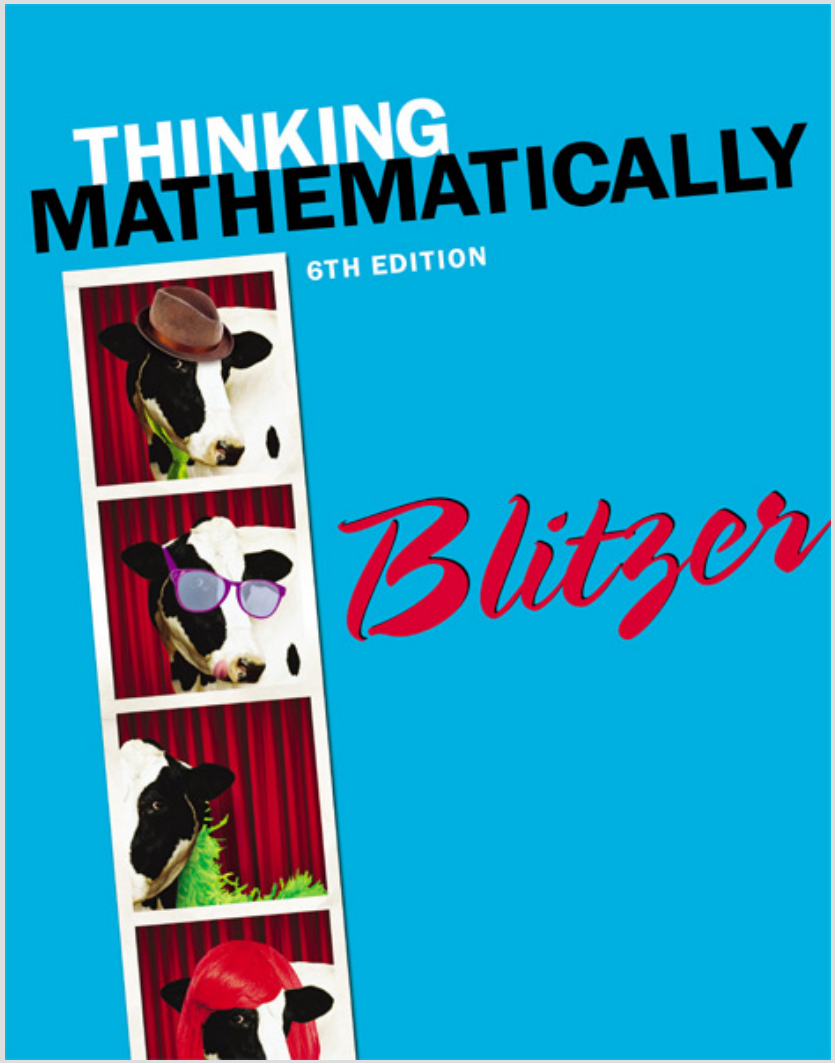
CHAPTER 9

Measurement



9.2

Measuring Area and Volume

Objectives

1. Use square units to measure area.

2. Use dimensional analysis to change units for area.

3. Use cubic units to measure volume.

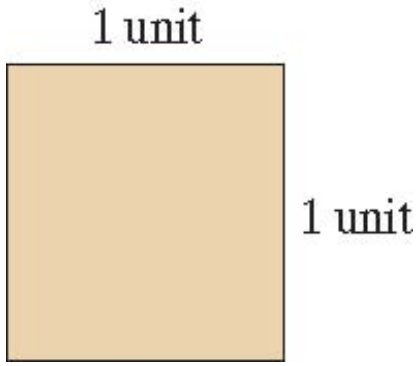
4. Use English and metric units to measure capacity.

Measuring Area

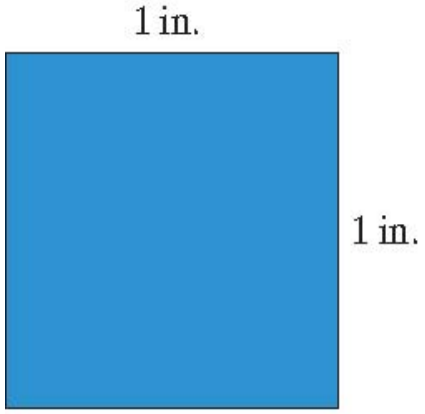
Square unit

A square, each of whose sides is one unit in length.

One square unit

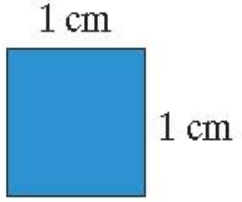


1 square inch,
1 in.^{2}



1 square inch,
symbolized 1 in.^{2}

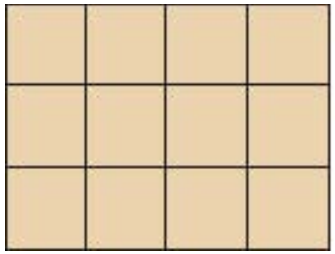
1 square centimeter,
1 cm^2



1 square centimeter,
symbolized 1 cm^2

Example: Measuring Area

What is the area of this region?



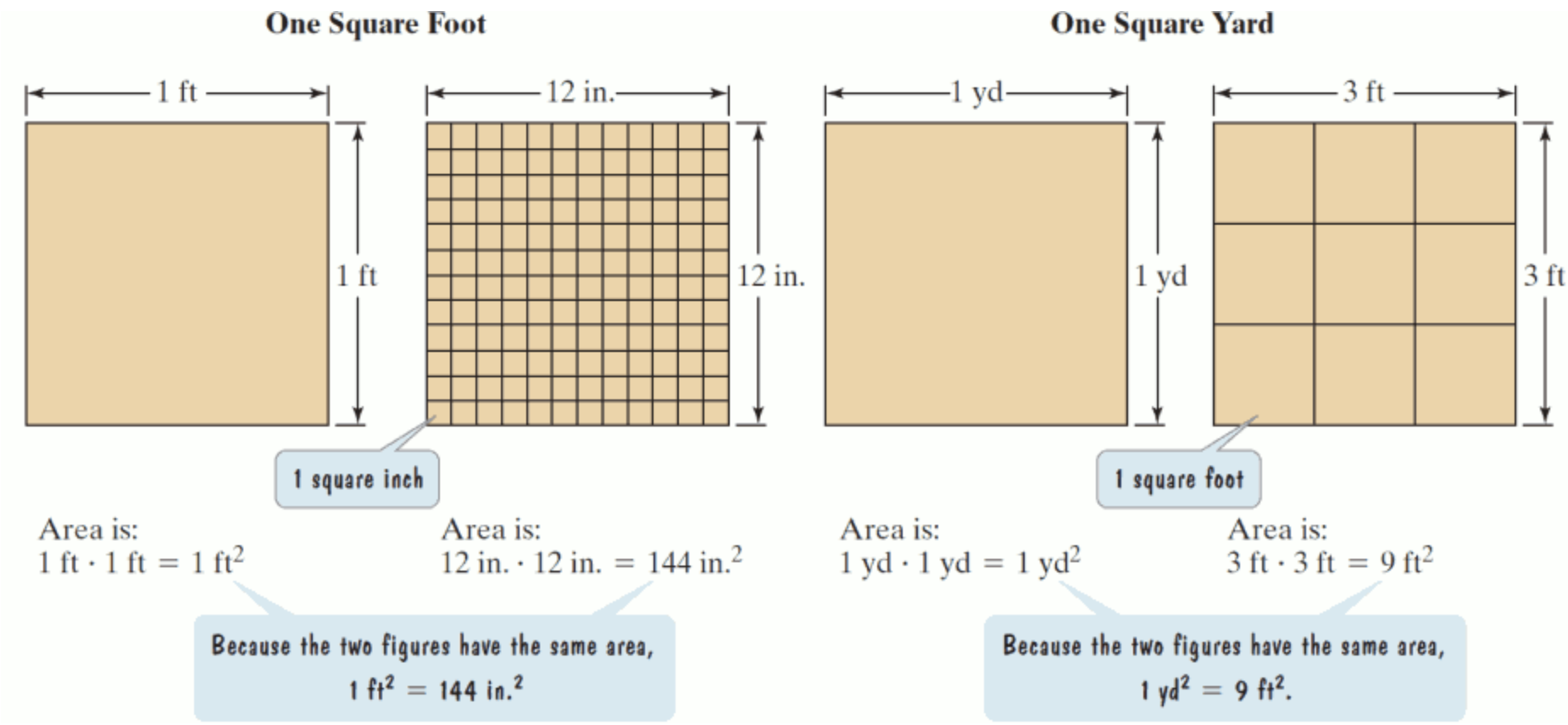
Square Unit

Solution:

We count 12 square units.

Therefore, the area is 12 square units.

Square Units of Measure: The English System



Square Units of Measure: The English System

1 square foot (ft^2) = 144 square inches ($in.^2$)

1 square yard (yd^2) = 9 square feet (ft^2)

1 acre (a) = 43,560 ft^2 or 4840 yd^2

1 square mile (mi^2) = 640 acres

Example: Using Square Units to Compute Population Density

After Alaska, Wyoming is the least densely populated state. The Population of Wyoming is 568,158 and its area is 97,814 square miles. What is Wyoming's population density?

Solution:

We compute the population density by dividing Wyoming's population by its area.

$$\text{Population density} = \frac{\text{population}}{\text{area}} = \frac{568{,}158 \text{ people}}{97{,}814 \text{ square miles}}$$

Using a calculator and rounding to the nearest tenth, we obtain a population density of 5.8 people per square mile of area.

English and Metric Equivalents for Area

1 square inch(in^2)	= 6.5 square centimeters (cm^2)
1 square foot (ft^2)	= 0.09 square meter (m^2)
1 square yard (yd^2)	= 0.8 square meter (m^2)
1 square mile (mi^2)	= 2.6 square kilometers (km^2)
1 acre	= 0.4 hectare (ha)

Example: Using Dimensional Analysis on Units of Area

A property in Italy is advertised at $545,000 for 6.8 hectares.

Find the area of the property in acres.

Solution:

To convert 6.8 hectares to area, we use a unit fraction with acres in the numerator and hectares in the denominator.

$$6.8 \text{ ha} \approx \frac{6.8 \text{ ha}}{1} \cdot \frac{1 \text{ acre}}{0.4 \text{ ha}} = \frac{6.8}{0.4} \text{ acres} = 17 \text{ acres}$$

Example Continued

What is the price per acre?

Solution:

The price per acres is the total price $545,000 divided by the number of acres, 17.

$$\text{price per acre} = \frac{\$545{,}000}{17 \text{ acres}} \approx \$32{,}059/\text{acre}$$

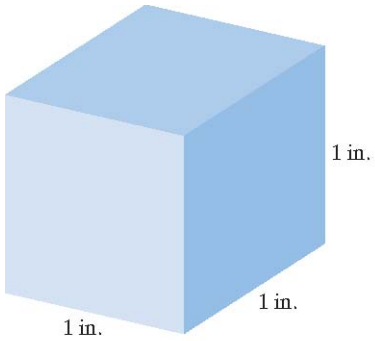
Measuring Volume

Volume refers to the amount of space occupied by a three-dimensional figure.

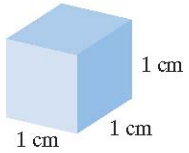
In order to measure this space, we begin by selecting a cubic unit.

1 cubic foot (ft^3) $= 12^3 = 1728$ cubic inches ($in.^3$)

1 cubic yard (yd^2) = 27 cubic feet (ft^3)

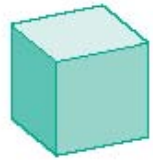


1 cubic inch, symbolized 1 $in.^3$

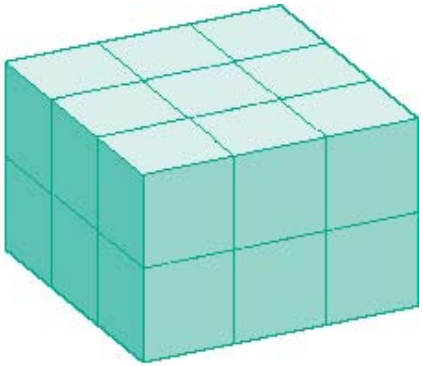


1 cubic centimeter, symbolized 1 cm^3

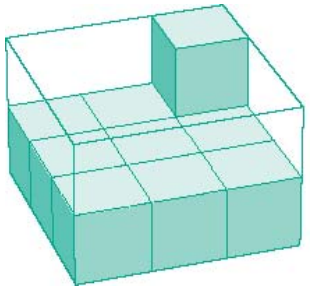
Example: Measuring Volume

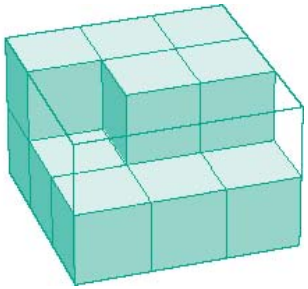


Cubic unit



Volume = ?





What is the volume of this solid?

Solution:

We determine the volume by counting the number of cubic units contained within the unit.

Here we see how the cubic units fit within the region.

There are 18 cubic units.

Capacity

Capacity : The amount of fluid that a three-dimensional object can hold.

English Units for Capacity

1 cup	= 8 ounces
4 quarts	= 1 gallon (gal)
1 gallon	= 128 ounces (oz)

Volume in Cubic Units	**Capacity**
1 cubic yard	about 200 gallons
1 cubic foot	about 7.48 gallons
231 cubic inches	about 1 gallon

Example: Volume and Capacity in the English System

A swimming pool has a volume of 22,500 cubic feet. How many gallons of water does the pool hold?

Solution:

We use the unit fraction: $\dfrac{7.48 \text{ gal}}{1 \text{ ft}^3}$.

$$22{,}500 \text{ ft}^3 = \frac{22{,}500 \cancel{\text{ft}^3}}{1} \cdot \frac{7.48 \text{ gal}}{1 \cancel{\text{ft}^3}} \approx 22{,}500 \,(\, 7.48 \,) \text{ gal} = 168{,}300 \text{ gal}$$

The pool holds approximately 168,300 gallons of water.

Units of Capacity in the Metric System

Symbol	Unit	Meaning
kL	kiloliter	1000 liters
hL	hectoliter	100 liters
daL	dekaliter	10 liters
L	liter	1 liter ≈ 1.06 quarts
dL	deciliter	0.1 liter
cL	centiliter	0.01 liters
mL	milliliter	0.001 liter

Volume and Capacity in the Metric System

Volume in Cubic Units		Capacity
$1\ cm^3$	=	1 mL
$1\ dm^3 = 1000\ cm^3$	=	1 L
$1\ m^3$	=	1 kL

Example: Volume and Capacity in the Metric System

An aquarium has a volume of 36,000 cubic centimeters. How many liters of water does the aquarium hold?

Solution:

We use the unit fraction: $\dfrac{1\text{L}}{1000\ \text{cm}^3}$

$$36{,}000\ \text{cm}^3 = \frac{36{,}000\ \cancel{\text{cm}^3}}{1} \cdot \frac{1\text{L}}{1000\ \cancel{\text{cm}^3}} = \frac{36{,}000}{1000}\ \text{L} = 36\ \text{L}$$