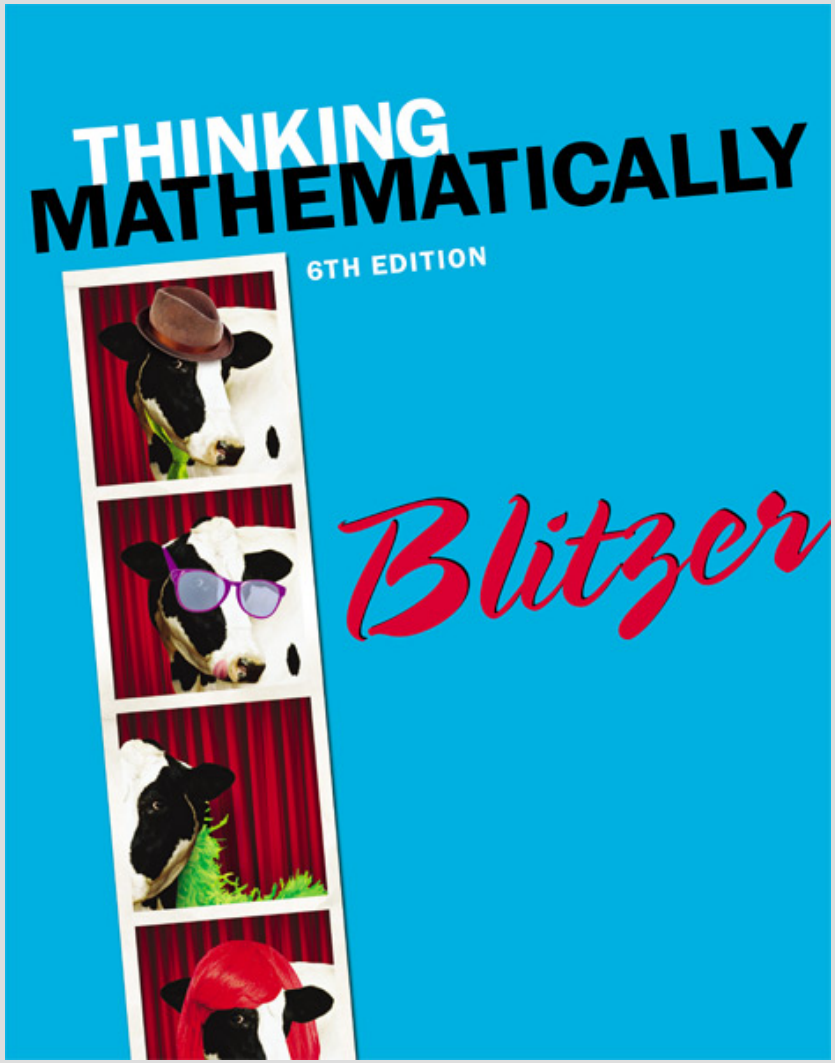
CHAPTER 3

Logic



3.5

Equivalent Statements and Variation of Conditional Statements

Objectives

1. Use a truth table to show that statements are equivalent.

2. Write the contrapositive for a conditional statement.

3. Write the converse and inverse of a conditional statement.

Equivalent Statements

Equivalent compound statements are made up of the same simple statements and have the same corresponding truth values for all true-false combinations of these simple statements.

If a compound statement is true, then its equivalent statement must also be true.

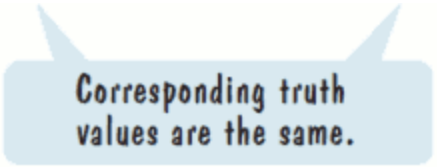
If a compound statement is false, its equivalent statement must also be false.

Example: Showing that Statements are Equivalent

Show that $p \vee \sim q$ and $\sim p \rightarrow \sim q$ are equivalent.

Solution: Construct a truth table and see if the corresponding truth values are the same:

p	q	$\sim q$	$p \vee \sim q$	$\sim p$	$\sim p \rightarrow \sim q$
T	T	F	T	F	T
T	F	T	T	F	T
F	T	F	F	T	F
F	F	T	T	T	T



Variations of the Conditional Statement

$p \rightarrow q$

A CONDITIONAL STATEMENT AND ITS EQUIVALENT CONTRAPOSITIVE

$$p \rightarrow q \equiv \sim q \rightarrow \sim p$$

The truth value of a conditional statement does not change if the antecedent and consequent are reversed and both are negated. The statement $\sim q \rightarrow \sim p$ is called the **contrapositive** of the conditional $p \rightarrow q$.

Example: Writing Equivalent Contrapositives

Write the equivalent contrapositive for:

If you live in Los Angeles, then you live in California.

p: You live in Los Angeles.

q: You live in California.

If you live in Los Angeles, then you live in California.

$$p \rightarrow q$$

$$\sim q \rightarrow \sim p$$

If you do not live in California, then you do not live in Los Angeles.

Variations of the Conditional Statement

VARIATIONS OF THE CONDITIONAL STATEMENT

Name	Symbolic Form	English Translation
Conditional	$p \rightarrow q$	If p, then q.
Converse	$q \rightarrow p$	If q, then p.
Inverse	$\sim p \rightarrow \sim q$	If not p, then not q.
Contrapositive	$\sim q \rightarrow \sim p$	If not q, then not p.

Conditional and Contrapositive are equivalent.

Converse and Inverse are equivalent.

Example: Writing Variations of a Conditional Statement

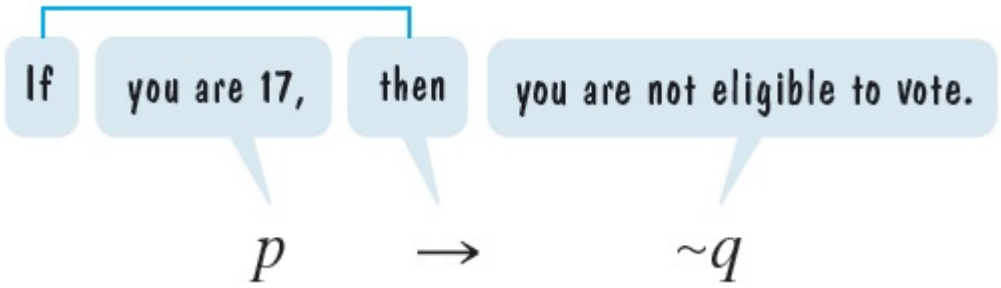
Write the converse, inverse, and contrapositive of the following **conditional** statement:

If you are 17, then you are not eligible to vote. (true)

Solution: Use the following representations:

p: You are 17.

q: You are eligible to vote.



Example continued

Now work with $p \rightarrow \sim q$ to form the converse, inverse and contrapositive.

Then translate the symbolic form of each statement back into English.

(see next slide)

Example continued

	Symbolic Statement	English Translation
Given Conditional Statement	$p \rightarrow \sim q$	If you are 17, then you are not eligible to vote. true
Converse: Reverse the components of $p \rightarrow \sim q$.	$\sim q \rightarrow p$	If you are not eligible to vote, then you are 17. not necessarily true
Inverse: Negate the components of $p \rightarrow \sim q$.	$\sim p \rightarrow \sim(\sim q)$ simplifies to $\sim p \rightarrow q$	If you are not 17, then you are eligible to vote. not necessarily true
Contrapositive: Reverse and negate the components of $p \rightarrow \sim q$.	$\sim(\sim q) \rightarrow \sim p$ simplifies to $q \rightarrow \sim p$	If you are eligible to vote, then you are not 17. true