CHAPTER 8

Personal Finance
8.4

Compound Interest
Objectives

1. Use compound interest formulas.
2. Calculate present value.
3. Understand and compute effective annual yield.
Compound Interest

*Compound interest* is interest computed on the original principal as well as on any accumulated interest.

To calculate the compound interest paid once a year we use

\[ A = P(1 + r)^t, \]

where \( A \) is called the account’s *future value*, the principal \( P \) is called its *present value*, \( r \) is the rate, and \( t \) is the number of years.
Example: Compound Interest

You deposit $2000 in a savings account at Hometown Bank, which has a rate of 6%.

a. Find the amount, $A$, of money in the account after 3 years subject to compound interest.

b. Find the interest.

Solution:

a. Principal $P$ is $2000$, $r$ is 6% or 0.06, and $t$ is 3. Substituting this into the compound interest formula, we get

$$A = P(1 + r)^t = 2000(1 + 0.06)^3 = 2000(1.06)^3 \approx 2382.03$$
Example: Compound Interest continued

Rounded to the nearest cent, the amount in the savings account after 3 years is $2382.03.

b. The amount in the account after 3 years is $2382.03. So, we take the difference of this amount and the principal to obtain the interest amount.

\[
2382.03 - 2000 = 382.03
\]

Thus, the interest you make after 3 years is $382.03.
Compound Interest

To calculate the compound interest paid more than once a year we use

\[ A = P \left(1 + \frac{r}{n}\right)^{nt}, \]

where \( A \) is called the account’s **future value**, the principal \( P \) is called its **present value**, \( r \) is the rate, \( n \) is the number of times the interest is compounded per year, and \( t \) is the number of years.
Example: Using the Compound Interest Formula

You deposit $7500 in a savings account that has a rate of 6%. The interest is compounded monthly.

a. How much money will you have after five years?

b. Find the interest after five years.

Solution:

a. Principal $P$ is $7500$, $r$ is 6% or 0.06, $t$ is 5, and $n$ is 12 since interest is being compounded monthly. Substituting this into the compound interest formula, we get

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} = 7500 \left(1 + \frac{0.06}{12}\right)^{12 \cdot 5} = 7500(1.005)^{60} \approx 10,116.38 \]
Example: Using the Compound Interest Formula

Rounded to the nearest cent, you will have $10,116.38 after five years.

b. The amount in the account after 5 years is $10,116.38. So, we take the difference of this amount and the principal to obtain the interest amount.

\[ 10,116.38 - 7500 = 2616.38 \]

Thus, the interest you make after 5 years is $2616.38.
Compound Interest
Continuous Compounding

Some banks use *continuous compounding*, where the compounding periods increase indefinitely.

After \( t \) years, the balance, \( A \), in an account with principal \( P \) and annual interest rate \( r \) (in decimal form) is given by the following formulas:

1. For \( n \) compounding periods per year: \( A = P \left( 1 + \frac{r}{n} \right)^{nt} \).

2. For continuous compounding: \( A = Pe^{rt} \).
Example: Choosing Between Investments

You decide to invest $8000 for 6 years and you have a choice between two accounts. The first pays 7% per year, compounded monthly. The second pays 6.85% per year, compounded continuously. Which is the better investment?

Solution: The better investment is the one with the greater balance at the end of 6 years.

7% account:

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} = 8000 \left(1 + \frac{0.07}{12}\right)^{12 \times 6} \approx 12,160.84 \]

The balance in this account after 6 years is $12,160.84.
Example: Choosing Between Investments continued

6.85% account:

\[ A = Pe^{rt} = 8000e^{0.0685(6)} \approx 12,066.60 \]

The balance in this account after 6 years is $12,066.60.

The better investment is the 7% monthly compounding option.
Calculating Present Value

If $A$ dollars are to be accumulated in $t$ years in an account that pays rate $r$ compounded $n$ times per year, then the present value $P$ that needs to be invested now is given by

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}.$$
Example: Calculating Present Value

How much money should be deposited in an account today that earns 6% compounded monthly so that it will accumulate to $20,000 in five years?

Solution: We use the present value formula, where $A$ is $20,000$, $r$ is 6% or 0.06, $n$ is 12, and $t$ is 5 years.

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}} = \frac{20,000}{\left(1 + \frac{0.06}{12}\right)^{12(5)}} \approx 14,827.4439$$

Approximately $14,827.45 should be invested today in order to accumulate to $20,000 in five years.
Effective Annual Yield

The effective annual yield, or the effective rate, is the simple interest rate that produces the same amount of money in an account at the end of one year as when the account is subjected to compound interest at a stated rate.
Example: Understanding Effective Annual Yield

You deposit $4000 in an account that pays 8% interest compounded monthly.

a. Find the future value after one year.
b. Use the future value formula for simple interest to determine the effective annual yield.

Solution:

a. We use the compound interest formula to find the account’s future value after one year.

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} = 4000 \left(1 + \frac{0.08}{12}\right)^{12 \cdot 1} \approx \$4332.00 \]
Example: Understanding Effective Annual Yield continued

b. The effective annual yield is the simple interest rate. So, we use the future value formula for simple interest to determine rate $r$.

Thus, the effective annual yield is 8.3%. This means that an account that earns 8% interest compounded monthly has an equivalent simple interest rate of 8.3%.

$$A = P(1 + rt)$$
$$4332 = 4000(1 + r \cdot 1)$$
$$4332 = 4000 + 4000r$$
$$332 = 4000r$$
$$r = \frac{332}{4000} = 0.083 = 8.3\%$$