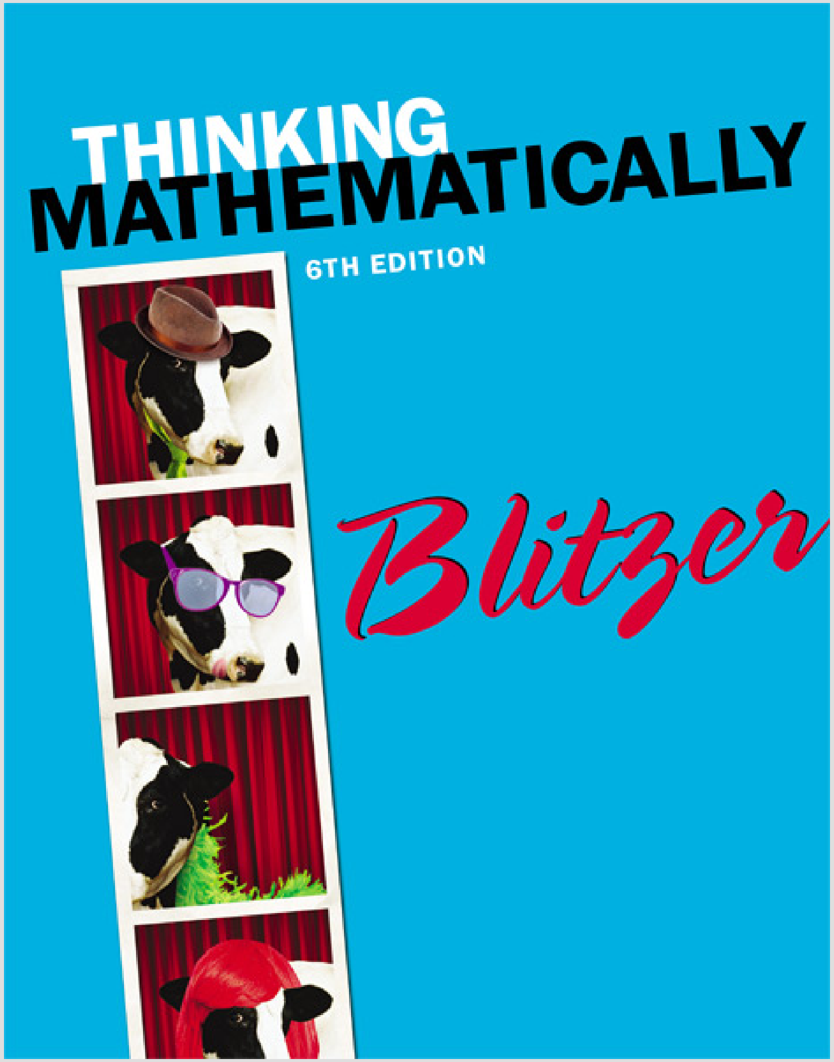
CHAPTER 3

Logic



3.7

Arguments and Truth Tables

Objectives

1. Use truth tables to determine validity.
2. Recognize and use forms of valid and invalid arguments.

Arguments

An **Argument** consists of two parts:

Premises: the given statements.

Conclusion: the result determined by the truth of the premises.

Valid Argument: The conclusion is true whenever the premises are assumed to be true.

Invalid Argument: Also called a **fallacy**

Truth tables can be used to test validity.

Testing the Validity of an Argument with a Truth Table

TESTING THE VALIDITY OF AN ARGUMENT WITH A TRUTH TABLE

1. Use a letter to represent each simple statement in the argument.
2. Express the premises and the conclusion symbolically.
3. Write a symbolic conditional statement of the form

$$[(\text{premise } 1) \wedge (\text{premise } 2) \wedge \cdots \wedge (\text{premise } n)] \rightarrow \text{conclusion},$$

   where n is the number of premises.
4. Construct a truth table for the conditional statement in step 3.
5. If the final column of the truth table has all trues, the conditional statement is a tautology and the argument is valid. If the final column does not have all trues, the conditional statement is not a tautology and the argument is invalid.

Example: Did the Pickiest Logician in the Galaxy Foul Up?

In Star Trek, the spaceship *Enterprise* is hit by an ion storm, causing the power to go out. Captain Cook wonders if Mr. Scott is aware of the problem. Mr. Spock replies, “If Mr. Scott is still with us, the power should be on momentarily.” Moments later the ship’s power comes back on.

Argument:

If Mr. Scott is still with us, then the power will come on.

The power comes on.

Therefore, Mr. Scott is still with us.

Determine whether the argument is valid or invalid.

Example continued

Solution

Step 1: p: Mr. Scott is still with us.

q: The power will come back on.

Step 2: Write the argument in symbolic form:

$p \rightarrow q$	If Mr. Scott is still with us, then the power will come on.
$\underline{q}$	The power comes on.
$\therefore \quad p$	Mr. Scott is still with us.

Step 3: Write the symbolic statement.

$$[(p \rightarrow \text{q}) \wedge q] \rightarrow p$$

Example continued

Step 4: Construct a truth table for the conditional statement.

p q	$p \to q$	$(p \to q) \land q$	$[(p \to q) \land q] \to p$
T T	T	T	T
T F	F	F	T
F T	T	T	F
F F	T	F	T

Step 5: Spock's argument is invalid, or a fallacy.

Example: Determining Validity with a Truth Table

Determine if the following argument is valid:

"I can't have anything more to do with the operation. If I did, I'd have to lie to the Ambassador. And I can't do that."

—Henry Bromell

Example continued

Solution:

We can express the argument as follows:

If I had anything more to do with the operation, I'd have to lie to the Ambassador.

I can't lie to the Ambassador.

Therefore, I can't have anything more to do with the operation.

Example continued

Step 1: Use a letter to represent each statement in the argument:

p: I have more to do with the operation

q: I have to lie to the Ambassador.

Example continued

Step 2: Express the premises and the conclusion symbolically.

$p \rightarrow q$	If I had anything more to do with the operation, I'd have to lie to the Ambassador.
$\underline{\sim q}$	<u>I can't lie to the Ambassador.</u>
$\therefore \quad \sim p$	Therefore, I can't have anything more to do with the operation.

Step 3: Write a symbolic statement:

$$[(p \rightarrow q) \wedge \sim q\,] \rightarrow \sim p$$

Example continued

Step 4: Construct the truth table.

p	q	$p \to q$	$\sim q$	$(p \to q) \wedge \sim q$	$\sim p$	$[(p \to q) \wedge \sim q] \to \sim p$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Step 5: The form of this argument is called **contrapositive reasoning.** It is a valid argument.

Standard Forms of Arguments

Valid Arguments			
Direct Reasoning	**Contrapositive Reasoning**	**Disjunctive Reasoning**	**Transitive Reasoning**
$p \to q$ p $\therefore q$	$p \to q$ $\sim q$ $\therefore \sim p$	$p \vee q$ $p \vee q$ $\sim p$ $\sim q$ $\therefore q$ $\therefore p$	$p \to q$ $q \to r$ $\therefore p \to r$ $\therefore \sim r \to \sim p$

Invalid Arguments			
Fallacy of the Converse	**Fallacy of the Inverse**	**Misuse of Disjunctive Reasoning**	**Misuse of Transitive Reasoning**
$p \to q$ q $\therefore p$	$p \to q$ $\sim p$ $\therefore \sim q$	$p \vee q$ $p \vee q$ p q $\therefore \sim q$ $\therefore \sim p$	$p \to q$ $q \to r$ $\therefore r \to p$ $\therefore \sim p \to \sim r$

Example: Determining Validity Without Truth Tables

Determine whether this argument is valid or invalid: Identify any sound arguments.

There is no need for surgery. I know this because if there is a tumor then there is need for surgery, but there is no tumor.

Solution:

p: There is a tumor

q: There is a need for surgery.

Example continued

Expressed symbolically:

If there is a tumor then there is need for surgery.	$p \to q$
There is no tumor.	$\sim p$
Therefore, there is no need for surgery.	$\therefore \sim q$

The argument is in the form of the fallacy of the inverse and is therefore, invalid.

Example: Drawing a Logical Conclusion

Draw a valid conclusion from the following premises:

If all students get requirements out of the way early, then no students take required courses in their last semester. Some students take required courses in their last semester.

Example (cont)

The form of the premises is

$p \rightarrow q$ If all students get requirements out of the way early, then no students take required courses in their last semester.

$\sim q$ Some students take required courses in their last semester. (Recall that the negation of *no* is *some*.)

$\therefore ?$

Example (cont)

The conclusion ~*p* is valid because it forms the contrapositive reasoning of a valid argument when it follows the given premises. The conclusion ~*p* translates as

Not all students get requirements out of the way early.

Because the negation of *all* is *some …not*, we can equivalently conclude that

Some students do not get requirements out of the way early.