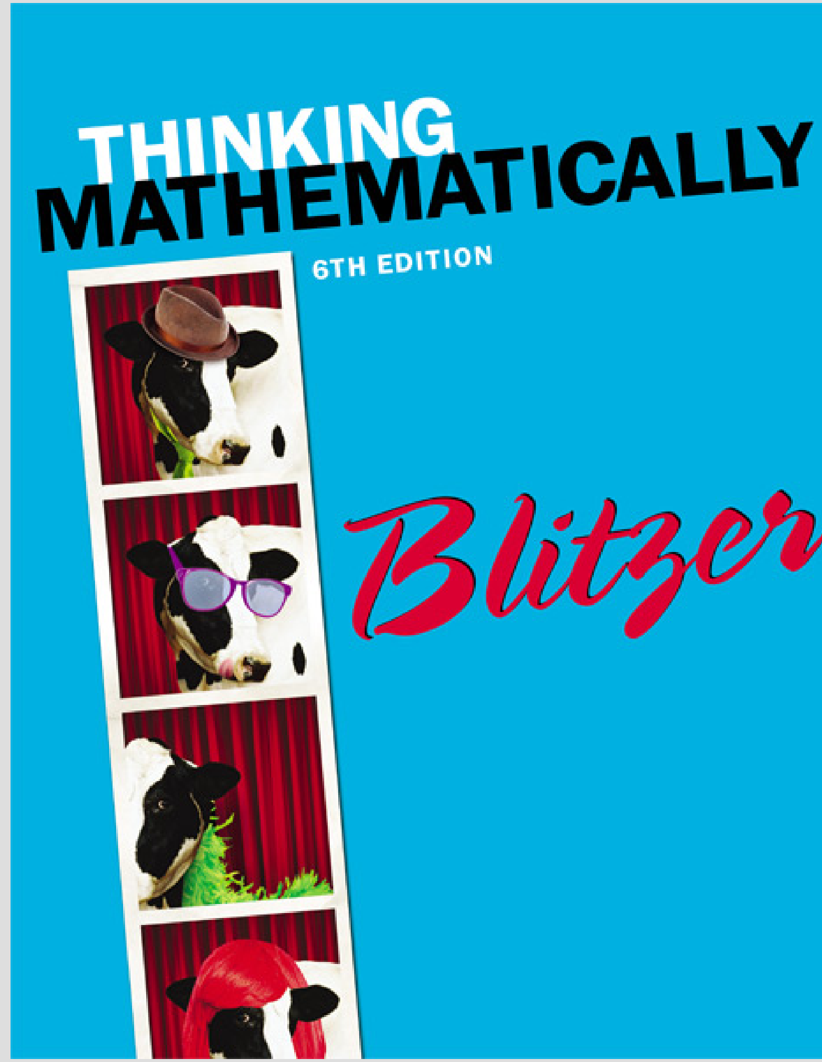
# CHAPTER 8

## Personal Finance



# 8.4

## Compound Interest

# Objectives

1. Use compound interest formulas.
2. Calculate present value.
3. Understand and compute effective annual yield.

# Compound Interest

***Compound interest*** is interest computed on the original principal as well as on any accumulated interest.

To calculate the compound interest paid once a year we use

$$A = P(1 + r)^t,$$

where $A$ is called the account's **future value**, the principal $P$ is called its **present value**, $r$ is the rate, and $t$ is the number of years.

# Example: Compound Interest

You deposit \$2000 in a savings account at Hometown Bank, which has a rate of 6%.

a. Find the amount, $A$, of money in the account after 3 years subject to compound interest.

b. Find the interest.

**Solution:**

a. Principal $P$ is \$2000, $r$ is 6% or 0.06, and $t$ is 3. Substituting this into the compound interest formula, we get

$$A = P(1 + r)^t = 2000(1 + 0.06)^3 = 2000(1.06)^3 \approx 2382.03$$

# Example: Compound Interest continued

Rounded to the nearest cent, the amount in the savings account after 3 years is \$2382.03.

b. The amount in the account after 3 years is \$2382.03. So, we take the difference of this amount and the principal to obtain the interest amount.

$$\$2382.03 - \$2000 = \$382.03$$

Thus, the interest you make after 3 years is \$382.03.

# Compound Interest

To calculate the compound interest paid more than once a year we use

$$A = P\left(1+\frac{r}{n}\right)^{nt},$$

where $A$ is called the account's **future value**, the principal $P$ is called its **present value**, $r$ is the rate, $n$ is the number of times the interest is compounded per year, and $t$ is the number of years.

# Example: Using the Compound Interest Formula

You deposit \$7500 in a savings account that has a rate of 6%. The interest is compounded monthly.

a. How much money will you have after five years?

b. Find the interest after five years.

**Solution:**

a. Principal $P$ is \$7500, $r$ is 6% or 0.06, $t$ is 5, and $n$ is 12 since interest is being compounded monthly. Substituting this into the compound interest formula, we get

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 7500\left(1+\frac{0.06}{12}\right)^{12\cdot 5} = 7500(1.005)^{60} \approx 10{,}116.38$$

# Example: Using the Compound Interest Formula

Rounded to the nearest cent, you will have $10,116.38 after five years.

b. The amount in the account after 5 years is $10,116.38. So, we take the difference of this amount and the principal to obtain the interest amount.

$$\$\,10{,}116.38 - \$7500 = \$2616.38$$

Thus, the interest you make after 5 years is $ 2616.38.

# Compound Interest
# Continuous Compounding

Some banks use ***continuous compounding***, where the compounding periods increase indefinitely.

After $t$ years, the balance, $A$, in an account with principal $P$ and annual interest rate $r$ (in decimal form) is given by the following formulas:

1. For $n$ compounding periods per year: $A = P\left(1+\frac{r}{n}\right)^{nt}$.

2. For continuous compounding: $A = Pe^{rt}$.

# Example: Choosing Between Investments

You decide to invest $8000 for 6 years and you have a choice between two accounts. The first pays 7% per year, compounded monthly. The second pays 6.85% per year, compounded continuously. Which is the better investment?

**Solution:** The better investment is the one with the greater balance at the end of 6 years.

7% account:

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 8000\left(1+\frac{0.07}{12}\right)^{12\cdot 6} \approx 12{,}160.84$$

The balance in this account after 6 years is $12,160.84.

# Example: Choosing Between Investments continued

6.85% account:

$$A = Pe^{rt} = 8000e^{0.0685(6)} \approx 12{,}066.60$$

The balance in this account after 6 years is $12,066.60.

The better investment is the 7% monthly compounding option.

# Planning for the Future with Compound Interest

Calculating Present Value

If $A$ dollars are to be accumulated in $t$ years in an account that pays rate $r$ compounded $n$ times per year, then the present value $P$ that needs to be invested now is given by

$$P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}.$$

# Example: Calculating Present Value

How much money should be deposited in an account today that earns 6% compounded monthly so that it will accumulate to $20,000 in five years?

**Solution:** We use the present value formula, where $A$ is $20,000, $r$ is 6% or 0.06, $n$ is 12, and $t$ is 5 years.

$$P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}} = \frac{20,000}{\left(1+\frac{0.06}{12}\right)^{12(5)}} \approx 14,827.4439$$

Approximately $14,827.45 should be invested today in order to accumulate to $20,000 in five years.

# Effective Annual Yield

The ***effective annual yield***, or the ***effective rate***, is the *simple interest rate* that produces the same amount of money in an account at the end of one year as when the account is subjected to compound interest at a stated rate.

# Example: Understanding Effective Annual Yield

You deposit $4000 in an account that pays 8% interest compounded monthly.

a. Find the future value after one year.

b. Use the future value formula for simple interest to determine the effective annual yield.

**Solution:**

a. We use the compound interest formula to find the account's future value after one year.

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = 4000\left(1 + \frac{0.08}{12}\right)^{12 \cdot 1} \approx \$4332.00$$

Principal is $4000.

Stated rate is 8% = 0.08.

Monthly compounding: $n = 12$
Time is one year: $t = 1$

# Example: Understanding Effective Annual Yield continued

b. The effective annual yield is the simple interest rate. So, we use the future value formula for simple interest to determine rate *r*.

Thus, the effective annual yield is 8.3%. This means that an account that earns 8% interest compounded monthly has an equivalent simple interest rate of 8.3%.

$$A = P(1 + rt)$$

$$4332 = 4000(1 + r \cdot 1)$$

$$4332 = 4000 + 4000r$$

$$332 = 4000r$$

$$r = \frac{332}{4000} = 0.083 = 8.3\%$$