# **CHAPTER 3**

## Logic



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# 3.6

### Negations of Conditional Statements and De Morgan's Laws

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## Objectives

- 1. Write the negation of a conditional statement.
- 2. Use De Morgan's laws.

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# The Negation of the Conditional Statement $p \rightarrow q$

#### THE NEGATION OF A CONDITIONAL STATEMENT

The negation of  $p \rightarrow q$  is  $p \land \sim q$ . This can be expressed as

 $\sim (p \rightarrow q) \equiv p \land \sim q.$ 

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# Example: The Negation of a Conditional Statement

Write the negation of:

If too much homework is given, a class should not be taken.

# **Solution:**

*p*: Too much homework is given, *q*: A class should be taken. The symbolic form is  $p \rightarrow \sim q$ .

## **Example continued**

The negation of  $p \rightarrow \neg q$  is  $p \land \neg(\neg q)$ which simplifies to  $p \land q$ .

Translating into English: Too much homework is given and the class should be taken.

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# **Summary of Conditional Statements**

#### THE CONDITIONAL STATEMENT $p \rightarrow q$

Contrapositive

 $p \rightarrow q$  is equivalent to  $\sim q \rightarrow \sim p$  (the contrapositive).

Converse and Inverse

- **1.**  $p \rightarrow q$  is not equivalent to  $q \rightarrow p$  (the converse).
- 2.  $p \rightarrow q$  is not equivalent to  $\sim p \rightarrow \sim q$  (the inverse).

#### Negation

The negation of  $p \rightarrow q$  is  $p \land \sim q$ .

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### **De Morgan's Laws**

#### **DE MORGAN'S LAWS**

**1.** 
$$\sim (p \land q) \equiv \sim p \lor \sim q$$
  
**2.**  $\sim (p \lor q) \equiv \sim p \land \sim q$ 

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# **De Morgan's Laws**

Proof of the first law is shown in the Truth Table below.

p q	$p \wedge q$	$\sim (p \land q)$	~p	~q	$\sim p \lor \sim q$
ТТ	Т	F	F	F	F
T F	F	Т	F	Т	Т
F T	F	Т	Т	F	Т
F F	F	Т	Т	Т	Т

Truth values are the same, proving that  $\sim$  ( $p \land q$ )  $\equiv \sim p \lor \sim q$ .

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## **De Morgan's Laws and Negations**

#### **DE MORGAN'S LAWS AND NEGATIONS**

**1.** 
$$\sim (p \land q) \equiv \sim p \lor \sim q$$

The negation of  $p \land q$  is  $\sim p \lor \sim q$ . To negate a conjunction, negate each component statement and change *and* to *or*.

**2.** 
$$\sim (p \lor q) \equiv \sim p \land \sim q$$

The negation of  $p \lor q$  is  $\sim p \land \sim q$ . To negate a disjunction, negate each component statement and change *or* to *and*.

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## Example: Negating Conjunctions and Disjunctions

Write the negation for each of the following statements:

**a.** All students do laundry on weekends and I do not.



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# **Example continued**

Write the negation for each of the following statements:

 b. Some college professors are entertaining lecturers or I'm bored.



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