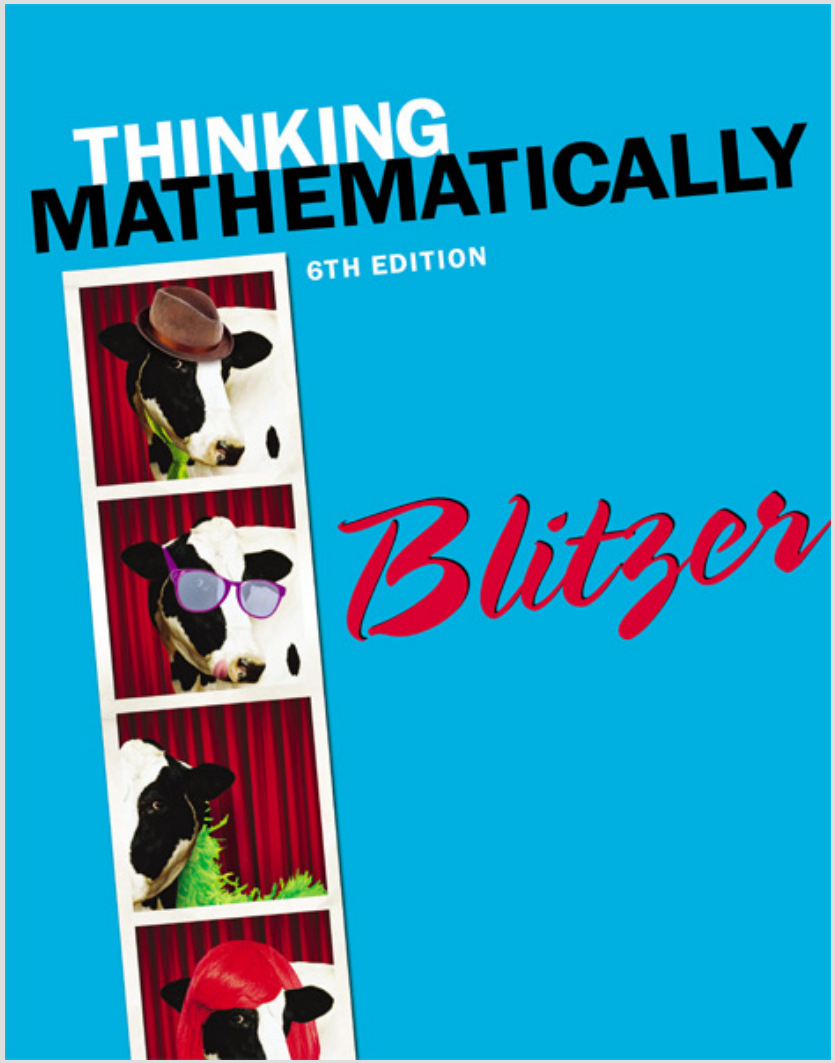
CHAPTER 1

Problem Solving and Critical Thinking



Problem Solving

1.3

Objective

1. Solve problems using the organization of the four-step problem solving process

Polya's Four Steps in Problem Solving
Step 1

Understand the problem.

Read the problem several times.

The first reading can serve as an overview.

In the second reading, write down what information is given and determine exactly what the problem requires you to find.

Polya's Four Steps in Problem Solving Step 2

Devise a Plan.

The plan for solving the problem might involve one or more of these problem solving strategies:

- Use inductive reasoning to look for a pattern.
- Make a systematic list or table.
- Use estimation to make an educated guess at the solution. Check the guess against the problem's conditions and work backward to eventually determine the solution.
- Try expressing the problem more simply and solve a similar simpler problem.
- Use trial and error.

Polya's Four Steps in Problem Solving
Step 2 continued

List the given information in a chart or table.

Try making a sketch or a diagram to illustrate the problem.

Relate the problem to similar problems that you have seen before. Try applying the procedures used to solve the similar problem to the new one.

Look for a "catch" if the answer seems too obvious. Perhaps the problem involves some sort of trick question deliberately intended to lead the problem solver in the wrong direction.

Use the given information to eliminate possibilities.

Use common sense.

Polya's Four Steps in Problem Solving Steps 3-4

Step 3: Carry out the plan and solve the problem.

Step 4: Look back and check the answer.

The answer should satisfy the conditions of the problem.

The answer should make sense and be reasonable. If this is not the case, recheck the method and any calculations. Perhaps there is an alternative way to arrive at a correct solution.

Example: Finding What is Missing

A man purchased five shirts, each at the same discount price. How much did he pay for them?

Solution:

Step 1: Understand the problem. Here's what is given: Number of shirts purchased: 5

We must find out how much the man paid for the five shirts.

Example: Finding What is Missing

A man purchased five shirts, each at the same discount price. How much did he pay for them?

Step 2: Devise a plan.

The amount that the man paid for the five shirts is the number of shirts, 5, times the cost of each shirt.

The discount price of each shirt is not given.

This missing piece of information makes it impossible to solve the problem.

Example: Applying the Four Step Procedure

By paying $100 cash up front and the balance at $20 a week, how long will it take to pay for a bicycle costing $680?

Solution:

Step 1: Understand the problem.

We are given:

Cost of the bicycle: $680

Amount paid in cash: $100

Weekly payments: $20

Example continued

Step 2: Devise a plan. Subtract the amount paid in cash from the cost of the bicycle. This results in the balance, the amount still to be paid. Divide this result by the weekly payment of $20 to find the number of weeks required to pay for the bicycle.

Step 3: Carry out the plan and solve the problem.

Begin by finding the balance:

$$\$680 - \$100 = \$580$$

Now, divide the balance by $20, the weekly payment to find the number of weeks:

$$\frac{\$580}{20} = 29 \text{ weeks}$$

Example continued

Step 4: Look back and check the answer.

We can double check the arithmetic. We can also see if the answer satisfies the conditions of the problem.

$20	weekly payment
× 29	number of weeks
$580	total of weekly payments

$580	total of weekly payments
+$100	amount paid in cash
$680	cost of the bicycle

The answer of 29 weeks satisfies the condition that the cost of the bicycle is $680.

Example: Making a List

You are an engineer programming the automatic gate for a 50-cent toll. The gate accepts exact change only and does not accept pennies. How many coin combinations must you program the gate to accept?

Solution

Step 1: Understand the problem. The total change must always be 50 cents. We need to find all combinations using half-dollars, quarters, dimes or nickels.

Example: Making a List

Step 2: Devise a plan. Make a list of all possible coin combinations. Begin with the coins of larger value and work toward coins of smaller value.

Step 3: Carry out the plan and solve the problem. We will set up the table using half-dollars, quarters, dimes and nickels as the table headings. Count the coin combinations in the table.

Half-Dollars	Quarters	Dimes	Nickels
1	0	0	0
0	2	0	0
0	1	2	1
0	1	1	3
0	1	0	5
0	0	5	0
0	0	4	2
0	0	3	4
0	0	2	6
0	0	1	8
0	0	0	10

Example: Making a List

Step 4: Look back and check the answer.
Double-check to make sure you have all possible combinations and that they add up to 50 cents. Double-check your total count.

Example: Using a Diagram

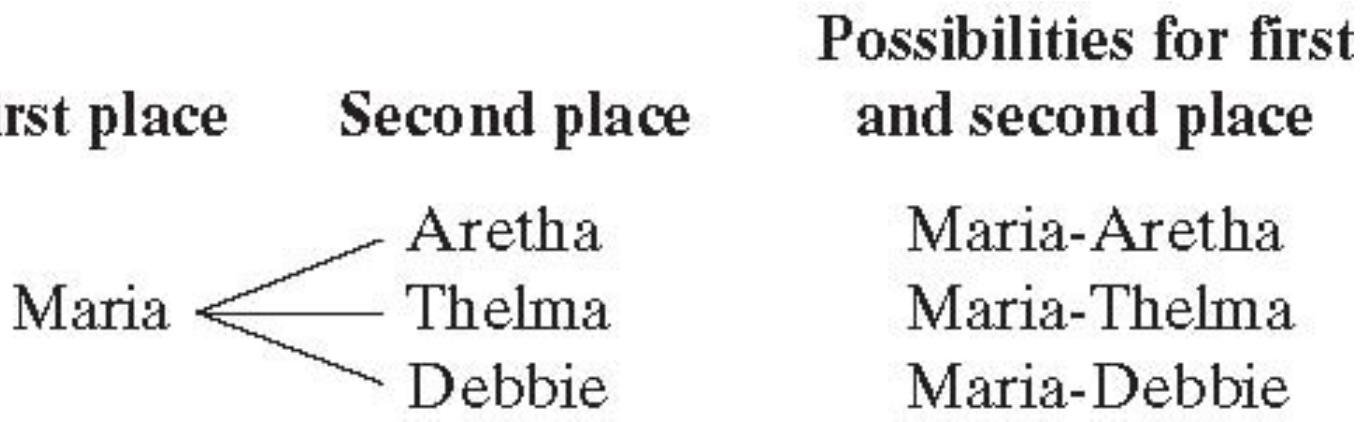
Four runners are in a one-mile race. Maria, Aretha, Thelma, and Debbie. Points are awarded only to the women finishing first or second. The first-place winner gets more points than the second-place winner.

How many different arrangements of first-and second-place winners are possible?

Example: Using a Diagram

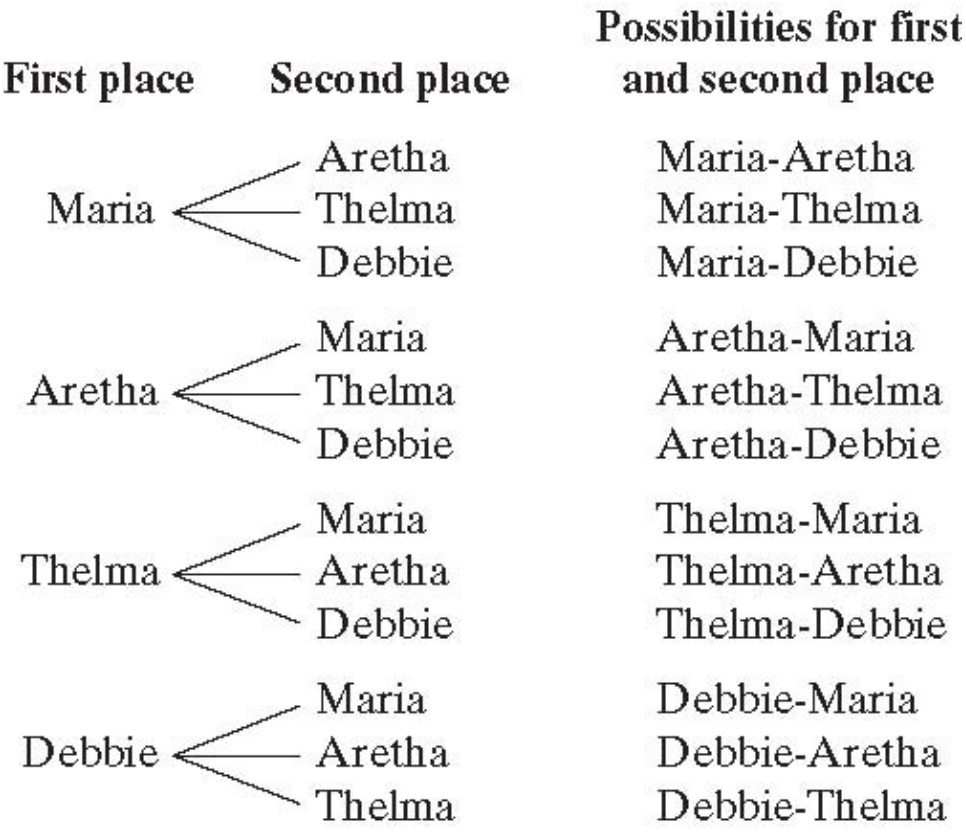
Step 1. Understand the problem.

Three possibilities for first and second position are:



Note that Maria finishing first and Aretha finishing second is a different outcome from Aretha finishing first and Maria finishing second since first place gets more points than second place.

Example continued

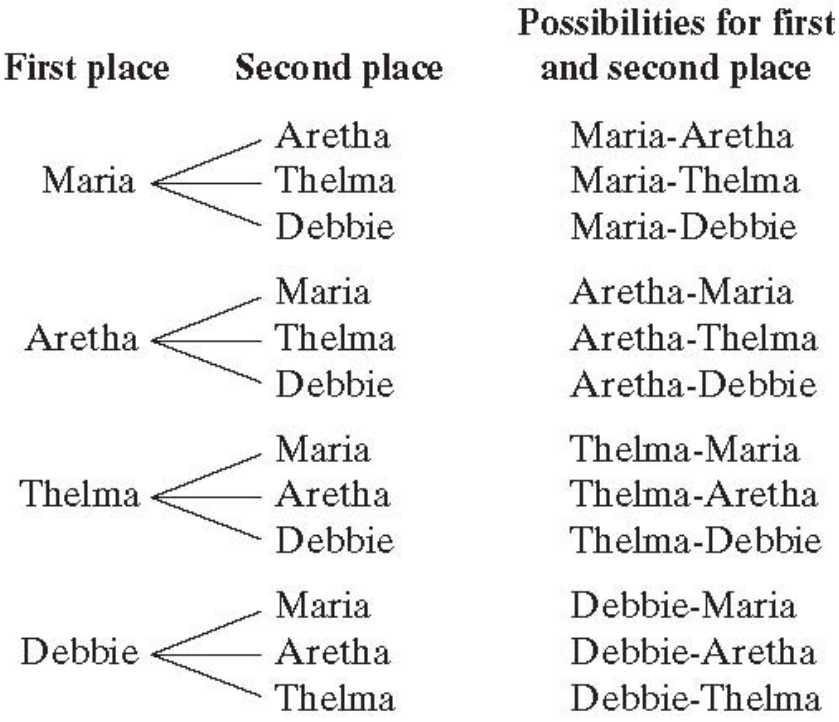


Listing each woman as the first place winner and pairing her with each of the 3 other women results in a tree diagram. The final column shows all the possible combinations.

Example continued

Step 4. Look back and check the answer.

Check the tree diagram to make sure that no possible first- and second-place outcomes have been left out.



Double-check your count for the winning pairs of runners.

We confirm there are 12 possible combinations.