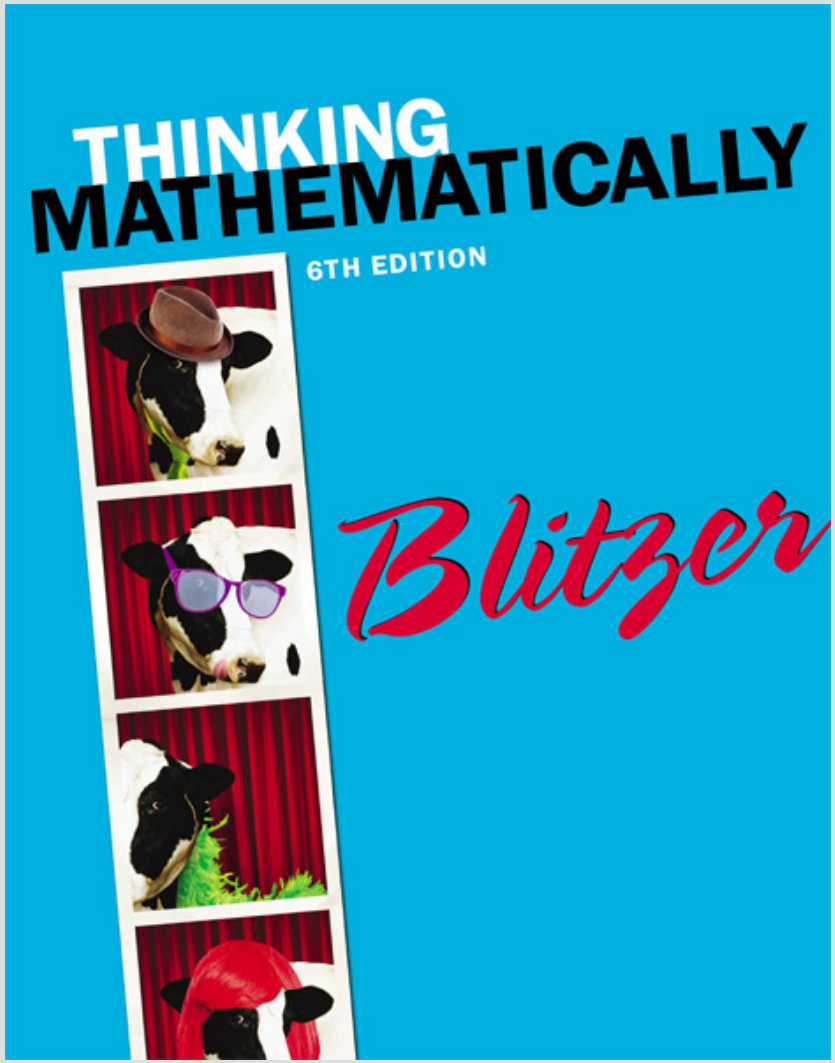
CHAPTER 4

Number Representation and Calculation



4.1

Our Hindu-Arabic System and Early Positional Systems

Objectives

1. Evaluate an exponential expression.
2. Write a Hindu-Arabic numeral in expanded form.
3. Express a number's expanded form as a Hindu-Arabic numeral.
4. Understand and use the Babylonian numeration system.
5. Understand and use the Mayan numeration system.

Our Hindu-Arabic Numeration System

An important characteristic is that we can write the numeral for any number, large or small, using only ten symbols called **digits**:

0,1,2,3,4,5,6,7,8, and 9

Hindu-Arabic numerals can be written in **expanded form**, in which the value of the digit in each position is made clear.

We can write 663 in an expanded form such that

$$663 = (6 \times 100) + (6 \times 10) + (3 \times 1)$$

$$= (6 \times 10^2) + (6 \times 10^1) + (3 \times 1)$$

Our Hindu-Arabic Numeration System

Hindu Arabic numeration system is called a *positional-value*, or *place-value*, system. The positional values in the system are based on the powers of ten:

$$\ldots, 10^5, 10^4, 10^3, 10^2, 10^1, 1$$

Example: Writing Hindu-Arabic Numerals in Expanded Form

Write 3407 in expanded form.

Solution:

$$3407 = (3 \times 10^3) + (4 \times 10^2) + (0 \times 10^1) + (7 \times 1)$$

or

$$= (3 \times 1000) + (4 \times 100) + (0 \times 10) + (7 \times 1)$$

Example: Expressing a Number's Expanded Form as a Hindu-Arabic Numeral

Express the expanded form as a Hindu-Arabic numeral: $(7 \times 10^3) + (5 \times 10^1) + (4 \times 1)$.

Solution: We start by showing all powers of 10, starting with the highest exponent given. Any power left out is expressed as 0 times that power of ten.

$$(7 \times 10^3) + (5 \times 10^1) + (4 \times 1)$$

$$= (7 \times 10^3) + (0 \times 10^2) + (5 \times 10^1) + (4 \times 1)$$

$$= 7054$$

The Babylonian Numeration System

Babylonian	∨	<
Hindu-Arabic	1	10

The place values in the Babylonian system use powers of 60. The place values are

$$\ldots, \quad 60^3, \quad 60^2, \quad 60^1, \quad 1.$$

$60^3 = 60 \times 60 \times 60 = 216{,}000$

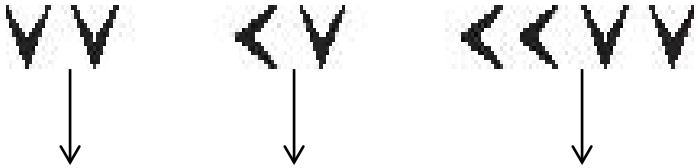
$60^2 = 60 \times 60 = 3600$

The Babylonians left a space to distinguish the various place values in a numeral from one another.

Example: Converting from a Babylonian Numeral to a Hindu-Arabic Numeral

Write  as a Hindu-Arabic numeral.

Solution: From left to right the place values are 60^2, 60^1, and 1.



$$(1+1)\times 60^2 + (10+1)\times 60^1 + (10+10+1+1)\times 1$$

Represent the numeral in each place as a familiar Hindu-Arabic numeral.

$$= \left(2\times 60^2\right) + \left(11\times 60^1\right) + \left(22\times 1\right)$$

Multiply each Hindu-Arabic numeral by its respective place value.

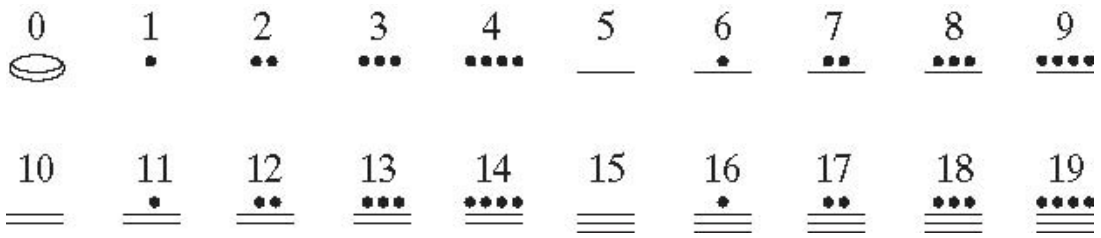
$$= \left(2\times 3600\right) + \left(11\times 60\right) + \left(22\times 1\right)$$

$$= 7200 + 660 + 22$$

Find the sum of these products

$$= 7882$$

The Mayan Numeration System



The place values in the Mayan system are

$$\ldots, \quad 18 \times 20^3, \quad 18 \times 20^2, \quad 18 \times 20, \quad 20, \quad 1$$

$18 \times 20 \times 20 \times 20 = 144{,}000$ $18 \times 20 \times 20 = 7200$ $18 \times 20 = 360$

Numerals in the Mayan system are expressed vertically. The place value at the bottom of the column is 1.

Example: Using the Mayan Numeration System

Write  as a Hindu-Arabic numeral.

Solution: The given Mayan numeral has four places. From top to bottom, the place values are 7200, 360, 20, and 1. Represent the numeral in each row as a familiar Hindu-Arabic numeral.

Mayan numeral		Hindu-Arabic numeral		Place value		
	=	14	×	7200	=	100,800
	=	0	×	360	=	0
	=	7	×	20	=	140
	=	12	×	1	=	12
						100,952

Multiply each Hindu-Arabic numeral by its respective place value.

Find the sum of these products.